

# Bio-inspired Algorithms for TSP and Generalized TSP

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## 1. Introduction

The Traveling Salesman Problem (TSP) is to find a Hamiltonian tour of minimal length on a fully connected graph. The TSP is a NP-Complete, and there is no polynomial algorithm to find the optimal result. Many bio-inspired algorithms has been proposed to address this problem. Generally, generic algorithm (GA), ant colony optimization (ACO) and particle swarm optimization (PSO) are three typical bio-inspired algorithm for TSP. In this section we will give a brief introduction to the above three bio-inspired algorithms and their application to the TSP.

### 1.1 GAs for TSP

GAs were introduced by Holland in the 1970s [1]. These algorithms are adaptive search techniques based on the mechanisms of natural selection and the survival of the fittest concept of biological evolution. By simulating biological evolution, GAs can solve searching problem domains effectively and easily apply to many of the current engineering problems. GAs have been widely used in many applications of TSP and its extensions throughout the literature [2-4].

A particularly nice introduction to GAs is given in Goldberg's book [5]. The main idea behind GAs is to start with randomly generating initial solutions and implements the "survival of the fittest" strategy to increasing better solutions through generations. A traditional GA process includes initial population generation, fitness evaluation, chromosome selection, applying genetic operators for reproduction, and suspension condition.

In designing a GA, how to encode a search solution is a primary and key issue [6]. Many optimization operators for TSP were proposed by Goldberg [5]. A commonly used encoding strategy is transposition expression [7]. In the transposition expression strategy, each city of the TSP is encoded as a gene of the chromosome with the constraint that each city appears once and only once in the chromosome. Transposition expression is the most nature expression for TSP which based on the order of tour. While such method may leads to infeasible tour after traditional crossover operator [7]. This is a common occurrence for TSP. Although feasibility can be maintained in many ways named 'repair algorithms', such algorithms can consume a considerable amount of time and can inhibit convergence.

Source: Travelling Salesman Problem, Book edited by: Federico Greco, ISBN 978-953-7619-10-7, pp. 202, September 2008, I-Tech, Vienna, Austria

3	2	1	5	4	6	8	7
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Figure 1. Transposition expression encoding method for TSP

Another typical encoding method is Random Keys encoding [8] which is introduced by Bean. In Random Keys encoding a random numbers encode the structure of the solution. Such representation ensures that feasible tours are maintained during the application of genetic operators.

In the GA, the crossover and mutation are two of most important method for the success of the algorithm. A crossover operator generates new individuals through recombining the current population hopefully to retain good features from the parents. Numbers of different crossovers have been proposed in the literatures to solve the TSP using a GA. The partially mapped crossover [5,11-12], linear order crossover [13] and order based crossover [5,8,11,12,14] are the commonly used crossover strategy in the TSP context. Expect the commonly used crossover strategy, many different crossover strategy are proposed for the TSP problem, for example: sub-tour crossover[15,16], edge recombination [17-20], distance preserving crossover [21-22], generic crossover [23], NGA [24], EAX [25-26], GSX [27-28], heuristic based crossover [29-35].

A mutation operator is used to enhance the diversity and provide a chance to escape from local optima. Many mutation operators were proposed such as inverse, insert, displace, swap, hybrid mutation [34], and heuristic mutation. The former five are realized by small alterations of genes. Heuristic mutation was proposed by Cheng and Gen [37-38], which adopts a neighborhood strategy to improve the solution.

At present, the genetic algorithm to solve the TSP has been to promote large-scale TSP, as well as a multiple TSP (MTSP) and generalized TSP. A lot of progress was made recently. Arthur E. Carter and Cliff T. Ragsdale propose a new GA chromosome and related operators for the MTSP [39]. H. D. Nguyen, et al described a hybrid GA based on a parallel implementation of a multi population steady-state GA involving local search heuristics [40]. Samanlioglu et.al proposes a methodology uses a "target-vector approach" in which the evaluation function is a weighted Tchebycheff metric with an ideal point for a symmetric multi-objective traveling salesman problem [41-43].

## 1.2 Ant colony optimization (ACO) for TSP

Ant Colony Optimization (ACO), first proposed by M. Dorigo et al. [44-46], is a population-based, general-purpose heuristic approach to combinatorial optimization problems. The earliest ACO algorithm [44-45], Ant System (AS), was applied to the TSP (mainly because the TSP is "a shortest path problem to which the ant colony metaphor is easily adapted and that it is a didactic problem" [4]. After that, most improved ACO algorithms also used the TSP as a test problem and the result is promising.

As the name suggests, ACO took inspiration from the foraging behavior of real ant colonies. Ants deposit pheromone on the ground they cover while working, forming a pheromone trail. Other ants tend to follow the pheromone trail. Consider an ant colony exploring the paths between their nest to a food source. At the beginning, the ants choose paths randomly in equal rate since there's no pheromone on the paths help them make the decision. Suppose that every ant walk in the same speed, shorter paths accumulate pheromone faster than longer paths because ants on those paths return earlier. A moment later, the difference in the

amount of pheromone on the paths becomes sufficient large so that the ants' decision are influenced and more ants select the shorter paths. Experiments show that this behavior can lead the ant colony to the shortest path.

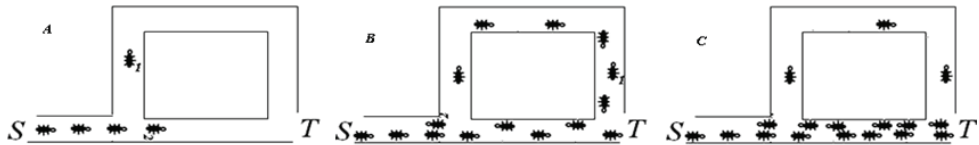


Figure 2. an ant colony exploits the paths between S and T. (A) The two paths are selected with the same probability at first. (B) Ant 2 choosing the lower path returns to S earlier. Thus pheromone on the lower path rises faster. (C) Most ants walk on the lower path after a minute.

Typical ant algorithms stimulate the above foraging behavior of ant colonies using a set of agents (artificial ants) and an indirect communication mechanism employing (artificial) pheromone. A simple framework may look like this:

```

Loop /* at this level each loop is called iteration */
    Each ant is positioned on a starting node.
    Loop /* at this level each loop is called a step */
        Each ant applies a stochastic state transition rule to incrementally build a solution
    Until all ants have built a complete solution
    A pheromone updating rule is applied
Until End condition
    
```

The stochastic state transition rule and the pheromone updating rule are two factor to the success of the ACO. And many strategies have been proposed for these two operators. The Ant Colony System (ACS) [48-50] and MAX-MIN Ant System (MMAS) [51] are among the most successful algorithms [52]. Recent researches focus most on extending the applications of ACO algorithms to more challenge problems. There're also some studies on the convergence theory of ACO algorithms too [47, 53-58].

### 1.3 PSO for TSP

The particle swarm optimization (PSO) was originally presented by Kennedy and Eberhart in 1995 [59]. It is an algorithm based stochastic optimization technique which inspired by social behavior among individuals. In the PSO system, individuals (we call them particles) move around a multidimensional search space. Each particle represents a potential solution of the problem, and can remember the best position (so1ution) it has reached. All the particles can share their information about the search space, so there is a global best solution.

In each iteration, every particle adjusts its velocity  $v_i$  and position  $x_i$  according to the following formulas:

$$\begin{aligned}
 v_i &= v_i + c_1 * r_1 * (x_{i,pbest} - x_i) + c_2 * r_2 * (x_{i,gbest} - x_i) \\
 x_i &= x_i + v_i
 \end{aligned}
 \tag{1}$$

Where  $w$  is inertia weight,  $c_1$  and  $c_2$  are acceleration coefficients,  $r_1$  and  $r_2$  are two independent random values between 0 and 1.  $x_{i,pbest}$  is the best solution this particle has reached;  $x_{i,gbest}$  is the global best solution of all the particles until now.

Due to the continuous characters of the position of particles in PSO, the standard encoding scheme of PSO can not be directly adopted for TSP. Much work was published to avoid such problem. Clerc adopted discrete particle swarm optimization (DPSO)[60] to make PSO suitable for solving TSP. Bo Liu, et al. proposes a PSO-based MA [61](PSOMA) for TSP, which combined evolutionary searching mechanism of PSO and adaptive local search. Yong-Qin Tao, et al. proposed a GRPSAC algorithm [62] combined ACO with PSO organically and adds gene regulation operator at the same time, which make solution of TSP problem more efficiency. Other recently work such as heuristic information method based on improved fuzzy discrete PSO [63] and chaotic PSO algorithm [64] were proved to be effective for TSP.

## 2. Ant colony optimization (ACO) for TSP

### 2.1 The method of ant colony optimization solving TSP

Among the bio-inspired algorithms, the ant colony optimization (ACO) is a popular approach for TSP since it's proposed by M.Dorigo in early nineties [65-66]. Ant colony optimization (ACO) takes inspiration from the foraging behavior of some ant species. These ants deposit pheromone on the ground in order to mark some favorable path that should be followed by other ants of the colony. Ant colony optimization exploits a similar mechanism for solving optimization problems.

In TSP, a set of cities is given and the distance between each of them is known. The goal is to find a Hamiltonian tour of minimal length on a fully connected graph. This goal is very similar with the ants to find the shortest path between the nest and the food source. In ant colony optimization, the problem is tackled by simulating a number of artificial ants moving on a graph that encodes the problem itself. A variable called pheromone is associated with each edge and can be read and modified by ants. The artificial ants explore the pheromone to find the most favorable path which is the shortest Hamiltonian Tour in TSP.

Ant colony optimization is an iterative algorithm. In an iterative step, each ant of the colony builds a solution by walking from vertex to vertex on the graph with the constraint of not visiting any vertex that has been visited before. The solution construction and the pheromone updating are two main steps for the ACO. In the solution construction step, an ant selects the next vertex to be visited according to a stochastic mechanism that is biased by the pheromone. After the solution construction step, the pheromone is updated on the basis of the quality of the solutions.

Under the above framework, many different version of the algorithm are proposed. According to the M.Dorigo's work [46,67], the Ant System (AS), MAX-MIN Ant System (MMAS) and Ant Colony System (ACS) are three of most popular ant algorithms. Following, we will give a short brief of those three algorithms on TSP.

#### 2.1.1 Ant System (AS)

Ant System is the first ACO algorithm proposed in the literature [44,65-66]. Its main characteristic is that, at each iteration, the pheromone values are updated by all the  $m$  ants that have built a solution in the iteration itself. The pheromone  $\tau_{ij}$ , associated with the edge joining cities  $i$  and  $j$ , is updated as follows:

$$\tau_{ij} \leftarrow (1 - \rho) \cdot \tau_{ij} + \sum_{k=1}^m \Delta \tau_{ij}^k \tag{1}$$

where  $\rho$  is the evaporation rate,  $m$  is the number of ants, and  $\Delta \tau_{ij}^k$  is the quantity of pheromone laid on edge  $(i, j)$  by ant  $k$ :

$$\Delta \tau_{ij}^k = \begin{cases} Q / L_k & \text{if ant } k \text{ used edge}(i, j) \text{ in its tour,} \\ 0 & \text{otherwise,} \end{cases} \tag{2}$$

where  $Q$  is a constant, and  $L_k$  is the length of the tour constructed by ant  $k$ .

In the construction of a solution, ants select the following city to be visited through a stochastic mechanism. When ant  $k$  is in city  $i$  and has so far constructed the partial solution  $S^p$ , the probability of going to city  $j$  is given by:

$$P_{gs}^k(t) = \begin{cases} \frac{[\tau_{gs}(t)]^\alpha [\eta_{gs}]^\beta}{\sum_{r \in J_k(g)} [\tau_{gr}(t)]^\alpha [\eta_{gr}]^\beta} & \text{if } s \in J_k(g) \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

where  $J_k(g)$  is the set of cities not visited yet by ant  $k$  when at city  $g$ . The parameters  $\alpha$  and  $\beta$  control the relative importance of the pheromone versus the heuristic information  $\eta_{ij}$ ,

which is given by  $\eta_{ij} = \frac{1}{d_{ij}}$ , where  $d_{ij}$  is the distance between cities  $i$  and  $j$ .

**2.1.2 MAX –MIN Ant System (MMAS)**

The MAX –MIN Ant System [51] is an improvement over the original Ant System. In the MMAS, only the best ant updates the pheromone trails and the value of the pheromone is bound. The pheromone update is implemented as follows:

$$\tau_{ij} = \begin{cases} \tau_{\max} & \text{if } (1 - \rho) \cdot \tau_{ij} + \Delta \tau_{ij}^{best} > \tau_{\max}, \\ \tau_{\min} & \text{if } (1 - \rho) \cdot \tau_{ij} + \Delta \tau_{ij}^{best} < \tau_{\min}, \\ (1 - \rho) \cdot \tau_{ij} + \Delta \tau_{ij}^{best} & \text{otherwise;} \end{cases} \tag{4}$$

where  $\tau_{\max}$  and  $\tau_{\min}$  are respectively the upper and lower bounds imposed on the pheromone and  $\Delta \tau_{ij}^{best}$  is:

$$\Delta \tau_{ij}^{best} = \begin{cases} 1/L_{best} & \text{if } (i, j) \text{ belongs to the best tour} \\ 0 & \text{otherwise;} \end{cases} \tag{5}$$

where  $L_{best}$  is the length of the tour of the best ant.

For the  $\tau_{max}$  and  $\tau_{min}$ , they are typically obtained empirically and tuned on the specific problem considered [68]. And some guidelines have been provided for defining  $\tau_{min}$  and  $\tau_{max}$  on the basis of analytical considerations [51].

### 2.1.3 Ant Colony System (ACS)

The ACS was considered the most efficient algorithm on the TSP problem. The main contribution of ACS [48, 50, 69] is introducing a novel local pheromone update in addition to the global pheromone.

The local pheromone update is performed by all the ants after each construction step. Each ant applies it only to the last edge traversed:

$$\tau_{ij} = (1 - \varphi) \cdot \tau_{ij} + \varphi \cdot \tau_0 \quad (6)$$

where  $\varphi \in (0, 1]$  is the pheromone decay coefficient, and  $\tau_0$  is the initial value of the pheromone.

Using the local update strategy, the pheromone concentration on the traversed edges is decreased. So, the subsequent ants are encouraged to choose other edges and to produce different solutions. This makes it less likely that several ants produce identical solutions during one iteration.

## 2.2 An adaptive strategy for weight parameter

Many strategies for ACO have been studied, but little theoretical work has been done on ACO's parameters  $\alpha$  and  $\beta$ , which control the relative weight of pheromone trail and heuristic value. In this part, we will theoretical show the importance and functioning of  $\alpha$  and  $\beta$ . The theoretical analysis show that a fixed  $\beta$  may not enable ACO to use both heuristic and pheromone information for solution when  $\alpha = 1$ . An adaptive  $\beta$  strategy and a new ACO called adaptive weight ant colony system (AWACS) with the adaptive  $\beta$  and  $\alpha = 1$  is introduced. The numerical experiment results show that the AWACS is more effective and steady than traditional ACS.

### 2.2.1 Theoretical analysis of the weight parameter

Given  $a, b \in J_k(g)$ , if  $P_{ga}^k(t) > P_{gb}^k(t)$ , which means that city  $a$  may be chosen by the ant  $k$  as the next city to city  $g$  with higher probability than city  $b$ , then  $\alpha$  and  $\beta$  satisfies the following formula:  $P_{ga}^k(t) > P_{gb}^k(t) \Leftrightarrow [\tau_{ga}(t)]^\alpha [\eta_{ga}]^\beta > [\tau_{gb}(t)]^\alpha [\eta_{gb}]^\beta$ .

When  $\tau_{ga}(t) = \tau_{gb}(t)$  or  $\eta_{ga} = \eta_{gb}$ , for  $\forall \alpha, \beta > 0$ , the formula above holds, so we have:

$$P_{ga}^k(t) > P_{gb}^k(t) \Leftrightarrow \begin{cases} \eta_{ga} > \eta_{gb} & \tau_{ga} = \tau_{gb} \\ \tau_{ga} > \tau_{gb} & \eta_{ga} = \eta_{gb} \end{cases} \quad (8)$$

However, when  $\tau_{ga}(t) \neq \tau_{gb}(t), \tau_{ga}(t) > 0, \tau_{gb}(t) > 0$  and  $\eta_{ga} \neq \eta_{gb} (\eta_{ga}(t) > 0, \eta_{gb}(t) > 0)$ , one has:  $P_{ga}^k(t) > P_{gb}^k(t) \Leftrightarrow [\tau_{ga}(t)]^\alpha [\eta_{ga}]^\beta > [\tau_{gb}(t)]^\alpha [\eta_{gb}]^\beta \Leftrightarrow \log \tau_{ga}(t) - \log \tau_{gb}(t) > \frac{\beta}{\alpha} (\log \eta_{gb} - \log \eta_{ga})$ .

And we have:

$$\begin{cases} \frac{\beta}{\alpha} < \frac{\log \tau_{ga}(t) - \log \tau_{gb}(t)}{\log \eta_{gb} - \log \eta_{ga}} & \eta_{gb} > \eta_{ga} \\ \frac{\beta}{\alpha} > \frac{\log \tau_{ga}(t) - \log \tau_{gb}(t)}{\log \eta_{gb} - \log \eta_{ga}} & \eta_{gb} < \eta_{ga} \end{cases} \quad (7)$$

Particularly, when  $\alpha=1$ , which exists in ACO algorithms like ACS, a conclusion can be drawn:

$$P_{ga}^k(t) > P_{gb}^k(t) \Leftrightarrow \begin{cases} \beta < \frac{\log \tau_{ga}(t) - \log \tau_{gb}(t)}{\log \eta_{gb} - \log \eta_{ga}} & \eta_{gb} > \eta_{ga} \\ \beta > \frac{\log \tau_{ga}(t) - \log \tau_{gb}(t)}{\log \eta_{gb} - \log \eta_{ga}} & \eta_{gb} < \eta_{ga} \end{cases} \quad (8)$$

For the sake of convenience, some symbols about the pheromone trail are defined as follows:  $\tau_g^{\max}(t) = \max_{r \in J_k(g)} \{\tau_{gr}(t)\}$  is the highest pheromone trail among all the cities

feasible to be selected as next stop to city  $g$ .  $\tau_g^{\min}(t) = \min_{r \in J_k(g)} \{\tau_{gr}(t)\}$  is the lowest one, and

$\tau_g^{ave}(t) = |J_k(g)|^{-1} \sum_{r \in J_k(g)} \tau_{gr}(t)$  is the average pheromone trail, where  $|J_k(g)|$  is the

number of elements in the set  $J_k(g)$ .  $\eta_g^{\max} = \max_{r \in J_k(g)} \{d_{gr}^{-1}\}$  is the highest heuristic value of elements in the set  $J_k(g)$ .  $\eta_g^{\min} = \min_{r \in J_k(g)} \{d_{gr}^{-1}\}$  stands for the lowest heuristic value, and

$\eta_g^{ave} = |J_k(g)|^{-1} \sum_{r \in J_k(g)} \eta_{gr} = |J_k(g)|^{-1} \sum_{r \in J_k(g)} d_{gr}^{-1}$  is the average heuristic value.

Let  $\alpha = 1$ , two cases are discussed in the following:

①  $[\tau_g^{\max}(t)]^\alpha [\eta_g^{\min}]^\beta > [\tau_g^{ave}(t)]^\alpha [\eta_g^{\max}]^\beta$ . It means that the ants will select the paths with the maximum pheromone trail with a very high probability ACS. According to Formula (3),

one has  $\beta < \frac{\log \tau_g^{\max}(t) - \log \tau_g^{ave}(t)}{\log \eta_g^{\max} - \log \eta_g^{\min}} = M_1(g, t)$ , because it is obvious that  $\eta_g^{\max} > \eta_g^{\min}$

holds in TSPLIB problems.

②  $[\tau_g^{\min}(t)]^\alpha [\eta_g^{\max}]^\beta > [\tau_g^{\max}(t)]^\alpha [\eta_g^{ave}]^\beta$ . It means that the ants will select the paths with the maximum heuristic value with a very high probability in ACS. It is

obvious that  $\beta > \frac{\log \tau_g^{\min}(t) - \log \tau_g^{\max}(t)}{\log \eta_g^{\text{ave}} - \log \eta_g^{\max}} = M_2(g, t)$  holds, when  $\eta_g^{\max} > \eta_g^{\text{ave}}$  and  $\tau_g^{\min}(t) > 0$ .

According to the analysis of case ① and ②, ACO may work as a non-heuristic searching when  $\beta < M_1(g, t)$ , and as a greedy searching without using pheromone trail when  $\beta > M_2(g, t)$ . Therefore, a fixed  $\beta$  may not enable ACO to find optimal solution by using both heuristic and pheromone information. However, the process of ACO will not be in the extreme as non-heuristic or greedy searching when  $M_1(g, t) \leq \beta \leq M_2(g, t)$ . So a new adaptive parameter  $\beta$  is designed as follows:

$$\alpha = 1, \beta(g, t) = \frac{\log \tau_g^{\text{ave}}(t) - \log \tau_g^{\max}(t)}{\log \eta_g^{\text{ave}} - \log \eta_g^{\max}} (\tau_g^{\min}(t) > 0) \quad (9)$$

where  $M_1(g, t) \leq \beta(g, t) \leq M_2(g, t)$  can be proved.

Based on the adaptive parameter  $\beta(g, t)$  strategy shown in Formula (4), a novel ACO algorithm, which is called adaptive weight ant colony system (AWACS) can be described as follows.

Initialize /\*  $\beta$  is chosen in  $[0, 5]$  randomly,  $q_0=0.6$  \*/  
**Loop** /\* at this level each loop is called iteration \*/  
 Each ant is positioned on a starting node.  
**Loop** /\* at this level each loop is called a step \*/  
 Each ant applies a stochastic state transition rule to incrementally build a solution and a local pheromone updating rule  
**Until** all ants have built a complete solution  
 A global pheromone updating rule is applied  
 $\beta(g, t)$  is updated ( $g = 1, \dots, n$ ) following Formula (11)  
**Until** End condition

The proof of its convergence ( $g = 1, \dots, n$ ) is the same as the one in Ref. [54]. According to the work of Ref. [54], it still holds that  $\tau_g^{\min}(t) > 0$  and  $\tau_g^{\max}(t) < +\infty$  ( $g = 1, \dots, n$ ) when the adaptive parameter  $\beta(g, t)$  strategy in Formula (4) is applied. Then, AWACS can be proved to find the optimal solution with probability one following the conclusion given by T. Stützle and M. Dorigo [54,69].

### 2.2.2 Numerical results and analyses

A comparison of the performance of ACS and AWACS is given in this section. In our experiments, the parameters are set as follows:  $m = 10$ ,  $\alpha = \rho = 0.1$ ,  $\tau_0 = (nL_m)^{-1}$ .  $q_0$  is set  $q_0 = 0.9$  in ACS, and  $q_0 = 0.6$  in AWACS, respectively. The initial value of  $\beta$  in AWACS is a



random figure changing in the interval [1,5]. The initial feasible solutions of TSP are generated in the way from Ref [49]. What's more, no local search strategy is used in experiment.

The experiments are conducted on two set of TSP problems. In the first set of 10 TSP, the distances between cities are measured by integers and in the left 10 TSP, and the distances are measured by real values. The datasets can be found in TSPLIB: <http://www.iwr.uni-heidelberg.de/~iwr/comopt/soft/TSPLIB95/TSPLIB.html>. The detail of the experiment result is given at table 1, table 2 and table 3.

Instance	Optimal	Best (ACS)	Best (AWACS)	Average (ACS)	Average (AWACS)	T <sub>avg</sub> (s) (ACS)	T <sub>avg</sub> (s) (AWACS)	Best $\beta$ (ACS)
st70	654	657	657	675.9	675.5	16.9	27.4	4
rat99	unknown	1188	1188	1211.7	1199.4	53.2	59.7	3
pr107	unknown	44539	44398	44906.3	44783.9	39.5	55.4	4
pr124	unknown	59205	59067	59819.9	59646.6	59.2	42.3	4
eil101	612	614	613	634.6	631.4	22.4	76.3	5
rd100	7858	7909	7861	8100.4	8066.2	59.5	54.1	3
eil51	415	415	415	423.9	423.7	6.7	7.8	3
lin105	14345	14376	14354	14509.3	14465.6	73.7	50.8	4, 5
kroD100	21249	21486	21265	21893	21628.2	25.8	60	5
kroC100	20703	20703	20703	21165.3	20914.9	29.5	67.7	4

Table 1. Comparison I of the results obtained by ACS and AWACS

Instance	Optimal	Best (ACS)	Best (AWACS)	Average (ACS)	Average (AWACS)	T <sub>avg</sub> (s) (ACS)	T <sub>avg</sub> (s) (AWACS)	Best $\beta$ (ACS)
kroA100	21282	21285.44	21285.44	21345.78	21286.33	51.3	51.8	2, 3
kroE100	22068	22078.66	22068.75	22206.62	22117.16	56.3	64.5	5
berlin52	7542	7544.36	7544.36	7544.36	7544.36	8.7	9.8	5
kroB150	26130	26127.35	26127.71	26332.75	26214.10	177.8	164.8	5
ch150	6528	6530.90	6530.90	6594.94	6559.66	373.6	118.1	2
kroB100	22141	22139.07	22139.07	22335.72	22177.47	55.5	68.6	4
kroA150	26524	26618.33	26524.86	26809.08	26685.73	204.5	242.9	5
u159	42080	42075.67	42075.67	42472.04	42168.54	356.7	80.2	1
pr76	108159	108159.4	108159.4	108610.6	108581.7	50.5	42.8	1
pr136	96772	96870.89	96785.86	97854.16	97236.61	344.3	158.9	14,5

Table 2. Comparison II of the results obtained by ACS and AWACS

Instance \Standard deviation	AWACS	ACS $\beta = 1$	ACS $\beta = 2$	ACS $\beta = 3$	ACS $\beta = 4$	ACS $\beta = 5$
kroA100	8.49	460.04	338.84	183.55	625.81	447.03
kroE100	123.82	327.01	467.75	529.73	330.12	366.49
berlin52	0.00	376.98	376.19	357.76	548.34	0.00
kroB150	221.98	447.98	652.57	821.48	664.54	486.91
ch150	54.50	114.76	153.71	109.36	171.66	54.81
kroB100	132.37	554.43	579.73	1091.25	558.86	233.01
kroA150	384.39	522.81	942.11	974.79	640.34	432.72
u159	623.16	726.99	3531.45	2458.43	1509.09	1661.63
pr76	1158.43	1180.56	5058.92	2088.68	1677.73	1411.15
pr136	1300.78	2386.53	5303.40	4572.69	3304.40	2173.27

Table 3. Comparison of standard deviations of the tour lengths obtained by AWACS and ACS

As shown in the above tables, there might be something like precision and time cost in the result of our experiments different from those in the former research because of the different program tools, systems and computing machines. Another possible reason is that the distances between cities in the first 10 instances are measured by integer numbers. But ACS and AWACS are running in the same setting, so the result remains helpful to compare the performance of these two algorithms.

From Table 1-2, it could be seen that AWACS performs better than ACS with the fixed  $\beta$ . The shortest lengths and the average lengths obtained by AWACS are shorter than those found by ACS in all of the TSP instances. As Table 3 shows, it can be concluded that the standard deviations of the tour lengths obtained by AWACS are smaller than those of ACS with the fixed  $\beta$ . Therefore, we can conclude that AWACS is proved to be more effective and steady than ACS.

ACS has to change the best integer value of parameter  $\beta$  with respect to different instances in the experiments. AWACS can avoid the difficulty about how to choose the experimental value of  $\beta$ , because its adaptive strategy can be considered as a function trying to find the best setting for each path search via meeting the request of Formula 4. Though, the time cost  $t_{avg}$  of AWACS is more than ACS in some case, it is less than the sum of time ACS costs with  $\beta = 1, 2, 3, 4, 5$  in all of the instances. As a result, the adaptive setting can save much time in choosing the experimental  $\beta$ . Item  $t_{avg}$  of AWACS is not less than ACS in all of the instances because it needs to compute the value of  $\beta n$  (number of cities) times in each iteration. However, the adaptive function of AWACS is feasible to use because of its acceptable time cost.

**2.3 Bi-directional searching ant colony system**

In 2.2, an adaptive strategy for the weight parameter is proposed by exploring the function of the parameter in the stochastic mechanism. In this section, we will further explore the stochastic mechanism and a bi-directional searching ant colony system is proposed.

**2.3.1 Bi-directional searching strategy using adaptive weight parameter**

In the proposed ACO algorithms, the state transition rule of the artificial ants is given as follows:

$$P_{gs}^k(t) = \begin{cases} \frac{[\tau_{gs}(t)]^\alpha [\eta_{gs}]^{\beta(g,t)}}{\sum_{r \in J_k(g)} [\tau_{gr}(t)]^\alpha [\eta_{gr}]^{\beta(g,t)}} & \text{if } s \in J_k(g) \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

The only difference between the (10) and (3) is the setting of the parameter  $\beta(g, t)$ . In the Bi-directional case, the parameter is set with one of the following two methods by probability 0.5

$$1). \beta(g,t) = \frac{\log \tau_g^{\max}(t) - \log \tau_g^{\text{ave}}(t)}{\log \eta_g^{\max} - \log \eta_g^{\min}} - \varepsilon_0 \quad 2). \beta(g,t) = \frac{\log \tau_g^{\min}(t) - \log \tau_g^{\max}(t)}{\log \eta_g^{\text{ave}} - \log \eta_g^{\max}} + \varepsilon_0.$$

where  $\tau_g^{\max}(t) = \max_{i \in J_k(g)} \{\tau_{gi}(t)\}$  is the highest pheromone trail among all the cities feasible to be selected as next stop to city  $g$ .  $\tau_g^{\min}(t) = \min_{i \in J_k(g)} \{\tau_{gi}(t)\}$  is the lowest one, and  $\tau_g^{\text{ave}}(t) = |J_k(g)|^{-1} \sum_{i \in J_k(g)} \tau_{gi}(t)$  is the average pheromone trail, where  $|J_k(g)|$  is the number of elements in the set  $J_k(g)$ .  $\eta_g^{\max} = \max_{i \in J_k(g)} \{d_{gi}^{-1}\}$  is the highest heuristic value of elements in the set  $J_k(g)$ .  $\eta_g^{\min} = \min_{i \in J_k(g)} \{d_{gi}^{-1}\}$  stands for the lowest heuristic value,  $\eta_g^{\text{ave}} = |J_k(g)|^{-1} \sum_{i \in J_k(g)} \eta_{gi} = |J_k(g)|^{-1} \sum_{i \in J_k(g)} d_{gi}^{-1}$  is the average heuristic value, and  $\varepsilon_0 > 0$ .

For the first method,

$$\beta(g,t) < \frac{\log \tau_g^{\max}(t) - \log \tau_g^{\text{ave}}(t)}{\log \eta_g^{\max} - \log \eta_g^{\min}} \Leftrightarrow \frac{[\tau_g^{\max}(t)]^\alpha [\eta_g^{\min}]^{\beta(g,t)}}{\sum_{r \in J_k(g)} [\tau_{gr}(t)]^\alpha [\eta_{gr}]^{\beta(g,t)}} > \frac{[\tau_g^{\text{ave}}(t)]^\alpha [\eta_g^{\max}]^{\beta(g,t)}}{\sum_{r \in J_k(g)} [\tau_{gr}(t)]^\alpha [\eta_{gr}]^{\beta(g,t)}}.$$

It means that the ants will select the paths with the maximum pheromone trail by the higher probability than most of the other feasible paths, even if they are paths with the highest heuristic value.

For the second one,

$$\beta(g,t) > \frac{\log \tau_g^{\min}(t) - \log \tau_g^{\max}(t)}{\log \eta_g^{\text{ave}} - \log \eta_g^{\max}} \Leftrightarrow \frac{[\tau_g^{\min}(t)]^\alpha [\eta_g^{\max}]^{\beta(g,t)}}{\sum_{r \in J_k(g)} [\tau_{gr}(t)]^\alpha [\eta_{gr}]^{\beta(g,t)}} > \frac{[\tau_g^{\max}(t)]^\alpha [\eta_g^{\text{ave}}]^{\beta(g,t)}}{\sum_{r \in J_k(g)} [\tau_{gr}(t)]^\alpha [\eta_{gr}]^{\beta(g,t)}}$$

It means that the ants will select the paths with the maximum heuristic value by the higher probability than most of the other feasible paths, even if they are paths with the highest pheromone trail.

Combing the above two methods of the parameter setting, the new ACO algorithm BSACS is designed as:

```

Initialize
/* $\beta$  is chosen between 0 and 5 randomly,  $q_0=0.6$  */
Loop /* at this level each loop is called iteration */
    Each ant is positioned on a starting node
    Loop /* at this level each loop is called a step */
        Each ant applies a state transition rule to incrementally build a solution and
        a local pheromone updating rule is applied
    Until all ants have built a complete solution
    A global pheromone updating rule is applied
 $\beta(g,t)$  is updated by either of the two methods by probability 0.5 ( $g=1,\dots,n$ )
Until End_condition
  
```

### 2.3.2 Numerical results and analyses

In this section, several large TSP instances of TSPIB [70] are tested by BSACS and ACS to show the efficiency of the BSACS. The parameters are set as follows:  $m = 10$ ,  $a = \rho = 0.1$ ,  $\tau_0 = (nL_{mn})^{-1}$ , and  $\alpha=1$ .  $q_0=0.9$  in ACS,  $\varepsilon_0=0.001$  and  $q_0=0.6$  in BSACS, respectively. All the instances are computed by BSACS 10 times, and so does ACS with each  $\beta(\beta=1,2,3,4,5)$ . As shown in Table 4 and Table 5, Item (1) is the length of the best tour obtained by ACS and BSACS. Item (6) is the length of optimal solution published in the TSPLIB: <http://www.iwr.uniheidelberg.de/iwr/comopt/soft/TSPLIB95/TSPLIB.html>. Item (2) is the relative error which can be computed by  $(2) = ((1) - (3)) \times (3)^{-1} \times 100\%$ . Item (1) and (2) show that BSACS can obtain better solution than ACS in all of the instances. Item (4) is the average length of the solutions found by both ACS and BSACS. Item (5) is the best value of  $\beta$  which can make ACS perform the best according to Item (1) or Item (4).

The experiment result shows that BSACS can perform better than ACS in every computation. What's more, ACS has to change the selection of  $\beta$  in different instances and cannot solve different large size TSP problems steadily with a fixed value of  $\beta$ . The reason is that ACS is not able to effectively use the pheromone trail and heuristic value in searching when  $\beta$  of the transition rule is fixed and unchanged in iterations. This disadvantage could be avoided by using BSACS because the new rule of BSACS (Formula 1) functions based on both pheromone trail and heuristic value adaptively. For the computational complexity, the BSACS need more time than ACS, because  $\beta(g,t)(g=1,\dots, n)$  has to be updated at each iteration. However, it doesn't mean that the cost of BSACS is more than ACS in the application, because the cost of ACS for the best parameter selection (Item (5) in Table 2) has not been calculated here. Therefore, BSACS can save the time in choosing the experimental value of the parameter. Generally, the BSACS improves the performance of ACS in solving large size TSP problems.

Instance	Algorithm	(1)Best	(2)%Error	(3)Optimal
bier127	ACS	118773	0.423	118282
	BSACS	118372	0.076	
d198	ACS	15888	0.68	15780
	BSACS	15780	0.00	
ts225	ACS	128829	1.734	126643
	BSACS	126905	0.206	
pcb442	ACS	51286	0.96	50799
	BSACS	51271	0.93	
att532	ACS	28147	1.67	27686
	BSACS	27939	0.91	
u574	ACS	38318	3.829	36905
	BSACS	37662	2.052	
rat783	ACS	9015	2.37	8806
	BSACS	8819	0.14	
fl1577	ACS	22977	3.27	222490.
	BSACS	22611	1.63	

Table 4. Comparison of the best solution obtained by ACS and BSACS

Instance	Algorithm	(4) Average	(5)Best $\beta$ of ACS	(6) $t_{avg}$ (second)
bier127	ACS	119185.3	2, 3	45.6
	BSACS	118826.8	-	91.0
d198	ACS	16054.8	2	97.8
	BSACS	15842.1	-	124.5
ts225	ACS	129102.5	4, 5	32.0
	BSACS	127262.8	-	67.0
pcb442	ACS	51690.2	2	281.6
	BSACS	51642.8	-	461.2
att532	ACS	28532.0	2	401.5
	BSACS	28163.7	-	539.7
u574	ACS	38657.8	1, 5	305.3
	BSACS	38291.9	-	504.3
rat783	ACS	9066.0	2	1185.4
	BSACS	8985.8	-	1559.8
fl1577	ACS	23163.5	2	3884.0
	BSACS	22680.3	-	6290.2

Table 5. Comparison of the average solution obtained by ACS and BSAC

**2.4 An adaptive volatility rate of pheromone trail**

The following presents a trial work of setting the parameters of ACO adaptively. First, a tuning rule for  $\rho$  is designed based on the quality of the solution constructed by artificial ants. Then, we introduce the adaptive  $\rho$  to form a new ACO algorithm, which is tested to compute several benchmark instances of traveling sales-man problem and film-copy deliverer problem.

### 2.4.1 An adaptive volatility rate setting strategy

After constructing its tour, an artificial ant also modifies the amount of pheromone on the visited edges by applying the pheromone updating rule. The rule is designed so that it tends to give more pheromone to the edges which should be visited by ants. The classical pheromone updating rule is as (1). And the optimal  $\rho$  was set  $\rho = 0.1$  experimentally [46, 49, 55], which means that 90 per cent of the original pheromone trail remains and its 10 per cent is replaced by the increment.

In order to update the pheromone according to the quality of solutions found by ants, an adaptive rule for volatility of the pheromone trail is designed as follows:

$$\rho_m = L_m^{-1} / (L_m^{-1} + L_p^{-1}) \quad (11)$$

where  $L_m$  is the length of the solution  $S_m$  found by ant  $m$ , and  $L_p$  is the length of the solution  $S_p$  built based on the pheromone matrix, shown as  $s = \arg \max_{u \in J_m(r)} \{\tau(r, u)\}$  where  $S$  is the

city selected as the next one to city  $r$  for any  $(r, s) \in S_p$ .

The motivation of the proposed rule is: better solutions should contribute more pheromone, and the worse ones contribute less. And a new ACO algorithm with the adaptive rule (shown as Equation 3) is introduced as follows:

```

Initialize
/* $\beta$  is chosen between 0 and 5 randomly,  $q_0=0.6$  */
Loop /* at this level each loop is called iteration */
  Each ant is positioned on a starting node
  Loop /* at this level each loop is called a step */
    Each ant applies a state transition rule to incrementally build a solution and
    a local pheromone updating rule is applied
    Each ant the calculate the  $\rho_i$  is based on its solution's length
  Until all ants have built a complete solution
   $\rho_{best}$  is calculated based on the best solution  $S_{best}$ .
  Carry out the pheromone updating rule with  $\rho_i$  ( $i=1, \dots, k$ ) and  $\rho_{best}$ .
Until End_condition
  
```

### 2.4.2 Numerical results

This section indicates the numerical results in the experiment that the proposed ACO algorithm is used to solve TSP problems [69]. Several TSP instances are computed by ACS [49], ACO [71] and the proposed ACO on a PC with an Intel Pentium 550MHz Processor and 256MB SDR Memory, and the results are shown in Table 1.

It should be noted that every instance is computed 20 times. The algorithms are both programmed in Visual C++6.0 for Windows System. They would not stop until a better solution could be found in 500 iterations, which is considered as a virtual convergence of the algorithms. Table 6 shows that the proposed ACO algorithm (PACO) performs better than ACS [49] and ACO [71]. The shortest lengths and the average lengths obtained by PACO are shorter than those found by ACS and ACO in all of the TSP instances. Furthermore, it can be concluded that the standard deviations of the tour lengths obtained by PACO are smaller than those of another algorithms. Therefore, we can conclude that PACO is proved to be

more effective and steady than ACS [49] and ACO [71]. Computation time cost of PACO is not less than ACS and ACO in all of the instances because it needs to compute the value of volatility rate  $k+1$  times per iteration. Although all optimal tours of TSP problems cannot be found by the tested algorithms, all of the errors for PACO are much less than that for another two ACO approaches. The algorithms may make improvement in solving TSP when reinforcing heuristic strategies like local search like ACS-3opt [49] and MMAS+rs [51] are used.

Problem	Algorithm	best	ave	time(s)	standard deviation
kroA100	ACS	21958	22088.8	65	1142.77
	ACO	21863	22082.5	94.6	1265.30
	PACO	21682	22076.2	117.2	549.85
ts225	ACS	130577	133195	430.6	7038.30
	ACO	130568	132984	439.3	7652.80
	PACO	130507	131560	419.4	1434.98
pr226	ACS	84534	86913.8	378.4	4065.25
	ACO	83659	87215.6	523.8	5206.70
	PACO	81967	83462.2	762.2	3103.41
lin105	ACS	14883	15125.4	88.8	475.37
	ACO	14795	15038.4	106.6	526.43
	PACO	14736	14888	112.2	211.34
kroB100	ACS	23014	23353.8	56.2	685.79
	ACO	22691	23468.1	102.9	702.46
	PACO	22289	22728	169.6	668.26
kroC100	ACS	21594	21942.6	54.8	509.77
	ACO	21236	21909.8	78.1	814.53
	PACO	20775	21598.4	114.8	414.62
lin318	ACS	48554	49224.4	849.2	1785.21
	ACO	48282	49196.7	902.7	2459.16
	PACO	47885	49172.8	866.8	1108.34

Table 6. Comparison of the ACS [49], ACO [51] and the proposed ACO (PACO) in TSP instances

### 3. Genetic algorithm for generalized TSP

#### 3.1 Generalized TSP (GTSP)

The generalized traveling salesman problem (GTSP) is a kind of combinatorial optimization problem, which has been introduced by Henry-Labordere [72] and Saksena [73] in the context of computer record balancing and of visit sequencing through welfare agencies since 1960s. The GTSP can be described as the problem of seeking a special Hamiltonian cycle with lowest cost in a complete weighted graph.

Let  $G=(V, E, M)$  be a complete weighted graph where  $V = \{v_1, v_2, \dots, v_n\}$  ( $n > 3$ ),  $E = \{e_{ij} \mid v_i, v_j \in V\}$  and  $W = \{w_{ij} \mid w_{ij} \geq 0 \text{ and } w_{ii} = 0, \forall i, j \in N(n)\}$  are vertex set, edge set, and cost set, respectively. The vertex set  $V$  is partitioned into  $m$  possibly intersecting groups  $V_1, V_2, \dots, V_m$  with  $|V_j| \geq 1$  and  $V = \bigcup_{j=1}^m V_j$ . The GTSP is required to pass through all of the groups, but not all of the vertices differing from that of TSP. For convenience, we also call  $W$  as the cost matrix and take it as  $W=(w_{ij})_{n \times n}$ . There are two different kinds of GTSP under the abovementioned framework of the special Hamiltonian cycle [75-76]: (1) the cycle passes exactly one vertex in each group (refer to Fig. 1) and (2) the cycle passes at least one vertex in each group (refer to Fig. 2). The first kind of GTSP is also known as E-GTSP, where E stands for equality [76]. In this paper we only discuss the GTSP for the first case and will still call it as GTSP for convenience.

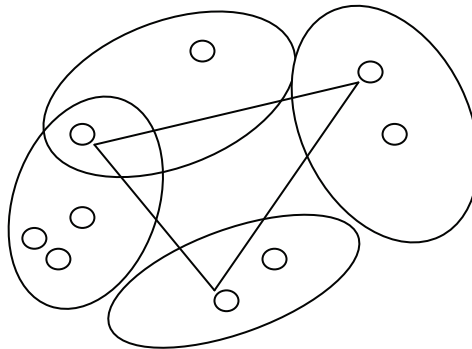


Figure 3. Exactly one vertex is visited in a GTSP cycle.

GTSP has extensive application fields. Laport et al. [75], Lien et al. [77], and Castelino et al. [78] reported the applications of GTSP. Just as mentioned in Ref. [77], "for many real-world problems that are inherently hierarchical, the GTSP offers a more accurate model than the TSP." Generally, GTSP provides a more ideal modeling tool for many real problems. Furthermore, GTSP can include the grouped and isolated vertices at the same time according to our present extension. Therefore, GTSP includes TSP theoretically (see Fig. 3) and application fields of GTSP are wider than those of TSP.

Although since late 1960s GTSP has been proposed [72-74], the related reported works are very limited compared with those on TSP [79-82] and the existing algorithms for GTSP are mainly based on dynamic programming techniques [72-74,76,83-84]. However, because of its NP-hard quality, only a few solutions of modest-size problems are supported by the current hardware technology and most of them fail to obtain the results due to the huge memory required in dynamic programming algorithms and the problem of lengthy computational time.

The main methodology of the dynamic programming algorithms is to transform the GTSP into TSP and then to solve the TSP using existing algorithms [76, 84-86]. The shortcomings of these methods are that the transformation increases the problem dimension dramatically and in some cases the dimension would expand up to more than three times of the original [77, 87-89]. Therefore, although theoretically the GTSP could be solved using the



corresponding transformed TSP, the technological limitation ruins its practical feasibility. Some studies have been performed to discuss and solve the problem [90–92]. This study, we will show some bio-inspired method on the GTSP problem.

### 3.2 Genetic algorithm for generalized TSP

Genetic algorithm (GA) is one of the powerful tools to deal with NP-hard combinatorial optimization problems and has been widely applied for finding the solution of TSP due to its high efficiency and strong searching ability. However, theoretical and application studies related to using GA methods to solve GTSP are very few. The [90] and [93] are two of most interesting work on this problem. In [90], a hybrid GTSP solving algorithm is proposed based on random-key GA and local search method, the main difficult of the method it is hard to handle large scale problems. In [93], a generalized chromosome is used and a generalized chromosome- based GA (GCGA) is proposed accordingly. The advantages of the GCGA are that it does not require the transformation from GTSP to TSP and remove the limitation of triangle inequality of the cost matrix, which enables the GCGA to be able to run with high efficiency.

#### 3.2.1 Generalized chromosome

The solution of GTSP is a special Hamiltonian cycle, which passes through all of the groups. The encoding for solution of GTSP is designed similarly to the one proposed by Huang et al. [94]. A hybrid encoding, which includes a head encoded with binary number and a body encoded with integer number, is given for the solution as figure 1 shows.

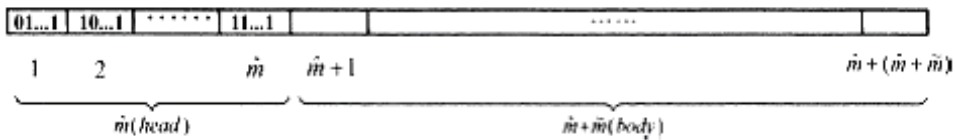


Figure 4. Hybrid Encoding for Solution of GTSP

In the body, there are  $m$  integer elements representing  $m$  groups ( $m = \hat{m} + \tilde{m}$ ,  $\hat{m}$  supper vertexes and  $\tilde{m}$  scattering vertexes[93]. In the head, there are  $\hat{m}$  binary elements representing vertexes in groups.

Let  $[e_1, \dots, e_i, \dots, e_{\hat{m}}, g_1, \dots, g_j, \dots, g_m]$  be a GTSP solution, where  $e_i$  (encoded in binary number) is the sequence number of the vertex in Group  $i$ , and  $g_j$  (encoded in integer number) is the sequence number of the  $j$ th group in the cycle. The set of hybrid encoding can be denoted by  $D = \{x | x = h \oplus b, h \in H, b \in B\}$ , and each solution for GTSP can be encoded as  $x \in D$ , where  $H = \{h | h = e_1, \dots, e_{\hat{m}}; e_i \leq |V_i|, i \leq \hat{m}\}$  represents the head and  $B = \{b | b = g_1, \dots, g_m\}$  represents the body.

#### 3.2.2 The framework of the GCGA

The special designed operators are needed to conduct random search on the generalized chromosome. The GCGA contains the following four operators: Initializing operator **P**, Generalized crossover operator **C**, Generalized crossover operator **C** and Generalized

reversion operator **R**. We give a brief introduction to these five steps in this section. More information about the GCGA can refer to [93].

### 1. Initializing operator **P**

Initializing operator **P** is used to generate an initial population. It is a two-element random operator. Its two variables are  $H$  and  $B$ , and its result is a subset of  $D$ . Denoting  $P$  as a population, then the initialization of  $P$  can be represented as  $P_N=(H,B)$ , where  $P_N$  is an operator to randomly generate an initial population with size  $N$ .

### 2. Generalized crossover operator **C**

To implement the crossover operation and generate new chromosomes, a generalized crossover operator is defined as  $C:D \times D \rightarrow D \times D$ . It is a two-element random operator. Its variables are the elements of  $D$ . The behavior of the operator is somewhat similar to the two-point crossover in the standard GA. Let the two crossover points selected randomly be  $i_1$  and  $i_2$  (assume  $i_1 < i_2$ ), where  $i_1 = \text{random}(\hat{2m} + \tilde{m})$ , and  $i_2 = \text{random}(\hat{2m} + \tilde{m})$ . If  $i_1 > \hat{m}$  then the crossover takes place in the body parts. In this case, the effect of crossover operator is equal to the conventional crossover in some extent, because the body parts of GC are equivalent to two normal chromosomes. If  $i_2 \leq \hat{m}$ , then the crossover takes place in the head parts. In this case, it is only needed to exchange the genes within the crossover segments. If  $i_1 \leq \hat{m} < i_2$ , then the generalized crossover can be treated as the combination of the above cases.

### 3. Generalized mutation operator **M**

To increase the diversity of the gene segments, the generalized mutation operator **M** is designed based on the insertion mutation used in standard GA. Preliminary gene  $i_2 = \text{random}(\hat{2m} + \tilde{m})$  is randomly selected, which is taken as the gene to be mutated. The difference between GCGA and standard GA is that if  $i < \hat{m}$  then the preliminary gene lies in the head part and its corresponding body part also need to be generated.

### 4. Generalized reversion operator **R**

To enhance the convergent speed of the GCGA, the generalized reversion operator is designed which is similar to the conventional reversion operation. Operator **R** can be used to select two reversion points  $i_1$  and  $i_2$  according to  $i_1 = \text{random}(\hat{m} + \tilde{m})$ , and  $i_2 = \text{random}(\hat{m} + \tilde{m})$ . If the solution generated after the reversion operator, then the operator **R** is taken, otherwise the operator won't taken.

## 3.3 Improved Evolutionary Algorithm (EA) for GTSP

### 3.3.1 The framework of EA for GTSP

In this section, an improved EA for the GTSP (EA-GTSP) has been proposed. In the EA-GTSP, the generalized chromosome described in 3.2 is used to encode the problem. And the following three operators are specially designed to improve the efficiency of the algorithm on the GTSP: crossover operator, mutation operator and local optimization strategy.

#### a. Crossover

At Step 3, pairs of solutions may be selected to carry out the crossover operator by the crossover probability  $P_c$ . Given two solutions  $S_x = h_x \oplus b_x$  and  $S_y = h_y \oplus b_y$  selected at Step 3 ( $h_x, h_y \in H, b_x, b_y \in B$ ), the process of crossover can be shown as follows:

Two integer numbers  $i_1, i_2 (i_1, i_2 \leq \hat{m} + (\hat{m} + \hat{m}), i_1 < i_2)$  are generated randomly to set the crossing position. If  $i_1 > \hat{m}$ , then,  $b_x \otimes b_y \rightarrow (b'_x, b'_y)$ , which is the same operator as the GCCA,  $x' = h_x \oplus b'_x, y' = h_y \oplus b'_y$ . If  $i_2 \leq \hat{m}$ ,  $h_x \otimes h_y \rightarrow (h'_x, h'_y)$ ,  $x' = h'_x \oplus b_x, y' = h'_y \oplus b_y$ . If  $i_1 < \hat{m}$  and  $i_2 \geq \hat{m}$ ,  $h_x \otimes h_y \rightarrow (h'_x, h'_y)$  and  $b_x \otimes b_y \rightarrow (b'_x, b'_y)$ ,  $x' = h'_x \oplus b'_x$  and  $y' = h'_y \oplus b'_y$ .

If the GTSP solution  $S_x$  costs less than  $S_y (i_2 \leq \hat{m})$ ,

$$h'_x = \{e_{x1}, \dots, e_{xi_1-1}, e_{xi_1} \cdot \text{AND} \cdot e_{yi_1}, \dots, e_{xi_2} \cdot \text{AND} \cdot e_{yi_2}, e_{xi_2+1}, \dots, e_{x\hat{m}}\}$$

$$h'_y = \{e_{x1}, \dots, e_{xi_1-1}, e_{xi_1} \cdot \text{OR} \cdot e_{yi_1}, \dots, e_{xi_2} \cdot \text{OR} \cdot e_{yi_2}, e_{xi_2+1}, \dots, e_{x\hat{m}}\};$$

otherwise,

$$h'_x = \{e_{y1}, \dots, e_{yi_1-1}, e_{xi_1} \cdot \text{AND} \cdot e_{yi_1}, \dots, e_{xi_2} \cdot \text{AND} \cdot e_{yi_2}, e_{yi_2+1}, \dots, e_{y\hat{m}}\}$$

$$h'_y = \{e_{y1}, \dots, e_{yi_1-1}, e_{xi_1} \cdot \text{OR} \cdot e_{yi_1}, \dots, e_{xi_2} \cdot \text{OR} \cdot e_{yi_2}, e_{yi_2+1}, \dots, e_{y\hat{m}}\}.$$

If the GTSP solution  $S_x$  costs less than  $S_y (i_1 < \hat{m}$  and  $i_2 \geq \hat{m})$ ,

$$h'_x = \{e_{x1}, \dots, e_{xi_1-1}, e_{xi_1} \cdot \text{AND} \cdot e_{yi_1}, \dots, e_{x\hat{m}} \cdot \text{AND} \cdot e_{y\hat{m}}\}$$

$$h'_y = \{e_{x1}, \dots, e_{xi_1-1}, e_{xi_1} \cdot \text{OR} \cdot e_{yi_1}, \dots, e_{x\hat{m}} \cdot \text{OR} \cdot e_{y\hat{m}}\};$$

otherwise,

$$h'_x = \{e_{y1}, \dots, e_{yi_1-1}, e_{xi_1} \cdot \text{AND} \cdot e_{yi_1}, \dots, e_{x\hat{m}} \cdot \text{AND} \cdot e_{y\hat{m}}\}$$

$$h'_y = \{e_{y1}, \dots, e_{yi_1-1}, e_{xi_1} \cdot \text{OR} \cdot e_{yi_1}, \dots, e_{x\hat{m}} \cdot \text{OR} \cdot e_{y\hat{m}}\}.$$

**b. Mutation**

The mutation operator is added to help EA-GTSP converge to the global optimal solution. Each solution is affected by the mutation operator by probability  $P_m$ . There are two procedures called head mutation and body mutation in the operator.

In the head mutation, given a head of a solution, the procedure of head mutation is:

Head mutation:  $h_z \rightarrow h'_z, h'_z = \{e_{z1}, \dots, e_{zi_3-1}, rebuild(e_{zi_3}), e_{zi_3+1}, \dots, e_{z\hat{m}}\}$

where  $h_z = \{e_{z1}, \dots, e_{zi-1}, e_{zi}, e_{zi+1}, \dots, e_{z\hat{m}}\}$ .  $rebuild(e_{zi})$  will generate a segment of binary bits randomly. Every binary element of solution  $S_z$  may be affected by  $rebuild(e_{zi})$  when  $r_{mh} < P_{mh}$  ( $r_{mh}$  is generated randomly in [0,1] for each binary element of the solution obtained at Steps 3 and 4).

In the body mutation, the procedure is described as follows:

Body mutation:  $b_z \rightarrow b'_z, b'_z = \{g'_{z1}, \dots, g'_{zi-1}, g'_{zi}, g'_{zi+1}, \dots, g'_{z\hat{m}}\}$

where  $b_z = \{g_{z1}, \dots, g_{zi}, \dots, g_{z\hat{m}}\}$  and  $r_{mb} < P_{mb}$  ( $r_{mb}$  is generated randomly in [0,1] for each solution obtained at Steps 3 and 4).

So the mutation operator of the EA-GTSP is defined as follows:

Mutation of EA-GTSP:  $S_z = h_z \oplus b_z \xrightarrow{\text{mutation}} S'_z = h'_z \oplus b'_z$ .

### c. Local Optimal Strategy

The local optimal strategy is helpful to find the best solution in a local searching space. Each solutions of the population are optimized according to a heuristic algorithm as follows:

Input: GTSP solution  $S_q$   
 For  $i=1$  to  $\hat{m}$  do //optimization for head  
   Choose a vertex in Group i to make  $S_q$  cost the lest  
 End for  
 For  $j=1$  to  $\hat{m} + \tilde{m} - 1$  do //optimization for body  
   Choose an order for  $g_{qj}$  and  $g_{qj+1}$  to make  $S_q$  cost the least.  
 End for  
 Output: a new solution  $S'_q$  ( $S_q$  is changed into  $S'_q$ .)

### d. Decoding for solution of GTSP

Because the head encoding is designed as binary number, it needs to be decoded in the following function.

$$h = [e_1, \dots, e_{\hat{m}}] \xrightarrow{\text{decoding}} [e_1 \cdot \text{MOD} \cdot |V_1|, \dots, e_i \cdot \text{MOD} \cdot |V_i|, \dots, e_{\hat{m}} \cdot \text{MOD} \cdot |V_{\hat{m}}|]$$

where  $|V_i|$  is the number of vertexes in Group i ( $V_i$ ).

Until now, we can summarize the algorithm of the improved EA for the GTSP as follows.

Initialize parameters.  
 Encode and initialize a population of solutions.  
 /\* $\beta$  is chosen between 0 and 5 randomly,  $q_0=0.6$  \*/  
**Loop** /\* at this level each loop is called iteration \*/  
   **Crossover Operator:** select pairs of solutions and change them into pairs of new Local solutions with the crossover operator by the crossover probability.  
   **Optimal Strategy:** optimize all of the solutions with a heuristic algorithm locally.  
   **Mutation Operator:** select several solutions by the mutation probability and change  
 End\_condition  
 Decoding for solution of GTSP

### 3.3.2 Numerical result

In this section, the efficiency of the EA-GTSP and other algorithms are compared on some benchmark problems [93].

Problem \ five runs	EA-GTSP Best	EA-GTSP Average	GCGA Best	GCGA Average	HCGA Best	HCGA Average
30KROA150	11018	11018	11018	11022	11018	11018
30KROB150	12195	12195	12196	12314	12195	12195
31PR152	51573	51573	51586	53376	51573	51573
32U159	22664	22664	22664	22685	22664	22664
40KROA200	13408	13408	13408	13617	13408	13408
40KROB200	13113	13114	13120	13352	13113	13119
45TS225	68340	68403	68340	68789	68340	68432
46PR226	64007	64007	64007	64574	64007	64007
53GIL262	1011	1011	1011	1057	1011	1011
53PR264	29546	29546	29549	29791	29546	29546
60PR299	22617	22631	22638	22996	22631	22638
64LIN318	20769	20799	20977	22115	20788	20914
80RD400	6446	6480	6465	6604	6456	6498
84FL417	9663	9663	9663	9725	9663	9663
88PR439	60099	60249	61273	62674	60184	60558
89PCB442	21695	21735	21978	22634	21768	21860

Table 7. Comparison of solution among EA-GTSP, GCGA and HCGA

The instances can be obtained from TSPLIB library which were originally generated for testing standard TSP algorithms. To test GTSP algorithms, Fischetti et al. [95] provided a partition algorithm to convert the TSP instances to GTSP instances.

In our experiments, we set the population size as 100 ( $pop\_size=100$ ), crossover probability as 0.5 ( $P_c = 0.5$ ), and mutation probability as 0.09 ( $P_m=0.09$ ,  $P_{mh}=0.001$ ,  $P_{mb}=0.005$ ). The algorithms would stop when no better solution could be found in 500 iterations. All of the instances are computed by EA-GTSP, HCGA [94] and GCGA [93] twenty times on a PC with 2.0 GHz processor and 256 MB SDR memory, and the results are shown in Table 1.

In Table 7, not only the best solution obtained by EA-GTSP is shorter than the one obtained by HCGA and GCGA does, but also the one on average, in all of the examples. It can show global optimal function of EA-GTSP. In order to show the performance of EA-GTSP, there is a comparison between it and several heuristic algorithms [96] by computing the same GTSP instances. As Table 2 shows, EA-GTSP is more efficient and steady than all of the test algorithms because it can get the best solution in most of the instances.

Problem\fi ve runs	EA- GTSP	NN (G-opt)	NN (G2-opt)	CI (G-opt)	CI (G2-opt)	MO (G-opt)	MO (G2-opt)	CI <sup>2</sup>	GI <sup>3</sup>
30KROA150	11018	11018	11018	11018	11018	11018	11018	11018	11018
30KROB150	12195	12196	12196	12196	12196	12196	12196	12196	12196
31PR152	51573	52506	52506	51915	51915	51820	51820	51820	51820
32U159	22664	23296	23296	22664	22664	22923	22923	23254	23254
40KROA200	13408	14110	14110	14059	14059	13887	13887	13406	13406
40KROB200	13113	13932	13111	13117	13117	13117	13117	13111	13111
45TS225	68340	68340	68340	69279	69279	68756	68756	68756	68756
46PR226	64007	65811	65395	65395	65395	64007	64007	64007	64007
53GIL262	1011	1077	1032	1036	1036	1021	1021	1064	1064
53PR264	29546	31241	31241	31056	31056	30779	30779	29655	29655
60PR299	22617	24163	23069	23119	23119	23129	23129	23119	23119
64LIN318	20769	22233	21787	21858	21858	22403	22403	21719	21719
80RD400	6446	7083	6614	6550	6550	6546	6546	6439	6439
84FL417	9663	9754	9754	9662	9662	9697	9697	9932	9697
88PR439	60099	63736	62514	61126	61126	62091	62091	62215	62215
89PCB442	21695	23364	21704	23307	23307	22697	22697	22936	22936

Table 8. Comparison of solution among EA-GTSP, GCGA and HCG

#### 4. Conclusion and discussions

The chapter introduces two examples of bio-inspired algorithm for traveling sales-man problems and its extended version. The first algorithm, named ant colony optimization (ACO) which is designed inspired by the natural ants' behavior, is a novel method to deal with TSPs. The experimental results prove the performance of ACO approach, which is feasible to solve TSP instances as well as the traditional method. The research results about the self-adaptive parameters of ACO are presented in the chapter, which indicates how to set an optimal ACO algorithm for different TSPs. Another algorithm is genetic algorithm, which is used to solve generalized traveling sales-man problem (GTSP) that is one extended style of TSPs. The best-so-far genetic algorithm for GTSP is introduced in the final subsection. Bio-inspired algorithms are the feasible methods for TSPs, and can attain better performance with the modified setting like self-adaptive parameters and hybrid coding, which are described in the chapter.

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## **Traveling Salesman Problem**

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The idea behind TSP was conceived by Austrian mathematician Karl Menger in mid 1930s who invited the research community to consider a problem from the everyday life from a mathematical point of view. A traveling salesman has to visit exactly once each one of a list of  $m$  cities and then return to the home city. He knows the cost of traveling from any city  $i$  to any other city  $j$ . Thus, which is the tour of least possible cost the salesman can take? In this book the problem of finding algorithmic technique leading to good/optimal solutions for TSP (or for some other strictly related problems) is considered. TSP is a very attractive problem for the research community because it arises as a natural subproblem in many applications concerning the every day life. Indeed, each application, in which an optimal ordering of a number of items has to be chosen in a way that the total cost of a solution is determined by adding up the costs arising from two successively items, can be modelled as a TSP instance. Thus, studying TSP can never be considered as an abstract research with no real importance.

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