The Feasibility Analysis of Available-to-Promise in Supply-Chain System under Fuzzy Environment

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1. Introduction

Today enterprises face many rigorous competitive market pressures and challenges, including the globalization of competition and cooperation, a variety of customer requirements and shortened product-life cycles. An enterprise has to seek effective methods to adjust the management strategy and to sustain competitive advantage. For example, Material Requirement Planning (MRP), Manufacturing Resource Planning (MRPII) and Enterprise Resource Planning (ERP) (Enns, 2002; Min & Zhou, 2002) are used to integrate the operation processes and resources for enterprises. The purpose of these tools is to reduce the response time to meet the market demand and increase customer satisfaction.

As the Internet and information technology grow rapidly, the industrial environment becomes more competitive for individual enterprises. In the global marketplace, no longer do individual enterprises compete as an independent entity but rather as an integral part of the supply chain (Enns, 2002). In other words, each enterprise will depend on its management ability to integrate and coordinate the complicated network of business relationships among supply-chain members (Cooper & Lambert, 2000). Therefore, supply-chain management (SCM) has become one of the most important issues for enterprises.

A supply chain may be viewed as an integrated system that performs the procurement of raw material, its transformation to intermediate and end-products, distributor and promoting of the end-products to either retailers or customers (Cooper & Lambert, 2000; Min & Zhou, 2002). In recent years, many researchers have become very interested in supply-chain management problems and the concepts of SCM from different viewpoints which have been presented (Christopher et al., 1998; Cooper & Lambert, 2000; Lee et al., 1997; Ross, 1997; Richard et al., 2003). Unfortunately, there is no explicit description of supply-chain management or its activities in the literature (Tan, 2001). For example, New and Payne (1995) described supply-chain management as the chain linking each element of the manufacturing and supply processes from raw materials to the end user, encompassing several organizational boundaries. Jukka et al., (2001) described supply-chain management as a new way to provide goods and services to the end-customer at the lowest cost and highest service level, for which centric approach to managing the supply chain is no longer appropriate. SCM is an integrated approach to increase the effectiveness of the logistics
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chain by improving cooperation between members in the chain. Figure 1 shows that the activities of a supply chain are comprised of two fundamental procedures: (1) the production layout and inventory control process, and (2) the distribution and logistics process (Beamon, 1998).

There is a growing interest in exploiting supply-chain models in real applications and in developing decision-support systems to enhance supply-chain management and control. However, a real supply chain operates in an uncertain environment. Different sources and types of uncertainty exist along the supply chain and these make supply-chain management problems more complex (Lee et al., 1997; Dejonckheere et al., 2002; Dolgui & Ould-Louly, 2002; Ouyang & Chang, 2002). In general, the supply-chain system is essential to provide the precise commitment of customers, promise the customers to deliver on time, and maintain the requirement of product and service quality. Facing the dynamic states of the external market, however, the various operation times are not easy to determine in real supply chains. Therefore, the enterprise cannot always fulfill the exact promised time for customers. In general, decreasing the inventory and shortening the operation time are the two main criteria for evaluating the performance and resolving the uncertainty problems in a supply-chain system (Petrovic et al., 1998; Petrovic, 2001). The operation time includes the time necessary for order processing, the production time and/or the transportation time. Traditionally, uncertain parameters in operation-time problems have been modeled by probability distributions in the literature (Mon et. al., 1995; Dolgui & Ould-Louly, 2002; Stevenson, 2002; Fatemi Ghomi & Rabbani, 2003; Dubois et. al., 2003). However, uncertain parameters can only be specified using the experience and subjective judgment of managers. The standard probabilistic reasoning methods are not appropriate to deal with this type of uncertainty problem (Petrovic et al., 1998; Petrovic, 2001). Therefore, in order to deal with the various uncertainties in operation time and the subjective judgment of decision makers, triangular fuzzy numbers will be used to express the lead-time for each member of the supply chain. Considering these uncertainties in operation time, we propose an order fulfillment analysis model for a supply-chain system by combining fuzzy-set theory with program evaluation and review technique (PERT).

Program evaluation and review technique (PERT) (Chen & Chang, 2001; Dubois et al., 2003; Fatemi Ghomi & Rabbani, 2003) is the most widely used management technique for planning and coordinating large-scale projects. By using PERT, managers are able to obtain (Stevenson, 2002):

1. a graphical display of project activities,
2. an estimate of how long the project will take,
3. an indication of which activities are the most critical to timely project completion,
4. an indication of how long any activity can be delayed without delaying the project.

Therefore, PERT is suitable for computing the order-completion time and forecasting the order-fulfillment ability in the supply-chain system. In traditional program evaluation and review techniques, various dynamic activity durations must be a crisp number or obedient to a certain probability distributions which usually are of Beta type. In PERT, we must specify an optimistic time, a most probable (or most likely) time, and a pessimistic time to estimate the duration of each activity. Optimistic time is the “best” activity completion time that could be obtained in a PERT network. Probable time is the most likely time to complete an activity in a PERT network. Pessimistic time is the “worst” activity time that could be expected in a PERT network. The three times are used to calculate an expected completion
time and variance for each activity (Dawson & Dawson, 1995 1998; Premachandrak, 2001). However, precise information about the durations of activities is seldom available in some contexts, like the early rough planning of long range projects.

In a real situation, the operation time of each activity is usually difficult to define and estimate. This implies that the activity time may be ill defined. Therefore, in recent years many researchers have developed the fuzzy PERT (FPERT) by combining the concepts of fuzzy-set theory with PERT to solve the project planning and control problem (Chen & Chang, 2001; Dubois et al., 2003; Fatemi Ghomi & Rabbani, 2003; Hapke & Sloinski, 1994; Hapke & Sloinski, 1996; Kuchta, 2001, Mon et al., 1995; Fatemi Ghomi & Teimouri, 2002). In FPERT, a fuzzy number can be used to express the duration time of each activity. The fuzzy PERT was first presented by Chanas & Kamburowski (1981). They use the fuzzy numbers to express the durations of all activities in project network. Based on the given possibility distributions of activity durations, the possibility distribution of the project completion time can be derived. In fact, the three time estimates are used to express the fuzzy duration of all activities in their method. And then, the $\alpha$-cut of fuzzy number is applied to calculate the interval of completion time of project network. However, it can obtain the different intervals of completion time with different $\alpha$ values. Under this condition, we cannot indicate the critical activities and paths effectively in a project network. Mon et al., (1995) assume that the duration of each activity is positive fuzzy number. Using the $\alpha$-cut of each fuzzy duration, the interval is denoted by $\tilde{A}^\alpha = [a_L^\alpha, a_R^\alpha]$. In this interval, $a_L^\alpha$ is the lower bound of duration and $a_R^\alpha$ is the upper bound of duration. Applying the linear combination of $\tilde{A}^\alpha$ to represent the operation time of each activity and determine the critical activities and paths. However, this approach will find the different critical activities and paths in accordance with $\alpha$ values. Chanas % Zielinski (2001) assume that the operation time of each activity can be represented as crisp, interval or fuzzy number. The operation procedure of this method like as the method of Mon et al., (1995). Dubois et al., (2003) indicated that each activity has different importance on a critical path. Therefore, they proposed a method to calculate the importance degree of each activity and path when choose a set of activities randomly. In these fuzzy PERT methods, fuzzy numbers are used to express the operation times of activities to reduce the uncertainties. However, these fuzzy PERT methods are complicated for calculating the critical degrees of each activity and path. Additionally, these methods cannot compute the possibility of meeting a certain requirement time effectively and easily.

For a supply-chain network, the most important issue for manager is to compute the fulfillment degree effectively when the due-date is given by customer. Therefore, in this paper, a revised FPERT method is proposed to calculate the fuzzy completion time and identify the fulfillment degree for the supply-chain system. In the calculation process, the generalized mean-values method (Lee & Li, 1988) is applied to rank the fuzzy operation times and determine the critical members of the supply chain. Then, an available-to-promise (ATP) index is defined by comparing the fuzzy completion time and required due date of the customer. According to the ATP value, the order fulfillment ability of a supply-chain system can be quickly identified in specific market conditions.
In this chapter, a revised FPERT method is applied to a cycle-time management problem in a supply-chain system. First, the fuzzy completion time of the supply-chain system is calculated with the Fuzzy PERT method. Second, the critical degrees of members and paths of the supply-chain system are identified in accordance with the fuzzy float time of members. Next, the index of available-to-promise (ATP) is defined and calculated to indicate the order-fulfillment degree of the supply-chain system. Finally, a simulation analysis is presented to illustrate the procedures for our proposed method at the end of this paper.

2. Fuzzy sets and notations

A fuzzy set can be defined mathematically by assigning to each possible element in the universe of discourse a value representing its grade of membership in the fuzzy set (Zadeh, 1965). This grade corresponds to the degree to which that element is similar to the concept represented by the fuzzy set. Thus, elements may belong in the fuzzy set to a greater or lesser degree as indicated by a larger or smaller membership grade. As already mentioned, these membership grades are very often represented by real number values ranging in the closed interval between 0 and 1 (Klir & Yuan, 1995).

2.1 Fuzzy number

The fuzzy number $\tilde{A}$ is a fuzzy set whose membership function $\mu_{\tilde{A}}(x)$ satisfies the following conditions (Klir & Yuan, 1995):

1. $\mu_{\tilde{A}}(x)$ is piecewise continuous;
2. $\mu_{\tilde{A}}(x)$ is a convex fuzzy subset;
3. $\mu_{\tilde{A}}(x)$ is normality of a fuzzy subset. It implies that at least one element $x_0$ must have a membership grade of 1 for the fuzzy set, i.e., $\mu_{\tilde{A}}(x_0) = 1$.

2.2 The $\alpha$-cut of fuzzy number

The $\alpha$-cut of fuzzy number $\tilde{A}$ is defined as

$$\tilde{A}^{\alpha} = \{x_i : \mu_{\tilde{A}}(x_i) \geq \alpha, x_i \in X\}$$

where $\alpha \in [0, 1]$.

The symbol $\tilde{A}^{\alpha}$ represents a non-empty bounded interval contained in $X$, which can be denoted by $\tilde{A}^{\alpha} = [a_L^{\alpha}, a_R^{\alpha}]$, $a_L^{\alpha}$ and $a_R^{\alpha}$, the lower and upper bounds of the closed interval, respectively (Kaufmann & Gupta, 1991; Zimmerman, 1991).

Give any two positive fuzzy numbers $\tilde{m}$, $\tilde{n}$ and a positive real number $r$, the $\alpha$-cut of two fuzzy numbers are $\tilde{m}^{\alpha} = [m_L^{\alpha}, m_R^{\alpha}]$ and $\tilde{n}^{\alpha} = [n_L^{\alpha}, n_R^{\alpha}]$ ($\alpha \in [0, 1]$), respectively. According to the interval of confidence (Kaufmann & Gupta, 1991), some main operations of positive fuzzy numbers $\tilde{m}$ and $\tilde{n}$ can be expressed as follows:
\[(\tilde{m} \oplus \tilde{n})^a = [m_i^a + n_i^a, m_u^a + n_u^a], \quad (2)\]
\[(\tilde{m} \Theta \tilde{n})^a = [m_i^a - n_u^a, m_u^a - n_i^a], \quad (3)\]

where \(\oplus\) and \(\Theta\) are fuzzy additive and subtractive operators, respectively.

### 2.3 Triangular Fuzzy Number (TFN)

A triangular fuzzy number is a popular type of fuzzy number, which can be expressed as \(\tilde{T} = (l, m, u)\). When \(l > 0\), then \(\tilde{T}\) is a positive triangular fuzzy number (PTFN) (Dubois & Prade, 1980; Zimmermann, 1991). The membership function of positive triangular fuzzy number \(\tilde{T}\) (shown in Figure 2) is defined as:

\[
\mu_{\tilde{T}}(x) = \begin{cases} 
\frac{x-l}{m-l}, & l \leq x \leq m \\
\frac{u-x}{u-m}, & m \leq x \leq u \\
0, & \text{otherwise}
\end{cases}
\quad (4)
\]

where \(l > 0\).

When given two positive triangular fuzzy numbers \(\tilde{T}_1 = (l_1, m_1, u_1)\) and \(\tilde{T}_2 = (l_2, m_2, u_2)\), the additive (\(\oplus\)) and subtractive (\(\Theta\)) operations between them can be expressed as follows (Kaufmann & Gupta, 1991):

\[
\tilde{T}_1 \oplus \tilde{T}_2 = (l_1 + l_2, m_1 + m_2, u_1 + u_2) \quad (5)
\]
\[
\tilde{T}_1 \Theta \tilde{T}_2 = (l_1 - u_2, m_1 - m_2, u_1 - l_2) \quad (6)
\]

### 2.4 Ranking method for fuzzy numbers

Many ranking methods have been developed to transform fuzzy numbers into crisp values (Chen & Hwang, 1992; Nicole et al., 2002). Lee & Li (1988) presented the so-called generalized mean value method. It is very easy to compare fuzzy numbers with this method. Let \(\tilde{T} = (l, m, u)\) be a triangular fuzzy number whose defuzzied value is easily computed (Lee & Li, 1988) as

\[
G(\tilde{T}) = \frac{l + m + u}{3} \quad (7)
\]

and
\[ S(\tilde{T}) = \frac{1}{18} [l^2 + m^2 + n^2 - ln - mn] \]  \hspace{1cm} (8)

where \( G(\tilde{T}) \) is the generalized mean value and \( S(\tilde{T}) \) is the deviation of fuzzy number \( \tilde{T} \), respectively.

Suppose \( \tilde{T}_1 = (l_1, m_1, u_1) \) and \( \tilde{T}_2 = (l_2, m_2, u_2) \) are two triangular fuzzy numbers. If \( G(T_1) > G(T_2) \), then \( \tilde{T}_1 > \tilde{T}_2 \); if \( G(T_1) = G(T_2) \) and \( S(\tilde{T}_1) < S(\tilde{T}_2) \), then \( \tilde{T}_1 > \tilde{T}_2 \); if \( G(T_1) = G(T_2) \) and \( S(\tilde{T}_1) = S(\tilde{T}_2) \), then \( \tilde{T}_1 \approx \tilde{T}_2 \).

3. Fuzzy PERT

Because the durations of activities in supply-chain network are usually difficult to estimate and determine exactly, it is reasonable way to use the fuzzy numbers to describe the durations. Therefore, positive triangular fuzzy numbers are used in this paper to express the operation time of each activity (member) on the supply-chain network. In general, there are two conventions for constructing a network graph. These conventions are referred to as activity-on-arrow (AOA) and activity-on-node (AON) (Stevenson, 2002). Both conventions are illustrated in Figures 3 and 4. In this paper, an activity-on-node (AON) graph is adopted to represent a project network. This graph includes two virtual activities of null duration, the initial node (I) and the ending node (E). According to the computation procedure of PERT, the forward pass is performed to calculate the fuzzy earliest-start-time and fuzzy earliest-finish-time as

\[ \tilde{S}_i^e = \max_{j \in P(i)} \{ \tilde{S}_j^e \oplus \tilde{d}_j \} \]  \hspace{1cm} (9)

\[ \tilde{F}_i^e = \tilde{S}_i^e \oplus \tilde{d}_i \]  \hspace{1cm} (10)

where \( \tilde{S}_i^e \) is the fuzzy earliest-start-time of activity \( i \) and \( \tilde{F}_i^e \) is the fuzzy earliest-finish-time of activity \( i \). If \( i = I \), then \( \tilde{S}_I^e = (0, 0, 0) \). If \( i = E \), then \( \tilde{F}_E^e = \tilde{T}_{end} \cdot \tilde{T}_{end} \) is the fuzzy completion time of supply-chain network. \( P(i) \) is the set of predecessors of activity \( i \). \( \tilde{S}_j^e \) is the fuzzy earliest-start-time of activity \( j \). \( \tilde{d}_j \) is the fuzzy operation time of activity \( j \). \( \tilde{d}_i \) is the fuzzy operation time of activity \( i \).

The backward pass is performed to calculate the fuzzy latest-start-time and fuzzy latest-finish-time as

\[ \tilde{F}_i^l = \min_{j \in S(i)} \{ \tilde{F}_j^l \oplus \tilde{d}_j \} \]  \hspace{1cm} (11)
\[ \tilde{S}_i^l = \tilde{F}_i^l \Theta \tilde{d}_i \]  

(12)

where \( \tilde{F}_i^l \) is the fuzzy latest-finish-time and \( \tilde{S}_i^l \) is the fuzzy latest-start-time of activity \( i \).

In the FPERT process, the generalized mean method (Lee & Li, 1988) is applied to compare the fuzzy numbers, and identify \( \tilde{S}_i^e, \tilde{S}_i^l, \tilde{F}_i^e \) and \( \tilde{F}_i^l \) of each activity \( i \).

In traditional PERT the float time (slack time) of each activity is difference between latest start time and earliest start time, or difference between latest finish time and earliest finish time (Stevenson, 2002). According to the \( \tilde{S}_i^e, \tilde{S}_i^l, \tilde{F}_i^e \) and \( \tilde{F}_i^l \) of each activity \( i \), the fuzzy float time can be calculated as

\[ \tilde{m}_i = \tilde{S}_i^l \Theta \tilde{S}_i^e \]  

(13)

or

\[ \tilde{m}_i = \tilde{F}_i^l \Theta \tilde{F}_i^e \]  

(14)

where

\( \tilde{m}_i \) is the fuzzy float time of activity \( i \).

\( \tilde{S}_i^l \) is the fuzzy latest-start-time of activity \( i \).

\( \tilde{S}_i^e \) is the fuzzy earliest-start-time of activity \( i \).

\( \tilde{F}_i^l \) is the fuzzy latest-finish-time of activity \( i \).

\( \tilde{F}_i^e \) is the fuzzy earliest-finish-time of activity \( i \).

Property:
According to the definition of \( \tilde{S}_i^e, \tilde{S}_i^l, \tilde{F}_i^e \) and \( \tilde{F}_i^l \), we know that the Equations (13) and (14) are identical.

Proof:
Suppose that the \( \alpha - cut \) of \( \tilde{S}_i^e, \tilde{S}_i^l, \tilde{F}_i^e, \tilde{F}_i^l \), and \( \tilde{d}_i \) for activity \( i \) can be represented as

\[ \tilde{S}_i^{e\alpha} = [s_i^{e\alpha L}, s_i^{e\alpha R}] \]

\[ \tilde{S}_i^{l\alpha} = [s_i^{l\alpha L}, s_i^{l\alpha R}] \]

\[ \tilde{F}_i^{e\alpha} = [f_i^{e\alpha L}, f_i^{e\alpha R}] \]
\[ \tilde{F}_i^\alpha = [f_i^L, f_i^R], \]
\[ \tilde{d}_i^\alpha = [d_{iL}, d_{iR}] \]
for \( \alpha \in [0, 1] \).

According to the Equations (10) and (12), we know that \( \tilde{F}_i^e = \tilde{S}_i^\alpha \oplus \tilde{d}_i^\alpha \) and \( \tilde{S}_i^\alpha = f_i^{l\alpha} \Theta \tilde{d}_i^\alpha \). Then, we have

\[ [f_i^{e\alpha}, f_i^{e\alpha}] = [s_i^{e\alpha}, s_i^{e\alpha}] \oplus [d_{iL}, d_{iR}] = [s_i^{e\alpha} + d_{iL}, s_i^{e\alpha} + d_{iR}] \]

and

\[ [s_i^{i\alpha}, s_i^{i\alpha}] = [f_i^{i\alpha}, f_i^{i\alpha}] \Theta [d_{iL}, d_{iR}] = [f_i^{i\alpha} - d_{iL}, f_i^{i\alpha} - d_{iR}] \]

It implies that

\[ f_i^{e\alpha} = s_i^{i\alpha} + d_{iL}, \quad f_i^{e\alpha} = s_i^{i\alpha} + d_{iR}, \quad s_i^{i\alpha} = f_i^{i\alpha} - d_{iL}, \quad s_i^{i\alpha} = f_i^{i\alpha} - d_{iR}. \]

Then,

\[ f_i^{e\alpha} + s_i^{i\alpha} = s_i^{e\alpha} + f_i^{i\alpha} \]
\[ s_i^{i\alpha} + f_i^{e\alpha} = f_i^{i\alpha} + s_i^{e\alpha} \]

It implies that

\[ s_i^{i\alpha} - s_i^{e\alpha} = f_i^{i\alpha} - f_i^{e\alpha} \]
\[ s_i^{L\alpha} - s_i^{e\alpha} = f_i^{L\alpha} - f_i^{e\alpha} \]

According to the definition of fuzzy float time, we have

\[ \tilde{m}_i^\alpha = (\tilde{S}_i^\alpha \Theta \tilde{s}_i^\alpha)^\alpha \]
\[ = [s_i^{i\alpha}, s_i^{i\alpha}] \Theta [s_i^{e\alpha}, s_i^{e\alpha}] \]
\[ = [s_i^{i\alpha} + s_i^{e\alpha}, s_i^{i\alpha} + s_i^{e\alpha}] \]
\[ = [s_i^{i\alpha} + s_i^{e\alpha} - s_i^{i\alpha} - s_i^{e\alpha}, s_i^{i\alpha} + s_i^{e\alpha} - s_i^{i\alpha} - s_i^{e\alpha}] \]
\[ = [s_i^{i\alpha} + s_i^{e\alpha} - s_i^{i\alpha} - s_i^{e\alpha}, s_i^{i\alpha} + s_i^{e\alpha} - s_i^{i\alpha} - s_i^{e\alpha}] \]
\[ = [s_i^{i\alpha} + s_i^{e\alpha}, s_i^{i\alpha} + s_i^{e\alpha}] \Theta [f_i^{e\alpha}, f_i^{e\alpha}] \]
\[ = (\tilde{F}_i^\alpha \Theta \tilde{F}_i^e)^\alpha \]

for \( \alpha \in [0, 1] \).
Therefore, one can prove that the Equations (13) and (14) are identical for the fuzzy float time of each activity in a supply-chain network.

In traditional PERT, activity $i$ is called a critical activity if its float time is zero. In general, the fuzzy float time is smaller; the degree of criticality is higher for each activity. Suppose that the fuzzy float time of activity $i$ is denoted by $\tilde{m}_i = (a_i, b_i, c_i)$, and then the degree of criticality can be defined as

$$CD_i = \begin{cases} 1, & \text{if } b_i \leq 0 \\ \mu_{\tilde{m}_i}(0), & \text{if } b_i > 0 \end{cases}$$

(15)

where $CD_i$ is the critical degree of activity $i$.

$\mu_{\tilde{m}_i}(0)$ is the membership degree of zero belongs to the fuzzy float time.

In a supply network, a path is a sequence of activities that leads from the initial node to the ending node. According to the critical degree of each activity, the degree of criticality of a path can be calculated as

$$\pi(P_k) = \min_{i \in P_k} \{CD_i\}$$

(16)

where $P_k$ is the k-th path in the network and $\pi(P_k)$ is the critical degree of k-th path.

If the path $P$ is critical path, then $\pi(P)$ must satisfy that $\pi(P) = \max_k \pi\{P_k\}$.

4. Order-fulfillment analysis model

Suppose that there are certain suppliers, manufacturers, distributors and final retailers in a supply chain. The operation time from receiving the order to delivering the materials to the manufacturer or suppliers $S_i$ ($i = 1, 2, ..., m$) is denoted by $\tilde{d}_{S_i} = (S_{i1}, S_{i2}, S_{i3})$; the operation of the production of manufacturer $M_k$ is denoted by $\tilde{d}_{M_k} = (M_{k1}, M_{k2}, M_{k3})$; the operation time of the assembly and distributor is denoted by $\tilde{d}_{D_i} = (D_{i1}, D_{i2}, D_{i3})$; the operation time of the final retailer is denoted by $\tilde{d}_{R_i} = (R_{i1}, R_{i2}, R_{i3})$.

Two basic possible cases in the supply-chain system are discussed below:

(1) One supplier delivers materials to one manufacturer; one manufacturer delivers products to one distributor, who sends the products to the final retailer (shown in Figure 5). In this case, the completion time ($\tilde{T}_{end}$) of the supply-chain system can be computed as:

$$\tilde{T}_{end} = \tilde{d}_S \oplus \tilde{d}_M \oplus \tilde{d}_D \oplus \tilde{d}_R$$

(17)

(2) Several suppliers deliver materials to one manufacturer, one manufacturer delivers products to one distributor, who sends the products to the final retail (shown in Figure
6. In this case, the completion time \((\tilde{T}_{\text{end}})\) of the supply-chain system may be computed as

\[
\tilde{T}_{\text{end}} = \max\{d_S, d_J\} \oplus \tilde{d}_M \oplus \tilde{d}_D \oplus \tilde{d}_R
\]

(18)

An actual supply-chain network may consist of two graphs of cases 1 and 2. Suppose the requirement due-date (RDD) of the customer is denoted by \(\tilde{R}\) and the completion time (CT) of the supply-chain system is denoted by \(\tilde{T}_{\text{end}}\). If \(\text{RDD} \geq \text{CT}\), then the supply-chain system can satisfy the requirement of the customer completely. However, it is difficult to make a comparison between RDD and CT directly when they are fuzzy numbers. Let \(\tilde{R} = (r_1, r_2, r_3)\) and \(\tilde{T}_{\text{end}} = (e_1, e_2, e_3)\), then \(\tilde{R} \Theta \tilde{T}_{\text{end}} = (r_1 - e_3, r_2 - e_2, r_3 - e_1)\).

The membership function of fuzzy number \(\tilde{R} \Theta \tilde{T}_{\text{end}}\) is shown in Figure 7. Therefore, the order-fulfillment ability analysis of the supply-chain system is described as follows:

1. If \(r_1 - e_3 \geq 0\), then the order-fulfillment degree is 100%.

2. If \(r_3 - e_1 \leq 0\), then the order-fulfillment degree is zero. In other words, the supply-chain system cannot deliver on time.

3. If \(r_1 - e_3 < 0 < r_3 - e_1\), then the order-fulfillment degree is denoted by the available-to-promise (ATP). The available-to-promise (ATP) can be defined as

\[
\text{ATP} = \begin{cases} 
1 & , \quad r_1 - e_3 \geq 0 \\
\frac{\delta_1}{\delta_1 + \delta_2} & , \quad r_1 - e_3 \leq 0 \leq r_3 - e_1 \\
0 & , \quad r_3 - e_1 \leq 0
\end{cases}
\]

(19)

where

\[
\delta_1 = \int_{0}^{\infty} \mu_{\tilde{R} \Theta \tilde{T}_{\text{end}}} (x) dx , \quad \delta_2 = \int_{0}^{\infty} \mu_{\tilde{R} \Theta \tilde{T}_{\text{end}}} (x) dx
\]

and \(\mu_{\tilde{R} \Theta \tilde{T}_{\text{end}}} (x)\) is the membership function of fuzzy number \(\tilde{R} \Theta \tilde{T}_{\text{end}}\).

In other words, when the value \(\delta_1\) is larger, the degree of order-fulfillment in the supply-chain system is higher. Consequently, the supply-chain system is more flexible. It implies that the supply-chain system can respond faster to meet the requirements of the customer.

In a supply-chain network, if the critical degrees of two activities (members) \(i\) and \(j\) are identical and \(CD_i = CD_j = 1\) then the generalized mean method (Lee & Li, 1988) can be used to determine the priority to shorten the duration and increase ATP value. The generalized mean value is smaller; the priority is higher to shorten the duration.
5. Analysis of simulation

In this paper, the following assumptions have been made in the supply-chain network.

1. The production facilities have unlimited capacities.
2. Customer demand is confined to a single product.
3. The raw material inventory is supplied from an external source.
4. External demand is fulfilled from the end-product inventory.

In order to implement the simulation analysis, the structure of the supply-chain system is assumed to be as shown in Figure 8. In Figure 8, \( S_1 \) and \( S_2 \) indicate two suppliers; \( M_1, M_2 \) and \( M_3 \) indicate three manufacturers; \( D_1, D_2 \) and \( D_3 \) indicate three distributors; \( R_1 \) indicates the final retailer. Furthermore, the operation times of the suppliers, the manufacturers, the distributor, and the retailer are expressed by triangular fuzzy numbers as shown in Table 1. According to the fuzzy PERT algorithm, the completion time of the supply-chain network is computed as \( \overline{T_{end}} = (13, 18, 25) \). According to the computation process of the proposed method, one can calculate the fuzzy earliest-start-time, fuzzy earliest-final-time, fuzzy latest-start-time, and fuzzy latest-final-time of each activity (shown in Table 1). The critical degree of each activity can be computed based on the fuzzy float time. For example, the degree of criticality of 2nd supplier \( (S_2) \) is denoted by \( CD_2 = 1 \). The critical degree of each path of the supply-chain network is shown in Table 2. According to Tables 1 and 2, the critical members are \( S_2, M_2, D_2, D_3, \) and \( R_1 \); therefore, it is known that the critical paths are \( I \rightarrow S_2 \rightarrow M_2 \rightarrow D_2 \rightarrow R_1 \rightarrow E \) and \( I \rightarrow S_2 \rightarrow M_2 \rightarrow D_2 \rightarrow D_3 \rightarrow R_1 \rightarrow E \) in this supply-chain network.

The available-to-promise (ATP) can be computed when the requirement due-date (RDD) is given by the customer. Given the RDD of the customer, the ATP of the supply-chain system can be identified and computed in accordance with equation (19). Suppose that the values of RDD are 13, 15, 18, 20 and 25. If \( \tilde{R} = (13, 13, 13) \) and \( \tilde{T}_{end} = (13, 18, 25) \), then \( \tilde{R} \Theta \tilde{T}_{end} = (-12, -5, 0) \). The ATP of the supply-chain system can be computed as

\[
ATP = \frac{\delta_1}{\delta_1 + \delta_2} = \frac{12*1 - 12*1}{\frac{2}{12*1}} = 0
\]

Under this condition, the order-fulfillment degree of the supply-chain system is zero. This implies that the supply chain system cannot deliver on time.

If \( \tilde{R} = (18, 18, 18) \) and \( \tilde{T}_{end} = (13, 18, 25) \), then \( \tilde{R} \Theta \tilde{T}_{end} = (-7, 0, 5) \). The ATP of the supply-chain system can be computed as

\[
ATP = \frac{\delta_1}{\delta_1 + \delta_2} = \frac{5*1}{\frac{2}{(7+5)*1}} = 0.42
\]
Under this condition, the order fulfillment degree of the supply-chain system is 42%.

According to the different RDD values, the ATP of the supply chain system can be computed as shown in Table 3. According to Table 3, one can observe that if \( \tilde{T}_{end} \) is fixed and RDD is smaller, then the ATP of the supply chain system is lower (shown in Figure 9). Sometimes customers find it is difficult to define a crisp due-date; thus, it is reasonable for customers to give a fuzzy due-date in this scenario. For example, if the expected due-date is \( t \), then the upper and lower dates will not be greater or smaller than 20% of the expected due-date. Thus, the fuzzy due-date is a triangular fuzzy number that can be denoted by \((0.8t, t, 1.2t)\). According to Table 3, the crisp RDD values can be extended to fuzzy numbers as shown in Table 4. When RDD are fuzzy numbers, the ATP can be computed respectively.

If \( \tilde{R}_1 = (10.4, 13, 15.6) \) and \( \tilde{T}_{end} = (13, 18, 25) \), then \( \tilde{R} \Theta \tilde{T}_{end} = (-14.6, -5, 2.6) \). The ATP of the supply-chain system may be computed as:

\[
ATP = \frac{\delta_1 \cdot 2.6 \cdot 0.34}{(2+14.6) \cdot 1} = 0.05
\]

Under this condition, the order-fulfillment degree of the supply-chain system is 5%.

If \( \tilde{R}_2 = (12, 15, 18) \) and \( \tilde{T}_{end} = (13, 18, 25) \), then \( \tilde{R} \Theta \tilde{T}_{end} = (-13, -3, 5) \). The ATP of the supply-chain system can be computed as:

\[
ATP = \frac{\delta_1 \cdot 5 \cdot 0.63}{(2+13) \cdot 1} = 0.18
\]

Under this condition, the order-fulfillment degree of the supply-chain system is 18%.

When \( \tilde{T}_{end} = (13, 18, 25) \) and RDD are fuzzy numbers, the ATP of the supply-chain system can be computed easily shown as Figure 10. If the completion time of the supply-chain system is sooner than the requirement due-date of the customer, the operation time of members on the critical path can be shortened to reduce the completion time and increase the ATP. In other words, the proposed method can highlight the critical members and critical path obviously.

### 6. Conclusions

Along with the growth of information technology, the competition between enterprises has transformed into a competition between supply-chains. Thus, a robust management and operation model of a supply-chain system will increase the competitiveness for the enterprise. In addition, the order-fulfillment ability of the supply chain is the key factor for competitive advantage.

In this chapter, triangular fuzzy numbers have been used to express the various uncertainties in operation times. Combining fuzzy set theory and PERT, a fuzzy PERT is proposed in this paper to compute the fuzzy earliest-start-time, fuzzy earliest-finish-time, fuzzy latest-start-time and fuzzy latest-finish-time of each activity (member) in a supply-
chain network. And then, a critical degree index based on the fuzzy float time is defined to calculate the critical degree of each activity and path of the supply-chain system easily. The fuzzy model has been proposed to analyze the order-fulfillment ability of a supply-chain system with uncertain operation time. According to the proposed model, the status of each critical member of the system can be immediately understood. If the completion time of the supply-chain system does not satisfy the requirement due-date of the customer, we may adjust the operation times of the critical members to increase the ATP value and customer satisfaction. The simulation results show that if an enterprise on the critical path can reduce his operation time effectively, the order-fulfillment ability of the supply-chain network system may be increased.

In the future, additional uncertain factors such as crash time and crash cost will be considered for developing a fuzzy model to analyze the order-fulfillment degree of the supply-chain system.

7. Acknowledgements

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8. References


Figure 1. Supply-chain process [Beamon, 1998]

\[
\mu_{\tilde{T}}(x)
\]

Figure 2. Positive triangular fuzzy number \( \tilde{T} \).
Figure 3. The graph of activity-on-arc (AOA).

Figure 4. The graph of activity-on-node (AON).

Figure 5. First type of supply chain.
Figure 6. Second type of supply chain.

Figure 7. Membership function of fuzzy number $\tilde{R} \Theta \tilde{T}_{end}$.
Figure 8. Graph of supply-chain network.

Figure 9. Crisp-value requirement due-date.
Figure 10. Fuzzy-number requirement due-date.

<table>
<thead>
<tr>
<th>Node</th>
<th>Operation time</th>
<th>Earliest start time (ES)</th>
<th>Earliest final time (EF)</th>
<th>Latest start time (LS)</th>
<th>Latest final time (LF)</th>
<th>Float time (m̅)</th>
<th>Critical degree</th>
</tr>
</thead>
<tbody>
<tr>
<td>S₁</td>
<td>(1,3,4)</td>
<td>(0,0,0)</td>
<td>(1,3,4)</td>
<td>(-5,7,19)</td>
<td>(-1,10,20)</td>
<td>(-5,7,19)</td>
<td>0.42</td>
</tr>
<tr>
<td>S₂</td>
<td>(2,4,5)</td>
<td>(0,0,0)</td>
<td>(2,4,5)</td>
<td>(-12,0,12)</td>
<td>(-7,4,14)</td>
<td>(-12,0,12)</td>
<td>1</td>
</tr>
<tr>
<td>M₁</td>
<td>(2,3,6)</td>
<td>(2,4,5)</td>
<td>(4,7,11)</td>
<td>(-1,10,20)</td>
<td>(5,13,22)</td>
<td>(-3,6,15)</td>
<td>0.33</td>
</tr>
<tr>
<td>M₂</td>
<td>(3,4,7)</td>
<td>(2,4,5)</td>
<td>(5,8,12)</td>
<td>(-7,4,14)</td>
<td>(0,8,17)</td>
<td>(-9,0,9)</td>
<td>1</td>
</tr>
<tr>
<td>M₃</td>
<td>(2,3,4)</td>
<td>(5,8,12)</td>
<td>(7,11,16)</td>
<td>(1,9,18)</td>
<td>(5,12,20)</td>
<td>(-4,1,6)</td>
<td>0.8</td>
</tr>
<tr>
<td>D₁</td>
<td>(2,3,4)</td>
<td>(5,8,12)</td>
<td>(7,11,16)</td>
<td>(5,13,22)</td>
<td>(9,16,24)</td>
<td>(0,5,10)</td>
<td>0</td>
</tr>
<tr>
<td>D₂</td>
<td>(3,4,5)</td>
<td>(5,8,12)</td>
<td>(8,12,17)</td>
<td>(0,8,17)</td>
<td>(5,12,20)</td>
<td>(-5,0,5)</td>
<td>1</td>
</tr>
<tr>
<td>D₃</td>
<td>(4,4,4)</td>
<td>(8,12,17)</td>
<td>(12,16,21)</td>
<td>(5,12,20)</td>
<td>(9,16,24)</td>
<td>(-3,0,3)</td>
<td>1</td>
</tr>
<tr>
<td>R₁</td>
<td>(1,2,4)</td>
<td>(12,16,21)</td>
<td>(13,18,25)</td>
<td>(9,16,24)</td>
<td>(13,18,25)</td>
<td>(-3,0,3)</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 1. Operation times for each member of supply-chain system.
<table>
<thead>
<tr>
<th>No.</th>
<th>Path</th>
<th>$\pi(P_k)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$I \rightarrow S_1 \rightarrow M_1 \rightarrow D_1 \rightarrow R_1 \rightarrow E$</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>$I \rightarrow S_2 \rightarrow M_1 \rightarrow D_1 \rightarrow R_1 \rightarrow E$</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>$I \rightarrow S_2 \rightarrow M_2 \rightarrow D_1 \rightarrow R_1 \rightarrow E$</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>$I \rightarrow S_2 \rightarrow M_2 \rightarrow D_2 \rightarrow R_1 \rightarrow E$</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>$I \rightarrow S_2 \rightarrow M_2 \rightarrow D_2 \rightarrow D_3 \rightarrow R_1 \rightarrow E$</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>$I \rightarrow S_2 \rightarrow M_2 \rightarrow M_3 \rightarrow D_3 \rightarrow R_1 \rightarrow E$</td>
<td>0.8</td>
</tr>
<tr>
<td>7</td>
<td>$I \rightarrow S_2 \rightarrow M_3 \rightarrow D_3 \rightarrow R_1 \rightarrow E$</td>
<td>0.8</td>
</tr>
</tbody>
</table>

Table 2. Critical degree of each path.

<table>
<thead>
<tr>
<th>$\tilde{R}$</th>
<th>$\tilde{T}_{end}$</th>
<th>$\tilde{R} \Theta \tilde{T}_{end}$</th>
<th>ATP</th>
</tr>
</thead>
<tbody>
<tr>
<td>(13,13,13)</td>
<td>(13,18,25)</td>
<td>(-12,-5,0)</td>
<td>0</td>
</tr>
<tr>
<td>(15,15,15)</td>
<td>(13,18,25)</td>
<td>(-10,-3,2)</td>
<td>0.07</td>
</tr>
<tr>
<td>(18,18,18)</td>
<td>(13,18,25)</td>
<td>(-7,0,5)</td>
<td>0.42</td>
</tr>
<tr>
<td>(20,20,20)</td>
<td>(13,18,25)</td>
<td>(-5,2,7)</td>
<td>0.7</td>
</tr>
<tr>
<td>(25,25,25)</td>
<td>(13,18,25)</td>
<td>(0,7,12)</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 3. ATP values when RDD are crisp values.

<table>
<thead>
<tr>
<th>$\tilde{R}$</th>
<th>$\tilde{T}_{end}$</th>
<th>$\tilde{R} \Theta \tilde{T}_{end}$</th>
<th>ATP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{R}_1 = (10.4,13,15.6)$</td>
<td>(13,18,25)</td>
<td>(-14.6,-5,2.6)</td>
<td>0.05</td>
</tr>
<tr>
<td>$\tilde{R}_2 = (12,15,18)$</td>
<td>(13,18,25)</td>
<td>(-13,-3,5)</td>
<td>0.18</td>
</tr>
<tr>
<td>$\tilde{R}_3 = (14.4,18,21.6)$</td>
<td>(13,18,25)</td>
<td>(-10.6,0,8.42)</td>
<td>0.44</td>
</tr>
<tr>
<td>$\tilde{R}_4 = (16,20,24)$</td>
<td>(13,18,25)</td>
<td>(-9,2,11)</td>
<td>0.63</td>
</tr>
<tr>
<td>$\tilde{R}_5 = (20,25,30)$</td>
<td>(13,18,25)</td>
<td>(-5,7,17)</td>
<td>0.90</td>
</tr>
<tr>
<td>$\tilde{R}_6 = (25.8,32,38.4)$</td>
<td>(13,18,25)</td>
<td>(0.8,14,25.4)</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 4. ATP values when RDD are fuzzy numbers.
Traditionally supply chain management has meant factories, assembly lines, warehouses, transportation vehicles, and time sheets. Modern supply chain management is a highly complex, multidimensional problem set with virtually endless number of variables for optimization. An Internet enabled supply chain may have just-in-time delivery, precise inventory visibility, and up-to-the-minute distribution-tracking capabilities. Technology advances have enabled supply chains to become strategic weapons that can help avoid disasters, lower costs, and make money. From internal enterprise processes to external business transactions with suppliers, transporters, channels and end-users marks the wide range of challenges researchers have to handle. The aim of this book is at revealing and illustrating this diversity in terms of scientific and theoretical fundamentals, prevailing concepts as well as current practical applications.

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