Position and Velocity Tracking in Cellular Networks Using the Kalman Filter

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1. Introduction

“Access to the right information anytime, anywhere” is becoming the new driving force for the information technology revolution. The “right” information’s relevance is based on the user’s profile and his/her current geographical position and/or time. Location Based Service (LBS) is an innovative technology that provides information or makes information available based on the geographical location of the mobile user. Analysts predict that LBSs will lead to new applications, generating billions of US dollars worldwide (Leite, 2001; Searle, 2001).

The need for an efficient and accurate mobile station (MS) positioning system is growing day by day. The ability to pinpoint the location of an individual has an obvious and vital value in the context of emergency services (Chan, 2003; Olama et al., 2008). Pinpointing the location of people and other valuable assets also opens the door to a new world of previously unimagined information services and m-commerce probabilities. For example, availability of services like “Where is the nearest ATM?”, “Check traffic conditions on the highway on my route”, “Find a parking lot nearby”, as well as answers to “Where is my advisor?”, and “Where is my car?” will be an everyday rule in our lives (Charalambous & Panayiotou, 2004).

A technology independent LBS architecture can be considered as comprised by three main parts (Girodon, 2002): A user requesting information, a mobile network operator and its partners, and several content providers (e.g. data, maps). The subscriber requests a personalized service dependant on his geographic location. The system will ask the Location Services Manager (which is in charge of handling requests, i.e., send/receive to the Location Calculator and the Content Providers) to pinpoint the location of the mobile. The Location Services Manager (LSM), using the Location Calculator, will ask the Content Provider (CP) to supply qualified information according to the mobile’s geographical position. The LSM will eventually receive the answer from the CP and send it to the mobile, performing the essential data translations. Fig. 1 outlines the precedent concept.
For effective provision of LBS, one has to provide an accurate location, as well as suitable information for users required by the corresponding service, with minimal expenditure. Thus, there are three main technology issues that have to be resolved for LBS: positioning technology, application technology, and location services (Dru & Saada, 2001).

A very important technology is of course the positioning technology, the way to find out the location of a mobile device accurately. Due to the unique characteristics of the cellular environment, it is a great challenge to locate the user precisely. However, in many cases, application technology and location services are important consideration of LBS. Application technology manages the geographic information and delivers the customer requests to the appropriate service provider, thus it constitutes the communication system involved. LBS uses the geographic information to provide geographically sensitive information and services. Location-based applications and services are not sensitive to the type of location technology that is used - they merely rely on reasonably accurate geographic coordinates (Chan, 2003).

This chapter is structured as follows: In Section 2, we describe the use and applications of LBSs. The current location determination technologies and standards are presented in Section 3. In Section 4 we describe the mathematical models used for the location and velocity estimation algorithms. An initial attempt for MS location estimation via received signal level using the maximum likelihood estimation (MLE) approach and triangulation is presented in Sections 5. Since the former approach lacks acceptable accuracy for demanding services as numerical results reveal, the extended Kalman filter (EKF) approach, which is the main topic in this chapter, is introduced in Sections 6. In Section 7 we present numerical results. Section 8 provides concluding remarks.

2. Location based services and applications

Several market studies predicted that mobile location services will grow highly in the next few years (Leite, 2001; Searle, 2001). There are three major market drivers for LBS. These can
be identified as commercial, technological, and regulatory drivers. Regulatory is primarily the driver for the US, whereas in Europe and elsewhere LBS is mainly commercially and technologically driven.

In the US, regulatory requirements for emergency calls in cellular systems were first established in 1996 with the Federal Communications Commission (FCC) mandating all wireless service providers to provide public safety answering points with information to locate an emergency 911 (E-911) caller with an accuracy of 100 meters for 67% of the cases (FCC Docket No.96-264, Revision of the Commission Rule to ensure compatibility with Enhanced 911 emergency calling system, FCC Reports and Orders, 1996.). It is also expected that the FCC will tighten its requirements in the near future (Reed et al., 1998). The E-911 mandate distinguishes between wireline and wireless calls and the wireless E-911 mandate is separated into two Phases. Phase I requires that the call taker automatically receives the wireless call-back number and delivers the location of the cell tower handling the call, and Phase II allows call takers to receive both the caller's wireless phone number and their location information with prescribed accuracy.

In Europe, LBS is mainly driven by location-based value-added services, with the E112 emergency service only appearing recently on the political agenda. In contrast to the US, there is neither a distinction between mobile and fixed operator obligations nor a mention of any prescribed accuracy levels. In 2000, the European Commission launched a Coordination Group on Access to Location Information by Emergency Services (CGALIES: http://www.telematica.de/cgalies/) and project Location of Cellular Users for Emergency Services (LOCUS: http://www.telematica.de/locus/) to advise the European Union on implementing 112 emergency calling services, to actively involve the relevant players, and to develop a consensus on relevant implementation issues. In 2002, the Council and the European Parliament adopted the new regulatory package to enter into force by 24 July 2003. Even though, a recent Recommendation (Commission Recommendation of 25 July 2003 (2003/558/EC) Official Journal of the European Union, Erkki Liikanen) recommended a review of the situation in 2005.

LBSs can be categorized in different ways depending on the classification condition. We propose four main categories of LBS: Information, Safety, Monitoring and Operator Services (Dru & Saada, 2001). Information services include, among many others, finding the nearest service, accessing traffic news, getting navigation help, advertising and locating individuals. They are considered to be one of the most promising services in terms of global revenue (i.e. operators, developers, providers). Information services can penetrate in three kinds of relationships. First, the “Business to Consumer” relationship is targeted in means of local product promotion or advertising promotions, perhaps in exchange with lower monthly subscriptions, as long as privacy is not violated. In “Consumer to Business” relationship, users might require information about local services (restaurants, gas stations, pharmacies, etc.) or local traffic information. Last, in “Consumer to Consumer” relationship the subscriber can locate friends, family members, or more generally members of a desirable community.

Safety services include public and private emergency services for both pedestrians and drivers. As previously mentioned, public emergency services have already been regulated in the US and in Europe. These services do not require a subscription, can be accessed by any mobile subscriber, and do not generate a profit for the operators. Emergency roadside assistance for drivers appears to be one of the most promising of the safety services in terms of operator revenue.
Monitoring mainly covers “Business to Business” services, e.g. operating fleet management applications, and tracking the location of external resources to optimize their use and control or ensure their safety. External resources include individuals (truck drivers, delivery personnel, maintenance technicians, etc) and objects (cars, trucks, trailers, containers).

Moreover, user location information can be used to improve the way that services are implemented in areas such as quality of service, optimization of radio resources (handover and channel allocation) and pricing. Location-based pricing has been identified as one of the most promising applications in Operator Services in Europe.

Many other applications, such as vehicle fleet management, location sensitive billing, intelligent transport systems, fraud protection, and mobile yellow pages have driven the cellular industry to research new and promising technologies for MS positioning (Olama et al., 2008).

3. Location determination technologies and standards

Location Determination Technologies (LDTs) are the heart of LBSs. They are methods that use the signals of the cellular system to find the location of a mobile station, thus they are used to solve the so-called Automatic Location Identification (ALI) problem. Since cellular systems were not originally designed for positioning, the implementation of different location techniques may require various hardware and/or software modifications to the handset, network or both.

Based on the functions of the MS and the network, implementation of a location method belongs to one of the following categories (Cellular Location Technology, IST-2000-25382-CELLO Project, 2001): Network-based, Mobile-based, Mobile-assisted, and Network-assisted. In network-based implementation one or several base stations (BSs) make the necessary measurements and send the measurement results to a location centre where the position is calculated. Network-based implementation does not require any changes to existing handsets, which is a significant advantage compared to mobile-based or most mobile-assisted solutions.

In mobile-based implementation the MS makes measurements and position determination. This allows positioning in idle mode by measuring control channels which are continuously transmitted. Some assisting information, e.g. BS coordinates, might be needed from the network to enable location determination in the MS. Mobile-based implementation does not support legacy handsets. Mobile-assisted implementation includes solutions where the MS makes measurements and sends the results to a location centre in the network for further processing. Thus, the computational burden is transferred to a location centre where powerful processors are available. However, signaling delay and signaling load increase compared to a mobile-based solution, especially if the location result is needed at MS. Although mobile-assisted solutions typically do not support legacy handsets, it is possible to use the measurement reports that are continuously sent by handsets to the network in active mode. Last, network-assisted methods include those where the main functions take place at the MS but there is also some assistance from the network.

LDTs are mainly separated into two categories: Satellite and Cellular LDTs (Cellular Location Technology, IST-2000-25382-CELLO Project, 2001). Satellite LDTs (see Fig. 2) are based on the principle of measuring the interval of time a set of signals spend travelling from a set of orbiting satellites to a receiver on or near the surface of earth. The main satellite LDTs are GPS, AGPS, DGPS, GLONASS and Galileo.
On the other hand, Cellular LDTs, which are addressed in this chapter, refer to the set of location techniques used by cellular networks, i.e. these methods use the signals of the cellular system to find the location of a MS. The main Cellular LDTs are Cell-ID (or Cell Of Origin (COO)), Received Signal Level (RSL), Angle Of Arrival (AOA), Uplink Time Difference of Arrival (TDOA), and Downlink Observed Time Differences (Cellular Location Technology, IST-2000-25382-CELLO Project, 2001). Additionally, there have been reported many hybrid solutions of the preceding methods, as well as a Database Correlation Method and a Signal Pattern Recognition method. Most of these LDTs make use of the triangulation concept, that is, they calculate the most possible MS location based on existing signal information for known locations (such as BSs). Such signal information might be the signal level from/to a BS or the propagation time from/to a BS (so-called Time of Arrival (TOA)). It is not the purpose of this chapter to separately address each one of these methods. On the contrary, we will focus on the general characteristics of the main methods and point out advantages and disadvantages of each method.

The simplest method for locating a MS is Cell-ID. If someone knows the cell area in which the MS is being used, then the position of the BS antenna can be used as an estimate of the MS location as described in Fig. 3. An advantage of this method is that no calculations are needed to obtain location information. Thus, Cell-ID based location is fast and suitable for applications requiring high capacity. The drawback is that accuracy is directly dependent on cell radius, which can be very large especially in rural areas. Accuracy can be improved using information of cell coverage area (e.g. sector cells), timing advance (TA) in GSM or round trip time (RTT) in UMTS and Network Measurement Reports (NMRs).

Angle of Arrival (AOA) technique is based on angle-calculation of the signal as it arrives at a base station. This angle defines a line out of each BS. A minimum of two BSs is required to determine the position of the mobile phone, which is located at the intersection of these lines as shown in Fig. 4. The technique relies on the technology of antenna arrays. In an array, the antennas are separated by a small distance and a measurable difference in arrival times and electrical phase received at each antenna are used to estimate the direction at which the
transmission is originating. Achieved accuracy depends on the number of available measurements, geometry of BSs around the MS and multipath propagation. Since AOA method needs line-of-sight propagation conditions to obtain correct location estimates, it is clearly not the method of choice in dense urban areas where line of sight to two BSs is seldom present. A major barrier to implement AOA method in existing 2G networks is the need for an antenna array at each BS. It would be very expensive to build an overlay of AOA sensors to existing cellular network. In addition to financial issues, AOA method may have a capacity problem as it requires the co-ordination of almost simultaneous measurements at several BS sites. However, AOA surpasses in supporting legacy handsets.

Fig. 3. Cell-ID LDT

Fig. 4. AOA LDT

Signal time of arrival (TOA) measurements, performed either at the BSs or at the MS, can be used for positioning. Absolute TOA measurements are directly related to the BS-MS distances and three measurements are needed for unique 2-dimensional (2-D) location. However, if the BSs and the MS are not synchronized (do not have a common time reference, such as GSM and UMTS FDD networks), absolute TOA is difficult to measure, thus TOA measurements can only be used in differential manner. Two such measurements then define a hyperbola, and four measurements are needed for unambiguous 2-D location.
But, in any case (uplink or downlink), a common time reference (e.g. a GPS receiver) is needed at all BSs for accurate measurements. Consider that 1 microsecond error equals 300 meters measurement error. Additionally, in any case, location measurement units (LMUs) are required in the network (Symmetricom, 2002), which accurately measure TOA in Uplink measurements or exact OTD in Downlink measurements, process the measurements, receive measurement requests and provide measurement data. Even the inherent synchronization of a network (e.g. existing CDMA systems) is not adequate for location purposes, and this additional timing equipment is required. The required infrastructure has an important cost effect.

In uplink Time Difference of Arrival (TDOA), measurements are performed at the BSs, thus it’s a network-based technique. This technique has two drawbacks compared to downlink method: It is only possible to perform the measurements in dedicated mode and there may be capacity problems. The advantage is that due to the network-based implementation, uplink TOA supports legacy phones. The location of a MS is accomplished by forcing the MS to request a handover to several neighbouring BSs. The MS then sends access bursts and TOA measurements are made from these bursts. As mentioned above, LMUs accurately measure the arrival time of the bursts. In the downlink time difference techniques, the MS observes time differences of signals from several BSs. These signals are typically control channel signals and therefore the MS can perform the measurements in idle mode as well as in dedicated mode. The clock differences of the BSs can be solved by LMUs of known location which continuously measure the observed time differences.

The accuracy of all time difference based techniques depends on several factors. The accuracy of an individual time difference measurement depends on signal bandwidth and multipath propagation. When the signal bandwidth is not large enough the time resolution in timing measurements is not adequate for the needed accuracy. On the other hand, multipath propagation imposes a significant difficulty on finding the earliest arriving signal component. In GSM and UMTS standardization, the downlink techniques are called Enhanced Observed Time Differences (E-OTD) and Observed Time Difference of Arrival (OTDOA), respectively.

The LDT we chose to present in this chapter is the Received Signal Level (RSL) method which makes use of power signal information. Using signal strength from the control channels of several BSs, the distances between the MS and the BSs can be estimated using a suitable propagation model. Assuming 2-D geometry, an omni-directional BS antenna, and free-space propagation conditions, signal level contours around BSs are circles. If signal levels from three different BSs are known, the location of the MS can be determined as the unique intersection point of the three circles. However, practical propagation conditions especially in urban areas are far from free-space propagation. Therefore, an environment-dependent propagation models should be used. In urban areas the received signal level decreases more rapidly with distance than in open areas.

Multipath fading and shadowing poses a problem for distance estimation based on signal level. The instantaneous, narrowband signal level may vary by as much as 30-40 dB over a distance of only a fraction of the wavelength. Random variations of this order of magnitude cause very large errors in distance estimates. However, fast fading can be smoothed out by averaging the signal strength over time and frequency band. Time-averaging only has a minor effect, due to the motion in the surrounding environment, if the MS is stationary. Contrary to fast fading, the random variations caused by shadowing can not be
compensated. Thus, the variations in antenna orientation and local shadowing conditions around the MS (indoors, inside a vehicle etc.) are seen as random errors in distance estimates and consequently in position estimate. Location accuracy also depends on the accuracy of the propagation model and the number of available measurements.

Signal strength method is easy to implement in GSM, based on measurement reports that are continuously transmitted from the MS back to the network in active mode. Therefore, it does not require any changes to existing phones, and is often called a network-based method although it is the MS that performs the measurements. An alternative implementation is to modify the MSs to enable sending measurement reports in idle mode also. GSM phones with this capability are already available. An advantage of this technique is that in the GSM network, every MS measures the signal levels from up to seven BSs at 0.48-second intervals to facilitate handover. Signal strength is an easy and low-cost method to enhance the accuracy of pure cell-ID based location. Table 1 serves as a comparison of the above mentioned techniques.

<table>
<thead>
<tr>
<th>Method</th>
<th>Accuracy Urban</th>
<th>Accuracy Rural</th>
<th>Response</th>
<th>Costs (Operator)</th>
<th>Modifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cell-ID</td>
<td>Moderate</td>
<td>Poor</td>
<td>Fast</td>
<td>Low</td>
<td>-</td>
</tr>
<tr>
<td>RSL</td>
<td>Good</td>
<td>Poor</td>
<td>Fast</td>
<td>Low</td>
<td>-</td>
</tr>
<tr>
<td>TDOA</td>
<td>Good</td>
<td>Poor</td>
<td>Moderate</td>
<td>Moderate</td>
<td>-</td>
</tr>
<tr>
<td>E-OTD</td>
<td>Good</td>
<td>Poor</td>
<td>Moderate</td>
<td>Moderate/High</td>
<td>SW</td>
</tr>
<tr>
<td>AOA</td>
<td>Good</td>
<td>Poor</td>
<td>Moderate</td>
<td>High</td>
<td>-</td>
</tr>
<tr>
<td>GPS</td>
<td>Moderate</td>
<td>Good</td>
<td>Slow</td>
<td>Low</td>
<td>SW/HW4</td>
</tr>
</tbody>
</table>

Table 1. LDTs comparison (Charalambous & Panayiotou, 2004).

In conclusion, no single technique is superior in terms of accuracy, response delay, coverage, capacity, and implementation costs. The choice of a LDT will depend on the importance of each parameter to the decision maker. There are considerable obstacles to location estimation (Romdhani & Trad, 2002). Achieving accurate location of a mobile system remains a challenge considering the sources of error in location estimation. The main obstacles are multipath propagation, non-line-of-sight (NLOS) conditions, geometric dilution of precision (GDOP), and lack of bandwidth. In the next section, we describe the mathematical models used for the location and velocity estimation algorithms which are based on RSL method.

4. System mathematical models

4.1 The lognormal propagation channel model

Here we consider a 2-D geometry with the MS located at \((x_0, y_0)\) and the BSs located at \((x_{BS_b,1}, y_{BS_b,1}), (x_{BS_b,2}, y_{BS_b,2}), ..., (x_{BS_b,S}, y_{BS_b,S})\). The general lognormal propagation channel model is described by (Rappaport, 2002)

\[
PL_b^s(d_b) = PL(d_{0b}) + 10\varepsilon_b \log \left( \frac{d_b}{d_{0b}} \right) + X_b^s
\]

(1)

where \(d_b \geq d_{0b}, s \in \{1,2,..,S\}, b \in \{1,2,..,B\}\), \(PL_b^s(d_b)\) is the path loss (PL) from the \(bth\) BS at distance \(d_b\) for the \(sth\) sample, \(d_{0b}\) is the reference distance, \(\varepsilon_b\) is the path loss exponent and
$X^n_b - \mathcal{N}(0;\sigma_b^2)$ is a Gaussian random variable (RV) represents the shadowing variance due to gross variations in the terrain profile and changes in the local topography. The reference distance $d_{bb}$ is necessary since the equation of PL is not valid for zero distance. It depends on the cell size and can be calculated through the free-space PL or through measurements. Thus, the reference distance must be in the far field of the transmitting antenna, for the free-space propagation to be valid. The path loss exponent $\varepsilon_b$ indicates the rate at which the PL increases with distance and it depends on the specific propagation environment. For example, in free space $\varepsilon_b = 2$, and when obstructions are present $\varepsilon_b$ has a larger value as described in Table 2.

<table>
<thead>
<tr>
<th>ENVIRONMENT</th>
<th>PATH LOSS EXPONENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free- space</td>
<td>2</td>
</tr>
<tr>
<td>Urban area cellular radio</td>
<td>2.7 to 3.5</td>
</tr>
<tr>
<td>Shadowed urban cellular radio</td>
<td>3 to 5</td>
</tr>
<tr>
<td>In-Building Line-of-Sight</td>
<td>1.6 to 1.8</td>
</tr>
<tr>
<td>Obstructed in building</td>
<td>4 to 6</td>
</tr>
<tr>
<td>Obstructed in factory</td>
<td>2 to 3</td>
</tr>
</tbody>
</table>

Table 2. Path loss exponents for different propagation environments (Rappaport, 2002)

In cellular networks, the MS preserves and frequently updates, in idle and active mode, the received power of the strongest non-serving BSs (e.g., in GSM the 6 strongest (3GPP TS 05.08 V8.19.0, 2004)) in addition to the one of the serving cell. Exploiting these measurements from surrounding BSs lead to estimate the location of the MS. The MLE approach described in Section 5 that employs this channel model is used to estimate the MS location.

Note that this channel model assumes there is always a line-of-sight (LOS) between the transmitting and receiving antennas, which are not the case in common wireless systems such as urban environments. In the next section, we consider a more realistic channel model (Aulin’s scattering model), which takes into account the multipath properties and NLOS condition usually encountered in wireless networks.

### 4.2 Aulin’s scattering model

The basic 3-dimensional (3-D) wireless scattering channel model described by (Aulin, 1979), which assumes that the electric field, denoted by $E(t)$, at any receiving point $(x_0, y_0, z_0)$ is the resultant of $P$ plane waves (see Fig. 5), in which the receiver moves in the X-Y plane having velocity $v$ in a direction making an angle $\gamma$ with the X-axis, is given by

$$E(t) = \sum_{n=1}^{P} E_n(t) = \sum_{n=1}^{P} r_n \cos(\omega_n t + \omega_n t + \theta_n) + e(t)$$

(2)

where

$$\omega_n = \frac{2\pi v}{\lambda} (\cos(\gamma - \alpha_n) \cos \beta_n),$$

$$\theta_n = \frac{2\pi}{\lambda} (x_0 \cos \alpha_n \cos \beta_n + y_0 \sin \alpha_n \cos \beta_n + z_0 \sin \beta_n) + \phi_n$$

(3)
and \(\alpha_n, \beta_n\) are spatial angles of arrival, \(\omega_n\) is the Doppler shift, \(\theta_n\) is the phase shift, \(r_n\) is the amplitude, \(\phi_n\) is the phase of the \(n\)th component, \(\lambda\) is the wavelength, \(\epsilon(t)\) is a white Gaussian noise, and \(P\) is the total number of paths. It can be seen from (3) that the Doppler and phase shifts depend on the velocity and location of the receiver, respectively. Aulin’s model postulates knowledge of the instantaneous received field at the MS, which is obtained through the circuitry of the mobile unit. It takes into account NLOS condition as well as multipath propagation environments. Clearly, (2) assumes transmission of a narrowband signal. This assumption is valid only when the signal bandwidth is smaller than the coherence bandwidth of the channel. Nevertheless, the above model is not restrictive since it can be modified to represent a wideband transmission by including multiple time-delayed echoes. In this case, the delay spread has to be estimated. A sounding device is usually dedicated to estimating the time delay of each discrete path such as the Rake receiver (Sklar, 2001).

![Fig. 5. Aulin’s 3-D multipath channel model (Aulin, 1979)](image)

It can be seen that the noisy instantaneous received field in (2) depends parametrically on the location and velocity of the receiver. Consequently, this expression is used to estimate the MS location and velocity by using the EKF. Next, we formulate the location estimation as a filtering problem in state-space form (Kailath, 1976). The general form, once discretized, is given by

\[
\begin{align*}
x_k &= f(x_{k-1}, w_{k-1}) \\
z_k &= h(x_k, v_k)
\end{align*}
\]

(4)

where \(f(\cdot, \cdot)\) and \(h(\cdot, \cdot)\) are known vector functions, \(k\) is the estimation step, \(z_k\) are the output measurements at time step \(k\), and \(x_k\) is the system state at time step \(k\) and must not be confused with location coordinates. Further, \(w_k\) and \(v_k\) are the discrete zero-mean, independent state and measurement noise processes, with covariance matrices \(Q\) and \(R\), respectively.
Now let $\mathbf{x}_k = \begin{bmatrix} x_k, \dot{x}_k, y_k, \dot{y}_k \end{bmatrix}^T$ denote the state of the MS at time $k$, where $x_k$ and $y_k$ are the Cartesian coordinates of the MS, $\dot{x}_k$ and $\dot{y}_k$ are the velocities of the MS in the X and Y directions, respectively. If we choose the case where the velocity of the MS is not known and is subject to unknown accelerations, then the dynamics of the MS can be written as (Gustafsson et al., 2002)

$$
\begin{bmatrix}
  x_k \\
  \dot{x}_k \\
  y_k \\
  \dot{y}_k
\end{bmatrix}
= 
\begin{bmatrix}
  1 & \Delta_k & 0 & 0 \\
  0 & 1 & 0 & 0 \\
  0 & 0 & 1 & \Delta_k \\
  0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
  x_{k-1} \\
  \dot{x}_{k-1} \\
  y_{k-1} \\
  \dot{y}_{k-1}
\end{bmatrix}
+ 
\begin{bmatrix}
  \Delta_k^2 / 2 & 0 \\
  \Delta_k & 0 \\
  0 & \Delta_k^2 / 2 \\
  0 & \Delta_k
\end{bmatrix}
\begin{bmatrix}
  w_{k-1,1} \\
  w_{k-1,2}
\end{bmatrix}
$$

(5)

where $\Delta_k$ is a (possibly non-uniform) measurement interval between time $k - 1$ and $k$. The measurement equation can be found from Aulin’s scattering model (2) and (3), which can be written in discrete form as

$$
z_k = h(\mathbf{x}_k, v_k) = \sum_{n=1}^{p} r_{n_k} \cos(\omega_n t_k + \omega_n t_k + \theta_n) + \nu(t_k)
$$

(6)

where

$$
\omega_n = \frac{2\pi}{\lambda} \sqrt{\dot{x}_k^2 + \dot{y}_k^2} \left( \cos(y_k - \alpha_n) \cos \beta_n \right),
$$

$$
\theta_n = -\frac{2\pi}{\lambda} \left( x_k \cos \alpha_n \cos \beta_n + y_k \sin \alpha_n \cos \beta_n + z_k \sin \beta_n \right) + \phi_n
$$

(7)

Clearly, the measurement equation $h(.,.)$ is a nonlinear function of the state-space vector, as observed in (6) and (7). If we assume perfect knowledge of the channel, which is attainable either through channel estimation at the receiver (e.g., GSM receiver), or through various estimation techniques (e.g., least-squares, ML), then this problem falls under the broad area of nonlinear parameter estimation from noisy data which can be solved using the EKF as described in Section 6. The MLE algorithm that employs the lognormal propagation channel model is discussed in the next section.

5. RSL location estimation via MLE

The main idea of the algorithm described in this section is to use MLE with the distances of the MS from the BSs as parameters. That is, based on the power measurements, which constitute the experiment sample, we will calculate the distances that maximize the likelihood function. As the size of the sample increases, the accuracy of the estimation increases. Further, triangulation is performed for the most possible MS location.

5.1 MLE Theory

Consider a random sample of the measured quantity $X = X_1, X_2, \ldots, X_N$ and let $\theta$ be the parameter to be estimated. The likelihood function

$$
L(X; \theta) = f(X_1, X_2, \ldots, X_N, \theta)
$$

(8)
is another way of writing the probability density function (PDF), but the observations are fixed and the parameter is freely varying. Thus, the likelihood function reflects the likelihood of a given $X$ arising for different values of $\theta$. Given the sample, we are looking for that parameter value $\hat{\theta}$ that maximizes the likelihood of the sample occurrence as (Eliason, 1993)

$$\hat{\theta} = \arg \max \theta L(X; \theta)$$

(9)

For convenience, we can maximize the so-called log-likelihood function $\log L(X; \theta)$. The derivative of the log-likelihood function

$$\frac{\partial}{\partial \theta} \left( \log L(X; \theta) \right)$$

(10)

is called score function. The score function must satisfy the sufficient first and second order conditions for a maximum.

### 5.2 Location estimation via MLE

In this section, the MLE method that employs the lognormal propagation channel model described in section 4.1 is considered for the MS location estimation (Olama et al., 2008). This method exploits the received power measurements at the MS which are available from network measurement reports (NMRs). Thus, we write the likelihood function and then maximize it with respect to the distances $\theta = d = (d_1, d_2, \ldots, d_B)$ from each BS, where $\theta$ is the parameter to be estimated. The ML estimator, denoted by $\hat{\theta} = \hat{d} = (\hat{d}_1, \hat{d}_2, \ldots, \hat{d}_B)$, represents the most possible MS/BS distances based on the measurements available at the MS.

Consider the measurement vector for the $s$th sample from all BSs, denoted by $PL'(d) = (PL'_1(d_1), PL'_2(d_2), \ldots, PL'_B(d_B))$. The distribution function for this vector is the $B$-variate normal distribution given by

$$p(PL'(d) | \theta) = (2\pi)^{-B/2} \left( \det(\Sigma_s) \right)^{-1/2} \exp \left\{ -\frac{1}{2} \left( PL'(d) - \overline{PL'}(d) \right)^T \Sigma_s^{-1} \left( PL'(d) - \overline{PL'}(d) \right) \right\}$$

(11)

where $\overline{PL'}(d) - N_B\left( \overline{PL'}(d) ; \Sigma_s \right)$, $\overline{PL'}(d) = (\overline{PL'_1}(d_1), \overline{PL'_2}(d_2), \ldots, \overline{PL'_B}(d_B))$ is the mean path loss from each BS, and $\Sigma_s$ is the covariance matrix. Assuming the noise is independent identically distributed (iid), then the logarithm likelihood function is the log product of the sample likelihood functions given by

$$L(\theta | PL'(d)) = \log \left( \frac{1}{(2\pi)^{S/2} \left( \det(\Sigma_s) \right)^{S/2}} \right) - \sum_{s=1}^{S} \left( PL'(d) - \overline{PL'}(d) \right)^T \Sigma_s^{-1} \left( PL'(d) - \overline{PL'}(d) \right)$$

(12)

where $S$ is the total number of samples. Maximizing (12) first with respect to $\overline{PL'}(d)$, the score function yields

$$\overline{PL'}_b(d_b) = \frac{1}{S} \sum_{s=1}^{S} PL'_s(d_b), \quad \forall b \in \{1, 2, \ldots, B\}$$

(13)
Solving for \( \hat{d} \) using the invariance property of the MLE (Zehna, 1966), it can be shown that

\[
\hat{d}_b = 10 \exp \left\{ \frac{1}{10 \epsilon_b} \left[ \frac{1}{S} \sum_{s=1}^{S} PL_s(d_b) - PL(d_0) \right] \right\}
\]

is the MLE for the distance of the \( b \)th BS from the MS. Next, we perform triangulation using the least squares error method (Wong et al., 2000) to estimate the MS location \((x_0, y_0)\), by solving

\[
\arg \min_{x_0, y_0} \left\{ \sum_{b=1}^{B} (d_b - \hat{d}_b)^2 \right\}
\]

5.3 Numerical results

In this example we employ a typical, yet realistic, simulation setup. The service area consists of a 19 cell cluster as configured in Fig. 6. The BSs are placed over a uniform hexagonal pattern of cells which are centrally equipped with omni-directional antennas. MSs are placed randomly in the central cell and the number of arranged users is 1000. The type of used environments is designated by the values of \( d_b, \sigma_b, \epsilon_b \) (all previously defined in section 4.1) and cell radii \( R_b \).

Path-loss exponent \( \epsilon_b \) and path-loss variance values \( \sigma_b^2 \) were taken the same (\( \sigma_b = 8 \text{ dB}, \epsilon_b = 3.5, \) for all \( b \) (ETSI TR 101 115 V8.2.0 (2000-04), Annex V.A)), though cell radii \( R_b \) and reference distance \( d_0 \) values are different for urban and suburban environments having values \( R_b = 500, 2500 \text{m}, \) \( d_0 = 50, 100 \text{m}, \) respectively (ETSI TR 101 115 V8.2.0 (2000-04), Annex V.A)). The number of samples \( S \) is 20, the number of BSs for triangulation is 3 to 7 and the radio-frequency is 900MHz. We illustrate the 67% and 95% cumulative distribution function (CDF) values for urban and suburban environments and different number of BSs. E.g., a 67% CDF value \( X \) (meters) is equal to the probability \( \Pr(\text{Error} < X(\text{meters})) = 0.67 \).

Fig. 6. Configuration of the cell arrangement

Fig. 7 shows numerical results in urban and suburban environments, respectively. It is observed that in urban environments the method’s accuracy is below the FCC mandate for
network-based solutions, and accuracy is improved as more BSs are incorporated. Results in suburban environments are also satisfactory; however accuracy degrades as the cell radii increases. This is due to the increasing error imposed by triangulation. It has been also observed that the accuracy increases as the number of samples, $S$, increases and as $\sigma_b$ and $\varepsilon_b$ decrease, as expected.

Fig. 7. 67% and 95% CDF of the ML estimate and 3 to 7 BSs for (a) Urban and (b) Suburban environments (Papageorgiou et al., 2005)

The maximum likelihood function concept is illustrated in Fig. 8. Instead of the separate likelihood functions, the overall likelihood function is considered. We show graphs of the likelihood function for one, two and three BSs. For one BS, the likelihood function is a 3-D Gaussian PDF which is maximized for distance $\hat{d}_1$ as shown in Fig. 8(a). For two BSs, the
Fig. 8. The overall likelihood function for (a) 1 BS, (b) 2 BSs, and (c) 3 BSs (Charalambous & Panayiotou, 2004)
likelihood function is the product of two such 3-D Gaussians with centres located at the BSs co-ordinates. This product yields two vertices at the intersection points of the likelihood functions as shown in Fig. 8(b). The true location is more likely to be near these vertices. With three BSs, there is only one vertex which represents the most probable MS location as shown in Fig. 8(c). In realistic NLOS and multipath conditions the method will not perform so well. Nevertheless, it can be used as the initial estimator for the EKF approach, discussed in the next section, to find a more accurate estimator. The EKF approach that employs the channel model of Aulin to estimate the MS location and velocity is discussed in the next section.

6. RSL location estimation via the EKF

Consider the general discrete-time dynamical system model described in (4) and rewritten herein as

$$x_k = f(x_{k-1}, w_{k-1})$$
$$z_k = h(x_k, v_k)$$

(16)

where $f(., .)$ and $h(., .)$ are known vector functions, $k$ is the estimation step, $z_k$ are the output measurements at time step $k$, and $x_k$ is the system state at time step $k$. Further, $w_k$ and $v_k$ are the discrete zero-mean, independent state and measurement noise processes, with covariance matrices $Q$ and $R$, respectively, and are assumed to be mutually independent. The set of entire measurements from the initial time step to time step $k$ is denoted by $Z_k = \{z_i\}_{i=1}^k$. The initial state of the system $x_0$ is given as a Gaussian random vector, with mean $m_0$ and covariance $V_0$, with $V_0$ symmetric and positive definite.

6.1 The EKF theory

The EKF (Anderson & Moore, 1979) is based on linearizing the nonlinear system models around the previous estimate. In other words, we only consider a linear Taylor approximation of the system function at the previous state estimate and that of the observation function at the corresponding predicted position. This approach gives a simple and efficient algorithm to handle a nonlinear model. However, convergence to a reasonable estimate may not be obtained if the initial guess is poor or if the disturbances are so large that the linearization is inadequate to describe the system. The general algorithm for the discrete EKF can be described by the time-update equations given as (Bishop & Welch, 2003)

$$\tilde{x}_k = f(\tilde{x}_{k-1}, 0)$$
$$\tilde{P}_k = A_k \tilde{P}_{k-1} A_k^T + W_k Q_k W_k^T$$

(17)

and the measurement-update equations given as

$$K_k = \tilde{P}_k H_k^T \left[ H_k \tilde{P}_k H_k^T + V_k R_k V_k^T \right]^{-1}$$
$$\hat{x}_k = \tilde{x}_k + K_k (z_k - h(\tilde{x}_k, 0))$$
$$\hat{P}_k = (I - K_k H_k) \tilde{P}_k$$

(18)

where
\[ A_k = \frac{\partial f}{\partial x}(\hat{x}_{k-1}, 0), \quad W_k = \frac{\partial f}{\partial w}(\hat{x}_{k-1}, 0), \]
\[ V_k = \frac{\partial h}{\partial v}(\hat{x}_k, 0), \quad H_k = \frac{\partial h}{\partial x}(\hat{x}_k, 0) \]

\( K \) is the gain matrix and \( P \) the estimation error covariance. The notation \( \hat{x}_k \) denotes the a priori state estimate at step \( k \) and \( \hat{x}_k \) the a posteriori state estimate given measurement \( z_k \). \( \hat{P}_k \) and \( \hat{P}_k \) are defined similarly.

### 6.2 Location estimation via the EKF

In this section, we employ the wave scattering model of Aulin (described in Section 4.2) in the EKF framework. We apply the general algorithm for the discrete EKF in (17)-(19) to our system model in (5)-(7) in which the state equation, \( x_k = f(x_{k-1}, w_{k-1}) \), is represented by the dynamics of the MS described in (5) and the measurement equation, \( z_k = h(x_k, v_k) \), is represented by the discrete-time Aulin’s scattering model described in (6) and (7). The result is given as (Olama et al., 2008)

\[ \hat{x}_k = A_k \hat{x}_{k-1} \]
\[ \hat{P}_k = A_k \hat{P}_{k-1} A_k^T + W_k Q_k, \quad W_k = \left[ \begin{array}{c} \Delta_k^2 / 2 \\ \Delta_k \\ 0 \\ 0 \\ 0 \end{array} \right] \]
\[ \hat{x}_k = \hat{x}_k + K_k (z_k - h(\hat{x}_k, 0)) \]
\[ \hat{P}_k = (1 - K_k H_k) \hat{P}_k \]

where

\[ A_k = \left[ \begin{array}{cccc} 1 & \Delta_k & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta_k \\ 0 & 0 & 0 & 1 \end{array} \right], \quad W_k = \left[ \begin{array}{c} \Delta_k^2 / 2 \\ \Delta_k \\ 0 \\ 0 \end{array} \right], \quad V_k = \left[ \begin{array}{c} 1 \\ \Delta_k \end{array} \right] \]

\[ H_k = \frac{\partial h}{\partial x}(\hat{x}_k, 0) = \left[ \begin{array}{c} H1_k \\ H2_k \\ H3_k \\ H4_k \end{array} \right], \]

\[ H1_k = \sum_{n=1}^{p} r_n \sin(\omega_n t_k + \omega_n t_k + \theta_n) \left( \frac{2\pi}{\lambda} \cos(\alpha_n) \cos(\beta_n) \right) \]

\[ H2_k = \sum_{n=1}^{p} -r_n \sin(\omega_n t_k + \omega_n t_k + \theta_n) \left( \frac{2\pi t}{\lambda \sqrt{\hat{x}_k^2 + \hat{y}_k^2}} \cos(\beta_n) \right) \]

\[ H3_k = \sum_{n=1}^{p} r_n \sin(\omega_n t_k + \omega_n t_k + \theta_n) \left( \frac{2\pi}{\lambda} \sin(\alpha_n) \cos(\beta_n) \right) \]

\[ H4_k = \sum_{n=1}^{p} -r_n \sin(\omega_n t_k + \omega_n t_k + \theta_n) \left( \frac{2\pi t}{\lambda \sqrt{\hat{x}_k^2 + \hat{y}_k^2}} \cos(\beta_n) \right) \]

\[ \left( \hat{y}_k \cos(\gamma_k - \alpha_n) - \hat{x}_k \sin(\gamma_k - \alpha_n) \right) \]
and $\gamma_k = \arctan(\hat{y}_k / \hat{x}_k)$. As in any nonlinear estimation problem, the convergence of the EKF to the true value of the location depends on the initial parameter value; therefore we first develop the MLE approach to obtain an initial estimator of adequate accuracy for the EKF. This hybrid algorithm, as numerical results indicate in the next section, has improved accuracy for the final MS location estimate.

The EKF approach takes into account NLOS condition as well as multipath propagation environments. It requires only one BS to estimate the MS location instead of at least three BSs as found in the MLE approach and in the literature (Hellebrandt & Scheibenbogen, 1999). However, an initial MS location estimate that requires at least three BSs, such as the MLE and triangulation method, will improve the convergence of the EKF.

7. Numerical results

In this numerical example, the EKF is employed utilizing the initial estimate as the first location estimate. Specifically, the first MLE algorithm passes its estimate to the EKF algorithm for final estimation and more precise location estimates. It will be shown that this approach corrects the initial estimate of the ML at a high level of accuracy.

The simulation setup for the initial estimate (MLE approach) remains the same; only now the number of BSs for triangulation is typically 5, and we are trying to locate a single MS. However, the simulated environment is determined by the environment-dependent parameters. All urban, suburban and rural ($R_b = 15000m$, $d_{0b} = 500m$ (ETSI TR 101 115 V8.2.0 (2000-04), Annex V.A.) environments have been considered, though only results for the rural case are illustrated due to space limitations. We choose the case when the velocity is not known to estimate the final location and velocity of the moving MS, and for simplicity, we assume zero acceleration.

As previously stated, we assume adequate channel knowledge, i.e., $\alpha_n$, $\beta_n$, $\phi_n$, $r_n$ are known. The number of paths $P$ and the distributions of the envelopes $r_n$ in (2) depend on the considered environment (Parsons, 1992). For urban areas $P \geq 6$ and the envelopes are Rayleigh distributed due to NLOS conditions. In urban and suburban areas typical values of $P$ are 2-6 and the envelopes are taken from the Nakagami distribution with appropriate parameter value (ETSI TR 101 115 V8.2.0 (2000-04), Annex V.A.). Lastly, $f_c = 2000Hz$ for simulation reasons.

Considering the rural environment, Fig. 9 illustrates the convergence of EKF to the real position and velocity of a moving MS. The relevant values are marked on the figure; these are the real position, estimated position, initial and final estimate errors. We observe that the final estimator is of high accuracy, if the initial estimate is used as the initial state of the algorithm. Specifically, the accuracy is below 10m most of the time; here it is 1.4m in comparison to the initial estimate accuracy of 1151m. This is due to the appropriateness of Aulin’s channel model and the efficiency of Kalman Filtering in this particular application.

In suburban and rural environments the results are even better, as expected. Moreover, it has been observed that the consistency and performance of the method are very high.

Fig. 10 illustrates the 67% and 95% CDF values of the final estimate in rural environment (the worst case of all), for different number of BSs used in triangulation. It is clear that the EKF approach achieves the FCC mandates for network-based solutions. The high accuracy, consistency and performance of the method, makes it suitable to be used in any LBSs, and particularly those which require high accuracy, such as emergency services.
Fig. 9. Mobile (a) location and (b) velocity estimation in rural environment (Papageorgiou et al., 2005)
8. Conclusion

In this chapter, two estimation approaches are introduced to track the position and velocity of a MS in a cellular network. They are based on lognormal shadowing and Aulin’s scattering models combined with the MLE and the EKF estimation algorithms, respectively. According to Aulin’s channel model, the instantaneous electric field is a nonlinear function of the MS location and velocity. Consequently, the EKF is employed for the estimation process. Since the EKF approach is sensitive to the initial condition, we propose to use the ML estimate that employs the lognormal channel model, as the initial EKF state. Numerical results for typical simulations show that they are highly accurate and consistent. These methods also excel in using inherent features of the cellular system, i.e., they support existing network infrastructure and channel signalling. The assumptions are knowledge of the channel and access to the instantaneous received field, which are obtained through channel sounding samples from the receiver circuitry. Future work will focus on generating efficient channel estimation algorithms, to remove the assumption on partial knowledge of the channel. Work on building a pilot application to test the performance of the EKF in realistic conditions is on-going together with the incorporation of channel model parameters estimation algorithms. Another direction in future work is to use more advanced filtering techniques such as the unscented Kalman filter (Julier & Uhlmann, 1997) and the particle filter (Arulampalam et al., 2002), which are not based upon the principal of linearising the nonlinear state and measurement models using Taylor series expansions as the EKF. Some preliminary results for MS location and velocity estimation algorithm based on particle filtering are presented in (Olama et al., 2007; Olama et al., 2008).

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10. References


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The aim of this book is to provide an overview of recent developments in Kalman filter theory and their applications in engineering and scientific fields. The book is divided into 24 chapters and organized in five blocks corresponding to recent advances in Kalman filtering theory, applications in medical and biological sciences, tracking and positioning systems, electrical engineering and, finally, industrial processes and communication networks.

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