Resolved Acceleration Control for Underwater Vehicle-Manipulator Systems: Continuous and Discrete Time Approach

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1. Introduction

Underwater robots, especially Underwater Vehicle-Manipulator Systems (UVMS), are expected to have important roles in ocean exploration (Yuh, 1995). Many studies about dynamics and control of UVMS have been reported (Maheshi et al., 1991; McMillan et al., 1995; McLain et al., 1996; Tarn et al., 1996; Antonelli & Chiaverini, 1998; McLain et al., 1998; Antonelli et al., 2000; Sarkar & Podder, 2001). However, there are only a few experimental studies. Most of the control methods of UVMS have been proposed based on the methods of Autonomous Underwater Vehicles. In these control methods, the desired accelerations and velocities of the end-tip of the manipulator are transformed to the desired manipulator’s joint accelerations and velocities only use of the kinematic relation, and the computed torque method with joint angle and angular velocity feedbacks are utilized. In other words, the control methods use errors consisting of task-space signals of vehicle and joint-space signals of manipulator. Therefore, the control performance of the end-effector depends on the vehicle’s control performance.

We have proposed continuous-time and discrete-time Resolved Acceleration Control (RAC) methods for UVMS (Yamada & Sagara, 2002; Sagara, 2003; Sagara et al., 2004; Sagara et al., 2006; Yatoh & Sagara, 2007; Yatoh & Sagara, 2008). In our proposed methods, the desired joint values are obtained by kinematic and momentum equations with feedback of task-space signals. From the viewpoint of underwater robot control, parameters and coefficients of hydrodynamic models are generally used as constant values that depend on the shape of the robots (Fossen, 1994). Our proposed methods described above can reduce the influence of the modelling errors of hydrodynamics by position and velocity feedbacks. The effectiveness of the RAC methods has been demonstrated by using a floating underwater robot with vertical planar 2-link manipulator shown in Figure 1.

In this chapter, our proposed continuous-time and discrete-time RAC methods are described and the both experimental results using a 2-link underwater robot are shown. First, we explain about a continuous-time RAC method and show that the RAC method has good control performance in comparison with a computed torque method. Next, to obtain higher control performance, we introduce a continuous-time RAC method with disturbance compensation. In practical systems digital computers are utilized for controllers, but there is no discrete-time control method for UVMS except our proposed methods. Then, we address
discrete time RAC methods including the ways of disturbance compensation and avoiding singular configuration.

Fig. 1. Vertical type 2-link underwater robot

2. Modelling

The UVMS model used in this chapter is shown in Figure 2. It has a robot base (vehicle) and an $n$-DOF manipulator.

Fig. 2. Model of underwater robot with n-link manipulator

The symbols used in this chapter are defined as follows:

- $n$: number of joints
- $\Sigma$: inertial coordinate frame
- $\Sigma_i$: link $i$ coordinate frame ($i = 0, 1, 2, \ldots, n$; link 0 means the vehicle)
- $^1R_i$: coordinate transformation matrix from $\Sigma_i$ to $\Sigma$
- $p_e$: position vector of the end-tip of the manipulator with respect to $\Sigma$
- $p_i$: position vector of the origin of $\Sigma_i$ with respect to $\Sigma$
- $r_i$: position vector of the center of gravity of link $i$ with respect to $\Sigma_i$
\( \phi_i \): relative angle of joint \( i \)

\( \Psi_0 \): roll-pitch-yaw attitude vector of \( \Sigma_0 \) with respect to \( \Sigma_i \)

\( \Psi_e \): roll-pitch-yaw attitude vector of the end-tip of the manipulator with respect to \( \Sigma_i \)

\( \omega_0 \): angular velocity vector of \( \Sigma_0 \) with respect to \( \Sigma_i \)

\( \omega_e \): angular velocity vector of the end-tip of the manipulator with respect to \( \Sigma_i \)

\( \phi \): relative joint angle vector \( (\phi_1 \ldots \phi_n)^T ) \)

\( i_k \): unit vector indicating a rotational axis of joint \( i \) \( (=[0 \ 0 \ 1]^T) \)

\( m_i \): mass of link \( i \)

\( iM_a \): added mass matrix of link \( i \) with respect to \( \Sigma_i \)

\( iI_i \): inertia tensor of link \( i \) with respect to \( \Sigma_i \)

\( iI_{ai} \): added inertia tensor of link \( i \) with respect to \( \Sigma_i \)

\( x_0 \): position and orientation vector of \( \Sigma_0 \) with respect to \( \Sigma_i \) \( (=[r_0^T \ \Psi_0^T]^T) \)

\( x_e \): position and orientation vector of the end-tip with respect to \( \Sigma_i \) \( (=[p_e^T \ \Psi_e^T]^T) \)

\( \nu_0 \): linear and angular vector of \( \Sigma_0 \) with respect to \( \Sigma_i \) \( (=[\dot{r}_0^T \ \omega_0^T]^T) \)

\( \nu_e \): linear and angular vector of the end-tip with respect to \( \Sigma_i \) \( (=[\dot{p}_e^T \ \omega_e^T]^T) \)

\( l_i \): length of link \( i \)

\( a_{gi} \): position vector from joint \( i \) to the center of gravity of link \( i \) with respect to \( \Sigma_i \)

\( a_{bi} \): position vector from joint \( i \) to the buoyancy center of link \( i \) with respect to \( \Sigma_i \)

\( D_i \): width of link \( i \)

\( V_i \): volume of link \( i \)

\( \rho \): fluid density

\( C_D_i \): drag coefficient of link \( i \)

\( g \): gravitational acceleration vector

\( E_j \): \( j \times j \) unit matrix

\( \tilde{\cdot} \): tilde operator stands for a cross product such that \( \tilde{r}a = r \times a \)

### 2.1 Kinematics

First, from Figure 2 a time derivative of the end-tip position vector \( \dot{p}_e \) is

\[
\dot{p}_e = \dot{r}_0 + \dot{\omega}_0 (p_e - r_0) + \sum_{i=1}^{n} \left[ k_i (p_e - p_i) \right] \dot{\phi}_i
\]

(1)

where \( k_i = I_j \ k_i \).

On the other hand, relationship between end-tip angular velocity and joint velocity is expressed with

\[
\omega_e = \omega_0 + \sum_{i=1}^{n} k_i \dot{\phi}_i.
\]

(2)
From Equations (1) and (2) the following equation is obtained:

$$\mathbf{v}_e = A \mathbf{v}_0 + B \dot{\phi}$$

where

$$A = \begin{bmatrix} E_3 & \tilde{p}_e - \tilde{r}_0 \\ 0 & E_3 \end{bmatrix},$$

$$B = \begin{bmatrix} k_1 (p_e - p_1) \\ k_2 (p_e - p_2) \\ \vdots \\ k_n (p_e - p_n) \end{bmatrix}.$$  

Next, let \( \eta \) and \( \mu \) be a linear and an angular momentum of the robot including hydrodynamic added mass tensor \( i M_{a_i} \) and added inertia tensor \( i I_{a_i} \) of link \( i \). Then

$$\eta = \sum_{i=0}^{n} M_{T_i} \dot{r}_i,$$  

$$\mu = \sum_{i=0}^{n} \left( I_{T_i} \omega_i + \tilde{r}_i M_{T_i} \dot{r}_i \right) - \mathbf{r}_0 \times \eta$$

where \( M_{T_i} = m_i E_3 + i M_{a_i} i R_i \) and \( I_{T_i} = i I_i (i I_i + i I_{a_i}) i R_i \). Here, linear and angular velocities of the center of gravity of link \( i \) are described as

$$\dot{r}_i = \dot{r}_0 + \tilde{a}_0 (r_i - r_0) + \sum_{j=1}^{i} \left( \tilde{k}_j (r_i - p_j) \right) \dot{\phi}_j,$$

$$\omega_i = \omega_0 + \sum_{j=1}^{i} k_j \dot{\phi}_j.$$  

Therefore, the following equation is obtained from Equations (4)-(7):

$$s = \begin{bmatrix} \eta \\ \mu \end{bmatrix} = C \mathbf{v}_0 + D \dot{\phi}$$

where

$$C = \begin{bmatrix} \sum_{i=0}^{n} M_{T_i} & -\sum_{i=0}^{n} M_{T_i} (\tilde{r}_i - \tilde{r}_0) \\ \sum_{i=0}^{n} (\tilde{r}_i - \tilde{r}_0) M_{T_i} & \sum_{i=0}^{n} (I_{T_i} - (\tilde{r}_i - \tilde{r}_0) M_{T_i} (\tilde{r}_i - \tilde{r}_0)) \end{bmatrix},$$

$$D = \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1n} \\ d_{21} & d_{22} & \cdots & d_{2n} \end{bmatrix}.$$
Here, we assume that the added mass and added inertia are constant. In reality, the added mass and inertia are variable, but the influence of the variation is compensated by a control method given in the following section.

### 2.2 Equation of motion

First, the drag force and the moment of joint $i$ can generally be represented as follows (Levesque & Richard, 1994):

$$f_{d_i} = \frac{\rho}{2} C_{D_i} D_i^l I_{i0}^l \left\| \omega_i \right\| \omega_i dx_i, \quad (9)$$

$$t_{d_i} = \frac{\rho}{2} C_{D_i} D_i^l I_{i0}^l \hat{x}_i \times \left\| \omega_i \right\| \omega_i dx_i \quad (10)$$

where

$$w_i = \begin{bmatrix} 0 & 0 \\ 0 & E_2 \end{bmatrix} R_i (\dot{\psi}_i + \ddot{\alpha}_i \dot{x}_i)$$

and

$$\hat{x}_i = [x_i \ 0 \ 0]^T.$$  

Next, the gravitational and buoyant forces acting on link $i$ are described as

$$f_{g_i} = (\rho V_i - m_i) g, \quad (11)$$

$$t_{g_i} = (\rho V_i \ddot{\alpha}_b_i - m_i \ddot{\alpha}_g_i) g. \quad (12)$$

Considering the hydrodynamic forces described above and using the Newton-Euler formulation, the following equation of motion can be obtained (Antonelli, 2003):

$$M(q) \ddot{\zeta} + N(q, \zeta) \zeta + f = u \quad (13)$$

where

$$q = \begin{bmatrix} x_0 \\ \phi \end{bmatrix}, \quad \zeta = \begin{bmatrix} v_0 \\ \phi \end{bmatrix}, \quad u = \begin{bmatrix} f_B \\ r_B \\ r_M \end{bmatrix},$$

and $M$ is the inertia matrix including the added mass $^{i} M_a$ and inertia $^{i} I_a$, $N \zeta$ is the vector of the Coriolis and centrifugal forces, $f$ is the vector consisting of the drag and gravitational and buoyant forces and moments, $f_B$ and $r_B$ are the force and torque vectors of the vehicle, respectively, and $r_M$ is the joint torque vector of manipulator. Moreover, the relationship between $\omega_*$ and $\psi_* = [\psi_{r_\ast} \ \psi_{p_\ast} \ \psi_{g_\ast}]^T \ (\ast = 0, e)$ is described as
\[ \bm{\omega}_* = S_{\psi_*} \dot{\psi}_* \]  
\[ S_{\psi_*} = \begin{bmatrix} \cos\psi_p \cos\psi_y & -\sin\psi_y \\ \cos\psi_p \sin\psi_y & \cos\psi_y \\ \sin\psi_p & 0 \end{bmatrix}. \]

Thus, the relationship between \( \dot{q} \) and \( \zeta \) is described as
\[ \zeta = \begin{bmatrix} E_3 \\ S_{\psi_0} \\ 0 \end{bmatrix} \dot{q}. \]

### 3. Continuous-time RAC

#### 3.1 RAC law

Differentiating Equations (3) and (8) with respect to time, the following equation can be obtained:
\[ W(t) \bm{\alpha}(t) = \bm{\beta}(t) + \gamma(t) - \dot{W}(t) \zeta(t) \]  
where
\[ W = \begin{bmatrix} C + E_6 \\ A \\ D \end{bmatrix}, \quad \alpha = \dot{\zeta}, \quad \beta = \begin{bmatrix} \dot{v}_0 \\ \dot{v}_e \end{bmatrix}, \quad \gamma = \begin{bmatrix} \dot{s} \\ 0 \end{bmatrix}, \]

and \( \dot{s} \) is the external force, including the hydrodynamic force and the thrust of the thruster, which acts on the vehicle.

For Equation (16), the reference acceleration is defined as
\[ \bm{\alpha}^\text{ref} = \begin{bmatrix} v_0^\text{ref} \\ \phi_0^\text{ref} \end{bmatrix} = W^\#(t)(\bm{\beta}^\text{ref} + \gamma(t) - \dot{W}(t)\zeta(t)) \]

where
\[ \bm{\beta}^\text{ref} = \begin{bmatrix} \dot{r}_0^\text{des} \\ \dot{\omega}_0^\text{des} \\ \dot{p}_e^\text{des} \end{bmatrix} + K_V \begin{bmatrix} \dot{r}_0^\text{des} - \dot{r}_0 \\ \dot{\omega}_0^\text{des} - \dot{\omega}_0 \\ \dot{p}_e^\text{des} - \dot{p}_e \end{bmatrix} + K_p \begin{bmatrix} e_{\phi_0} \\ e_{\omega_0} \\ e_{p_e} \end{bmatrix}, \]

and \( W^\# \) is the pseudoinverse of \( W \), i.e. \( W^\# = W^T(WW^T)^{-1} \), and \( \dot{\ast} \) is the desired value of \( \ast \), \( K_V \) and \( K_p \) are diagonal matrices consisting of scalar gain constants. Moreover,
\[ e_{\omega_0} = \begin{bmatrix} \frac{1}{2} \left( i_s \times i_s_d + j_s \times j_s_d + k_s \times k_s_d \right) \ast \end{bmatrix} (\ast = 0, e) \]
where \( i^*, j^*, k^* \) are unit vectors along the axes of \( \Sigma_s \) with respect to \( \Sigma_I \), and these vectors can be obtained from the rotational matrix (Luh et al., 1980):

\[
[i^* \ j^* \ k^*] = R_s.
\]

Using Equations (13) and (17) actual control input for UVMS is calculated by

\[
u = M(q)\dot{\alpha}^{ref} + N(q, \zeta)\dot{\zeta} + f.
\] (20)

Here, we represent the matrices and vectors of Equation (13) to the block matrix form:

\[
M = \begin{bmatrix} M_{BB} & M_{BM} \\ M_{MB} & M_{MM} \end{bmatrix}, \quad N = \begin{bmatrix} N_{BB} & N_{BM} \\ N_{MB} & N_{MM} \end{bmatrix}, \quad f_D = \begin{bmatrix} f_B \\ f_M \end{bmatrix}, \quad u = \begin{bmatrix} u_B \\ u_M \end{bmatrix}.
\]

Then we have the following equation with respect to the input of the vehicle:

\[
M_{BB}\ddot{v}_0 + M_{BM}\dot{\phi} + N_{BB}\dot{v}_0 + M_{BM}\dot{\phi} + f_B = u_B.
\] (21)

And the time derivative of Equation (8) is

\[
\dot{s} = C\dot{v}_0 + D\dot{\phi} + \dot{C}\dot{v}_0 + \dot{D}\dot{\phi}.
\] (22)

Comparing with Equations (21) and (22), \( C = M_{BB}, \quad D = M_{BM}, \quad \dot{C} = N_{BB}, \quad \dot{D} = N_{BM} \) and \( \dot{s} = u_B - f_B \) are obtained. Moreover,

\[
\dot{A} = \begin{bmatrix} 0 & -\left(\bar{p}_c - \bar{r}_0\right) \\ 0 & 0 \end{bmatrix},
\]

\[
\dot{B} = \begin{bmatrix} b_1 & b_2 & \cdots & b_n \\ \omega_1 \times k_1 & \omega_1 \times k_2 & \cdots & \omega_1 \times k_n \end{bmatrix}
\]

where \( b_i = (\omega_i \times k_i) \times (p_c - p_i) + \bar{k}_i (\bar{p}_c - \bar{p}_i) \). Therefore, all elements of \( W \) and \( \dot{W} \) in Equation (16) can be calculated.

### 3.2 Disturbance compensation of vehicle

From the viewpoint of underwater robot control, parameters and coefficients of hydrodynamic models are generally used as constant values that depend on the shape of robots (Fossen, 1995). The RAC law (17) can reduce the influence of the modelling errors of hydrodynamics by position and velocity feedbacks. Here, to obtain higher control performance, the influence of hydrodynamic modelling error with respect to the vehicle is treated as a disturbance and a disturbance compensation method is introduced.

First, the basic disturbance compensation is described. For \( M_{BB} \) in Equation (21) the nominal model using constant values of added mass, added moment of inertia and drag coefficient is defined as \( \bar{M}_{BB} \). Moreover, the basic disturbance is defined as

\[
f_L = u_B - \bar{M}_{BB}\dot{v}_0,
\] (23)
and the estimated value is calculated by

$$\hat{f}_E = F(p)(u_B - \overrightarrow{M}_{BB}\ddot{\nu}_0)$$

(24)

where $F(p) = 1/(T_f p + 1)$ is a low pass filter with a time constant $T_f$ and $p$ is the time differentiation operator.

Therefore, for the reference acceleration of vehicle $\nu_0^{ref}$, the control input with disturbance compensation becomes

$$u_B = \overrightarrow{M}_{BB}\ddot{\nu}_0^{ref} + \hat{f}_L,$$

(25)

and the configuration of the basic disturbance compensation is shown in Figure 3(a).

![Basic disturbance compensation diagram](image)

Fig. 3. Configuration of disturbance compensation

Next, the basic disturbance compensation is modified. For $\overrightarrow{M}_{BM}$, $\overrightarrow{N}_{BB}$, $\overrightarrow{N}_{BM}$ and $f_B$ in Equation (21) the nominal models using constant values of added mass, added moment of inertia and drag coefficient are defined as $\overrightarrow{M}_{BM}$, $\overrightarrow{N}_{BB}$, $\overrightarrow{N}_{BM}$ and $\overrightarrow{f}_B$, respectively. Then the vehicle control input with these nominal models and the reference acceleration $\alpha^{ref}$ becomes

$$\overrightarrow{u}_B = \overrightarrow{M}_{BB}\ddot{\nu}_0^{ref} + \ddot{\nu}_0$$

(26)

where

$$\ddot{\nu}_0 = \overrightarrow{M}_{BM}\ddot{\phi}^{ref} + \overrightarrow{N}_{BB}\ddot{\nu}_0 + \overrightarrow{N}_{BM}\ddot{\phi} + \overrightarrow{f}_B.$$  

(27)

From Equations (23) and (26) the modelling error with respect to the hydrodynamics can be defined as

$$f_E = u_B - \overrightarrow{M}_{BB}\ddot{\nu}_0 - f_l,$$

(28)

and the estimated value is calculated by

$$\hat{f}_E = F(p)(u_B - \overrightarrow{M}_{BB}\ddot{\nu}_0 - f_l).$$

(29)
Therefore, the control input with disturbance compensation becomes

\[ u_b = \overline{M}_{bb} \nu_{0}^{\text{ref}} + f_t + \hat{f}_L \]  

(30)

and the configuration of the modified disturbance compensation is shown in Figure 3(b).

4. Experiment of continuous-time RAC

In this section, some experiments of the RAC method are done for the vertical type 2-link underwater robot shown in Figure 1.

4.1 Experimental system

Figure 4 shows the configuration of the experimental system. A robot has a 2-DOF manipulator with joints that are actively rotated by velocity and torque control type servo actuators consisting of servo motors and incremental type encoders. The physical parameters of the underwater robot are shown in Table 1. Moreover, four 40[W] thrusters are attached to the vertical and horizontal directions on the robot base to provide propulsion for controlling the position and attitude angle of the base. The forward and reverse propulsion generated by the thruster are calculated by

\[ F = \begin{cases} 
1.341v^2 - 1.363v - 0.026 & (1.2 \leq v \leq 4) \\
-0.763v^2 - 0.835v + 0.019 & (-4 \leq v \leq -1.2) 
\end{cases} \]  

(31)

where \( v \) is the input voltage to the power amplifier of the thruster. Note that Equation (31) were obtained from the experiments (Yamada & Sagara, 2002).
The measurement and control system consist of a CCD camera, a video tracker, and a personal computer (PC). Two LEDs are attached to the base, and their motion is monitored by the CCD camera. Video signals of the LED markers are transformed into position data by the video tracker, and put into the PC via a GPIB communication line. Using the position data and the rotational angle of each joint measured by the encoder, the positions and attitude angles of the robot base and manipulator are computed in the PC. The PC is also used as a controller.

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<th>Base</th>
<th>Link 1</th>
<th>Link 2</th>
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</thead>
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<td>Mass (kg)</td>
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<td>4.25</td>
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</tr>
<tr>
<td>Moment of inertia (kg \cdot m^2)</td>
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<td>0.19</td>
<td>0.012</td>
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<tr>
<td>Link length (z_i direction) (m)</td>
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<td>Link width (m)</td>
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<td>0.12</td>
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<tr>
<td>Added mass (x_i direction) (kg)</td>
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<tr>
<td>Added mass (z_i direction) (kg)</td>
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<td>Added moment of inertia (kg \cdot m^2)</td>
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<td>Drag coefficient (x_i direction)</td>
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<tr>
<td>Drag coefficient (z_i direction)</td>
<td>1.2</td>
<td>1.2</td>
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</tr>
</tbody>
</table>

Table 1. Physical parameters of underwater robot

4.1 Comparison of control performance of RAC and computed torque methods

In this subsection, to compare the control performances of the RAC method and a computed torque method that is generally used to control of UVMS, simulations and experiments are done. Note that joint torque control type servo actuators are used in the experiments.

Model of vertical type 2-link underwater robot is shown in Figure 5. In this figure \( F_i \) \((i = 1, 2, 3)\) is the thrust of thruster and \( R \) is a distance from the origin of \( \Sigma_0 \) to the thruster. For the model shown in Figure 5 kinematic, momentum and dynamic Equations (3), (8) and (13) are reduced to

\[
\dot{p}_{ev} = A_V \dot{x}_{0v} + B_V \dot{\phi}_V, \tag{32}
\]

\[
s_V = C_V \dot{x}_{0v} + D_V \dot{\phi}_V, \tag{33}
\]

\[
M_V(q_V) \ddot{q}_V + N_V(q_V, \dot{q}_V) \dot{q}_V + f_V = u_V \tag{34}
\]

where

\[
p_{ev} = \begin{bmatrix} p_{e1} \\ p_{e2} \end{bmatrix}, \quad x_{0v} = \begin{bmatrix} x_{01} \\ x_{02} \end{bmatrix}, \quad \phi_V = \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix}, \quad q_V = \begin{bmatrix} q_{01} \\ q_{02} \end{bmatrix}, \quad u_V = \begin{bmatrix} u_{B_V} \\ u_{M_V} \end{bmatrix}, \quad u_{B_V} = \begin{bmatrix} f_{01} \\ f_{02} \end{bmatrix}, \quad u_{M_V} = \begin{bmatrix} r_{01} \\ r_{02} \end{bmatrix},
\]

and \( A_V, B_V, D_V, C_V \) and \( s_V \) are appropriate matrices and vector.
Similarly, the RAC law (17) is reduced to

\[
\alpha_v^{\text{ref}}(t) = W_v \dot{\theta}(t) | \beta_v^{\text{ref}}(t) + \gamma_v(t) - \dot{W}_v(t) \dot{q}_v(t) |
\]

where

\[
\beta_v^{\text{ref}} = \begin{bmatrix} \dot{x}_{0v}^{\text{des}} \\ \dot{p}_{e_v}^{\text{des}} \end{bmatrix} + K_{V_v} \begin{bmatrix} \dot{x}_{0v}^{\text{des}} - x_{0v} \\ \dot{p}_{e_v}^{\text{des}} - p_{e_v} \end{bmatrix} + K_{P_v} \begin{bmatrix} x_{0v}^{\text{des}} - x_{0v} \\ p_{e_v}^{\text{des}} - p_{e_v} \end{bmatrix},
\]

and \( \alpha_v^{\text{ref}} \) is the reference of \( \alpha_v (\approx \dot{q}_v) \), \( K_{V_v} \) and \( K_{P_v} \) are positive diagonal matrices.

On the other hand, a computed torque method is briefly described as follows. From Equation (32) the task-space velocity \( \nu_v = [x_v^T \quad p_v^T]^T \) and joint-space velocity \( \dot{q}_v \) are related as

\[
\nu_v(t) = J(t) \dot{q}_v(t)
\]

where

\[
J = \begin{bmatrix} E_v & 0 \\ A_v & B_v \end{bmatrix}.
\]

From Equation (37) the following equation can be obtained:
\[ \ddot{q}_V(t) = J^\#(t)(\dot{\mathbf{v}}_V(t) - \dot{J}(t)\dot{q}_V(t)) . \] (38)

For Equation (38) reference joint-space acceleration is defined as

\[ \ddot{q}_{V_{\text{ref}}}(t) = J^\#(t)(\dot{\mathbf{v}}^\text{ref}_{V}(t) - \dot{J}(t)\dot{q}_{V_{\text{ref}}}(t)) . \] (39)

Based on Equations (34) and (39) actual control input is calculated by using the following equation:

\[ M_V \ddot{q}^\text{des}_V + N_V \dot{q}_V + f_V = u_V \] (40)

where

\[ q_{V_{\text{des}}} = q_{V_{\text{ref}}} + \overline{K}_{V_V} (q_{V_{\text{ref}}} - \dot{q}_{V}) + \overline{K}_{P_V} (q_{V_{\text{ref}}} - q_{V}) , \] (41)

and \( \overline{K}_{V_V} \) and \( \overline{K}_{P_V} \) are positive diagonal matrices.

Both simulations and experiments are carried out under the following condition. The desired end-tip position is set up along a straight path from the initial position to the target. On the other hand, the desired position and attitude of the base are set up the initial values. The feedback gains are \( K_{V_V} = \overline{K}_{V_V} = \text{diag}[10 10 10 10 10] \) and \( K_{P_V} = \overline{K}_{P_V} = \text{diag} \{100 100 100 50 50\} \). The initial relative joint angles are \( \phi_0 = -\pi/2 \) [rad], \( \phi_1 = \pi/3 \) [rad] and \( \phi_2 = -5\pi/18 \) [rad].

Fig. 6. Simulation results of computed torque method and RAC method

First, simulation results of the computed torque method and the RAC method are shown in Figure 6(a) and (b). From Figure 6 we can see that both control methods have similar performance.

Next, we show the experimental results. As a computer is used for a controller in experiments, the sampling period for the controller is set up to \( T = 1/60 \) [s]. Figure 7 shows the both experimental results. From this figure, we can see that the performance of the computed torque method becomes worse. Since the computed torque method only uses joint-space errors, the control performance of the end-tip of the manipulator depends on the
robot base (vehicle) control performance. Therefore, if the acceleration and velocity relations between the end-tip and joints are inaccurate or the control performance of the vehicle is not better, good control performance of the end-tip cannot be obtained. On the other hand, from Figure 7 it can be seen that the RAC method has good control performance.

![Figure 7](image)

(a) Computed torque method  
(b) RAC method

**4.2 RAC method with disturbance compensation of vehicle**

Experiments are carried out under the following condition. The desired end-tip position is set up along a straight path from the initial position to the target. On the other hand, the desired position and attitude of the base are set up the initial values. The feedback gains are
$K_{V_v} = \text{diag}[10 \ 10 \ 10 \ 20 \ 20]$ and $K_{P_v} = \text{diag}[100 \ 100 \ 100 \ 100 \ 100]$. The time constant of filter is $T_f = 1\, [s]$. In this case joint velocity control type actuators are used.

Figure 8 shows the motion of the robot and estimated disturbance of the RAC with disturbance compensation, and Figure 9 shows the time histories of experimental results with and without disturbance compensation. From Figures 8 and 9, it can be seen that the end-tip of manipulator follows the desired trajectory. Moreover, since the robot base position and attitude errors become small values using the disturbance compensation, the end-tip position error is also reduced.

Fig. 8. Experimental result of RAC method with disturbance compensation
(a) RAC with disturbance compensation         (b) RAC without disturbance compensation

Fig. 9. Experimental results of RAC method with and without disturbance compensation
5. Discrete-time RAC

In practical systems digital computers are utilized for controllers, but there is no discrete-time control method for UVMS except our proposed methods (Sagara, 2003; Sagara et al., 2004; Sagara et al., 2006; Yatoh & Sagara, 2008). In this section, we address discrete time RAC methods including the ways of disturbance compensation of the vehicle and avoidance of singular configuration of the manipulator.

5.1 Discrete-time RAC law

Discretizing Equation (16) by a sampling period \( T \), and applying \( \beta(k) \) and \( W(k) \) to the backward Euler approximation, the following equation can be obtained:

\[
W(k)\alpha(k-1) = \frac{1}{T}\{\nu(k) - \nu(k-1) + T\gamma(k) - [W(k) - W(k-1)]\zeta(t)\} \tag{42}
\]

where \( \nu = [\nu_0^T \quad \nu_e^T]^T \). Note that a computational time delay is introduced into Equation (42), and the discrete time \( kT \) is abbreviated to \( k \).

For Equation (42), the desired acceleration is defined as

\[
\alpha_d(k) = \frac{1}{T}W^\#(k)[\nu_d(k+1) - \nu_d(k) + Ae_\nu(k) + T\gamma(k)] \tag{43}
\]

where

\[
e_\nu(k) = \nu_d(k) - \nu(k) \tag{44}
\]

and \( \nu_d(k) \) is the desired value of \( \nu(k) \), \( A = \text{diag}\{\lambda_i\} \ (i = 1, \ldots, 12) \) is the velocity feedback gain matrix.

From Equations (42) and (43) we have

\[
TW(k)e_d(k-1) = e_\nu(k) - e_\nu(k-1) + Ae_\nu(k) - T[\gamma(k) - \gamma(k-1)] + [W(k) - W(k-1)]\zeta(t) \tag{45}
\]

where

\[
e_\alpha(k) = \alpha_d(k) - \alpha(k). \tag{46}
\]

Assuming \( W(k) \approx W(k-1) \) and \( \gamma(k) \approx \gamma(k-1) \) for one sampling period, Equation (45) can be rewritten as

\[
TW(k)e_\alpha(k-1) = ((q-1)E_{12} + A)e_\nu(k) \tag{46}
\]

where \( q \) is the forward shift operator. Since all elements of \( W(k) \) are bounded, if \( \lambda_i \) is selected to satisfy \( 0 < \lambda_i < 1 \) and the convergence of \( e_\alpha(k) \) tends to zero as \( k \) tends to infinity, the convergence of \( e_\nu(k) \) to zero as \( k \) tends to infinity can be ensured from Equation (46).

Moreover, the desired velocity of \( \nu(k) \) is defined as

\[
\nu_d(k) = \frac{1}{T}S_{0\nu}(k)[x_d(k) - x_d(k-1) + Fx_d(k-1)] \tag{47}
\]

where
\[
S_{0e} = \begin{bmatrix}
E_3 & 0 \\
S_{\varphi_0} & E_3 \\
0 & S_{\psi_0}
\end{bmatrix}, \quad e_x(k) = x_d(k) - x(k), \quad x = \begin{bmatrix}
n_0 \\
\psi_0 \\
p_e \\
\psi_e
\end{bmatrix},
\]
and \(x_d(k)\) is the desired value of \(x(k) = [x_0^T, x_\varphi^T]^T\), \(\Gamma = \text{diag}\{\gamma_i\} \quad (i = 1, \ldots, 12)\) is the position feedback gain matrix.

From Equations (44) - (47) the following equation can be obtained:
\[
Te_x(k) = S_{0e}[E_{12} - (E_{12} - \Gamma)\eta^{-1}]e_x(k)
\tag{48}
\]
where \(\nu(k)\) is applied to the backward Euler approximation. From Equation (48), if \(\gamma_i\) is selected to satisfy \(0 < \gamma_i < 1\) and the convergence of \(e_x(k)\) tends to zero as \(k\) tends to infinity, the convergence of \(e_x(k)\) to zero as \(k\) tends to infinity can be ensured.

The configuration of the control system described in this subsection is shown in Figure 10.

### 5.2 Disturbance compensation of vehicle

Discretizing the low pass filter, \(F(p) = 1/(T_f p + 1)\), shown in Figure 3(b) (Godler et al., 2002), a digital version of disturbance compensation can be obtained. Figure 11 shows the digital version where \(h = e^{-T_f}/T\).

![Digital type disturbance compensation](image.png)

Fig. 11. Digital type disturbance compensation

### 5.3 Avoidance of a singular configuration

In much work on UVMS it is considered that the vehicle is keeping its initial state during the manipulation. In order to avoid the singular configuration of the manipulator in such case, the desired value of the vehicle is modified by using the determinant of the manipulator’s Jacobian matrix \(J(k) = \det J(k)\) (Sagara et al., 2006).

The desired linear acceleration of the vehicle is defined as...
where $\dot{p}_{e_d}$ is the desired linear velocity of the end-tip of the manipulator, and $k_aT$ and $k_sT$ are the time when $|J(k)|$ becomes less or greater than a threshold $J_s$, respectively, and $n_aT$ is the acceleration time.

5.4 Experiment of discrete-time RAC

In this subsection, some experiments of the discrete-time RAC method described above are done for the underwater robot shown in Figures 1 and 4.

All experiments are carried out under the following condition. The desired end-tip position is set up along a straight path from the initial position to the target. On the other hand, the
desired position and attitude of the base are set up the initial values. The sampling period is $T = 1/60$ [s] based on the processing time of video tracker.

First, a basic discrete-time RAC experiment is done. In this case, the feedback gains are $\Lambda = \text{diag}[0.6, 0.6, 0.25, 0.25, 0.25]$ and $\Gamma = \text{diag}[0.3, 0.3, 0.2, 0.2, 0.2]$. Figure 12 shows the experimental result. From this figure, it can be seen that the discrete-time RAC method has good control performance and the performance is similar to that of the continuous-time version shown in Figure 7(b).

![Fig. 12: Experimental result of discrete-time RAC method.](image1)

Next, experiments of discrete-time RAC with and without disturbance compensation of the base are done. To validate the performance of disturbance compensation, the feedback gains of the RAC are $\Lambda = \Gamma = \text{diag}[0.3, 0.3, 0.2, 0.2, 0.2]$. Using these values of the gains the basic control performance of the RAC becomes worse. The time constant of the filter for the disturbance compensation is $T_f = 0.1$ [s]. The experimental results of the RAC with and without disturbance compensation are shown in Figure 13(a) and (b), respectively. And Figure 14 shows the time history of the estimated disturbance. From Figures 13 and 14, it
can be seen that the position and attitude errors of the base are reduced by using the disturbance compensation.

Fig. 14. Estimated disturbance (digital version)

Fig. 15. Experimental result of discrete-time RAC considering singular configuration
Finally, an experiment of avoidance of singular configuration is done. In this case, the basic desired position and attitude of the base (vehicle) is set as the initial values, and the threshold of the determinant of the Jacobian matrix is $J_s = 0.45$. And the feedback gains are $A = \Gamma = \text{diag}[0.6, 0.6, 0.25, 0.25, 0.25]$. The experimental result is shown in Figure 15. From Figure 15, we can see that the end-tip of the manipulator and base follow the desired trajectories avoiding the singular configuration of the manipulator and the tracking errors are very small.

6. Conclusion

In this chapter, our proposed continuous-time and discrete-time RAC methods was described and the both experimental results using a 2-link underwater robot were shown. For the continuous-time RAC method, experimental results showed that the RAC method has good control performance in comparison with a computed torque method and the RAC method with disturbance compensation can reduce the influence of the hydrodynamic modelling error. In practical systems digital computers are utilized for controllers. Then, we addressed discrete-time RAC methods including the ways of disturbance compensation and avoidance of singular configuration. Experimental results show that the control performance of the discrete-time RAC method is similar to the continuous version. Our future work is to carry out experiments in 3-dimensional space to evaluate the validity of the RAC methods.

7. References


For the latest twenty to thirty years, a significant number of AUVs has been created for the solving of wide spectrum of scientific and applied tasks of ocean development and research. For the short time period the AUVs have shown the efficiency at performance of complex search and inspection works and opened a number of new important applications. Initially the information about AUVs had mainly review-advertising character but now more attention is paid to practical achievements, problems and systems technologies. AUVs are losing their prototype status and have become a fully operational, reliable and effective tool and modern multi-purpose AUVs represent the new class of underwater robotic objects with inherent tasks and practical applications, particular features of technology, systems structure and functional properties.

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