A Physics Approach to Supply Chain Oscillations and Their Control

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1. Introduction

Virtually all manufacturing involves supply chains, in which value is added by each entity in the chain until a finished product emerges at the end. A serious problem that plagues all supply chains is unwanted fluctuation in the inventories along the chain. This disrupts the product output, is costly in capital, and can result in considerable disruption and hardship in personal lives. J.D. Sterman and his colleagues at MIT (Sterman & Fiddaman, 1993) have performed a useful service by providing business schools with a widely used simulation game that demonstrates for a beer distribution supply chain how easily the fluctuations can arise, and how difficult they can be to control. Their simulation results indicate that the oscillations are due to the over-reaction to input fluctuations by the individual entities in the chain, and the results strongly suggest that the situation could be improved by information technology that enables real-time feedback.

The purpose of this chapter is to describe a statistical physics approach to understanding the supply chain oscillations. Standard techniques from statistical physics can lead to insights on the nature of the oscillations and on means for their control. Much of the supply chain work has been reported earlier, primarily at a number of conferences (Dozier & Chang, 2004a, 2004b, 2005, 2006a, 2006b, 2007), but this chapter is the first time that the general approach together with its applications have been assembled in one place, along with a number of possible extensions.

Section 2 provides some background on the application of statistical physics techniques to manufacturing issues. The focus in this section is on a pseudo-thermodynamics description of a manufacturing sector, and on the nature of effective intervention forces that can be uniquely derived from this thermodynamics.

Section 3 demonstrates how the statistical physics approach leads directly to the existence of supply chain normal mode oscillations. It is shown that the nature of the normal modes depends on the extent of the information exchange between the entities in the supply chain. When the information exchange occurs only between an entity and the two entities immediately below and above it in the chain, the normal mode oscillation frequencies depend strongly on the way the oscillation amplitudes change along the chain: the normal modes resemble sound waves. On the other hand, when the information exchange occurs between an entity and all the other entities in the chain, the normal mode oscillation...
frequencies depend more weakly on the way the oscillation amplitudes change along the chain: the normal modes resemble plasma oscillations.

Section 4 discusses possible interventions that can increase the production rate of a supply chain, utilizing the effective pseudo-thermodynamic forces discussed in Section 2. It demonstrates that both the time scale and the position focus of the intervention are important for determining the effectiveness of the intervention. An example is given of a useful application to supply chains of a quasilinear approximation technique that is often used in plasma physics problems.

Section 5 summarizes the approach and results to date, and suggests several possible extensions.

2. Background on statistical physics modeling in manufacturing

There has been a long-term association between economics and one particular aspect of statistical physics: thermodynamics. This association can be found in both neoclassical economics and modern new growth economics. For example, Krugman (1995) points out that economics is based on physics, and one of his favorite examples is that of the thermodynamics of economics. Even systems far from economic equilibrium can be treated by (open system) thermodynamics (Thorne & London, 2000). Costanza, et. al. (1997) have noted that biology has also played an important role, emphasizing that both thermodynamics and biology drove the development of new growth economic models: Smith and Foley (2002) have remarked that both neoclassical economics and classical thermodynamics seek to describe natural systems in terms of solutions to constrained optimization problems.

The foregoing observations about the role of thermodynamics in economics are in the quasi-static realm: the variables involved are relatively slowly changing in time. However, statistical physics also treats dynamic (time-dependent) phenomena, and can be used to understand important time-dependent processes in economics. Koehler (2001, 2003) has been studying the interactions between the private and public sectors, and has emphasized the need to understand how best to conduct interactions between sectors that can have quite drastically different intrinsic time scales.

2.1 Rationale for application of statistical physics

Why does statistical physics provide a good framework for analyzing static (time-independent) and dynamic (time-dependent) aspects of cybernetic-based management of manufacturing? There are two reasons:

1. For the time-independent aspects, the formalism is designed to display the implications of systematically focusing on a few macroscopic variables of interest, an advantage for understanding complex systems such as manufacturing.
2. For time-dependent aspects, the formalism naturally takes into account the impact of time-dependent collective (cooperative) effects on the systems, of importance for designing effective cybernetic management control practices for complex situations.

These two reasons are considered further below.

2.1(i). Time-independent aspects: systematic focus on macroscopic variables

The systematic focusing on a few macroscopic variables of interest is important for understanding the relationships that can exist between the variables, and the associated implications for how the associated microscopic variables are distributed in an industrial
sector. For example, suppose the objectives of policies and management actions on a particular industrial sector are to increase the productivity and the sales for the sector. Statistical physics can explicitly exhibit the relationships between the two variables and show how the impact of policies and management on the two are reflected by changes in the distribution of productivities and sales of the companies in the sector.

The systematic focusing on a few macroscopic variables of interest is accomplished by the constrained optimization approach of statistical physics. The approach is “constrained” by the requirement that specific measured values can be assigned to the few chosen relevant variables, and it is “optimized” by deriving the probability distribution that corresponds to the maximum possible number of possible arrangements of the entities comprising the system. This approach leads to well-defined “management forces” that can change the distribution of the variables within an industrial sector.

2.1(ii). Time-dependent aspects: collective effects in a sector

An understanding of the collective effects in an industrial sector is important for optimizing the focus and timing of both policy and management efforts. In particular, it is well known that companies in supply chains exhibit inventory oscillations that can be larger than the oscillations in market demands. This is wasteful of resources, and impacts costs and profits in a negative manner. Properly focused policy and cybernetic-based management can reduce these oscillations, and statistical physics can provide a systematic guide on how best to focus and time these efforts to perform this cost-saving reduction.

2.2 Effective pseudo-thermodynamic forces

In this chapter we would like to consider the impact of interventional control on reducing the resource-wasteful inventory oscillations in supply chains. This involves the application of effective pseudo-thermodynamic forces to the entities in the supply chains. These forces can be most easily understood by considering a thermodynamic description of the quasistatic state of a manufacturing sector. In this subsection, the thermodynamics approach – with its definition of an effective force - is briefly summarized, and some validating examples are given of its successful application to real data.

2.2(i). Conservation laws for information transfer involving effective “information force”

Thermodynamics can be applied to manufacturing in a number of ways. As an illustration of a general approach, let us suppose that a particular industrial sector has an objective of decreasing the average unit cost of production, and wishes to do this by implementing information technology transfer from an external source. (For example, this external source could be the government to which the companies in the sector have provided real-time data.) The statistical physics approach will produce a concrete measurable value-added for that sector due to the information transfer.

This objective can be posed as a straightforward constrained optimization problem in statistical physics, and the problem can be solved by maximizing a probability distribution subject to taking into account the constraints by the use of Lagrange multipliers. As an example, suppose that the total costs incurred in some length of time, $C(\text{total}) = \sum C(i)n(i)$, is known and that the total number of units produced in that time, $N(\text{total}) = \sum n(i)$, is known, where in both expressions the summation is from $i = 0$ to $i = N_T$, the total number of companies in the sector. With this knowledge, we then ask what the most likely distribution
of production is over $i$. Since the total number of allowable ways that $N(\text{total})$ can be arranged subject to an assumed distribution $n(i) = P[N(\text{total}), n(i)] = N(\text{total})! / [n(1)!n(2)! \ldots n(N_{\text{T}})!]$, the most likely distribution $n(i)$ can be determined by maximizing $\ln P$ subject to the constraints that $N(\text{total})$ and $C(\text{total})$ are known. Introducing the Lagrange multipliers $\alpha$ and $\beta$ to form $F(n(i)) = \ln [P[N(\text{total}), n(i)]] - \alpha [\sum n(i) - N(\text{total})] - \beta [\sum C(i)n(i) - C(\text{total})]$, we find on setting $dF(n(i)) / dn(i) = 0$ and using the Stirling large number approximation for a factorial, $-\ln{n(i)} - 1 - \alpha - \beta C(i) = 0$, i.e. $n(i) = A \exp[-\beta C(i)]$, the familiar Maxwell-Boltzmann distribution. (A more detailed derivation of the relevant equations is contained in Dozier and Chang (2004).)

Given the Maxwell-Boltzmann distribution function, it is straightforward to define quantities that are analogous to the conventional thermodynamic quantities of thermodynamics. These are shown in Table 1:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Physics</th>
<th>Economics</th>
</tr>
</thead>
<tbody>
<tr>
<td>State</td>
<td>Hamiltonian eigenfunction</td>
<td>Production site</td>
</tr>
<tr>
<td>Energy</td>
<td>Hamiltonian eigenvalue $E_i$</td>
<td>Unit production cost $C_i$</td>
</tr>
<tr>
<td>Occupation number</td>
<td>Number in state $N_i$</td>
<td>Sales output $N_i$</td>
</tr>
<tr>
<td>Partition function</td>
<td>$\sum \exp[-(1/k_B T)E_i]$</td>
<td>$\sum \exp[-\beta C_i]$</td>
</tr>
<tr>
<td>Free energy $F$</td>
<td>$k_B T \ln Z$</td>
<td>$(1/\beta) \ln Z$</td>
</tr>
<tr>
<td>Generalized force</td>
<td>$\xi = \text{Pressure}$</td>
<td>$\xi = \text{Information technology}$</td>
</tr>
<tr>
<td>Example</td>
<td>$\xi = \text{Electric field x charge}$</td>
<td>$\xi = \text{Mfg. technology}$</td>
</tr>
<tr>
<td>Entropy (randomness)</td>
<td>$-\partial F / \partial T$</td>
<td>$k_B \beta^2 \partial F / \partial \beta$</td>
</tr>
</tbody>
</table>

Table 1. Comparison of statistical formalism in physics and in economics (based on Dozier and Chang (2004a))

In this Table, the statistical physics formalism has naturally replaced the product of Boltzmann’s constant $k_B$ and the temperature $T$ in the physics realm by the inverse of a “bureaucratic factor $\beta$” in the economic realm.

Note that the constrained optimization approach leads directly to a generalized pressure-like force corresponding to information technology transfer. (A second example is also provided in Table 1 of an effective force due to the introduction of new manufacturing technology.) For the purposes of this chapter, the important result is that an effective “force” can be derived from a partition function or associated free energy provided by a constrained optimization pseudo-thermodynamics of manufacturing.

As a further illustration of the result of applying a statistical physics technique to model a manufacturing sector, a conservation equation relating pseudo-thermodynamic quantities can be obtained by differentiating the expression for the total cost of production. This is shown in Table 2.

We note that the conservation equation (that corresponds to the First Law of Thermodynamics) involves the effective force $\xi$ that is uniquely determined by the “free energy function” $F$. It is also interesting to note that if the two terms in the conservation equation have opposite signs, then the effect of information technology transfer is not as large as it could be. On the other hand, if the two terms have the same sign, then the impact of information technology transfer on costs for the sector are maximized.
Table 2. Conservation law for information technology transfer (based on Dozier and Chang (2004a))

As a preliminary data comparison, these results were applied to two particular sectors (the semiconductor sector and the heavy springs sector) in the U.S. Economic Census Data for the Los Angeles Metropolitan Statistical Area for 1992 and 1997 (Dozier & Chang, 2006b). The per capita information technology investments were known for both of these sectors: they were high for the semiconductor sector and low for the heavy springs sector.

The semiconductor sector experienced both a decrease in the average unit cost of production and a decrease in the entropy term (a lower effective temperature). In this case, both terms in the conservation equation were operating synergistically. The sector also displayed a marked increase in output.

The heavy spring sector (the information-poor sector), on the other hand, displayed little change in output. Its entropy term increased and its average unit cost of production also increased. In this case, both terms acted in the wrong direction.

Although this preliminary examination of data is not directly applicable to information transfer in general, and although it is not possible to attribute all the changes to information technology investments since several other factors were operating, the examples suggest that both the force and entropy terms (due to all the relevant factors) in the conservation equation can act synergistically in the desired direction.

Accordingly, this suggests that information technology transfer can act to both decrease the effective temperature of a sector and to reduce costs. The former might be achieved by focusing information technology transfer efforts on companies that have unit costs that are far out of line with the average unit costs. It might also be achieved by focusing information technology transfer efforts on companies that have the best unit costs, letting the other companies drop by the wayside. Another possible method of decreasing entropy might be to provide market incentives for “heat flow” from the sector to sectors (including the government itself) that have lower temperatures.
2.2(ii). Static phenomena: Impact of information on productivity in the Los Angeles region

As a second example of the application of the constrained optimization technique of statistical physics, the impact of information technology on the improvement of productivity (i.e. on improving the output per employee) was considered. A detailed account of this example is contained in Dozier and Chang (2006b) and is described in the proceedings of the CITSA 04 conference (Dozier & Chang 2004a).

The constrained optimization formalism is applied here also, except that now the focus is not on the unit cost of production but rather on the shipments per company. In this example, the technique is used to derive the most likely distribution of the number of companies having a particular value of shipments per company vs. the shipments per company.

![Graph of cumulative number of companies vs shipments per company](image)

Fig. 1. Comparison of U.S. economic census cumulative number of companies vs. shipments/company (diamond points) in LACMSA in 1992 and the statistical physics cumulative distribution curve (square points) with $\beta = 0.167$ per $10^6$.

The figure above (taken from Dozier and Chang, 2006b) compares the predicted cumulative distribution for a value of “inverse temperature” $\beta = 0.167$ per million dollars with actual data for the manufacturing sectors in the Los Angeles Metropolitan Statistical Area for 1992. As can be seen from the Figure, the comparison is quite good, giving good validation for the statistical physics constrained optimization applicability to an economics problem.

In this preliminary study, the interest was in the impact of information technology (IT) on various statistical physics parameters of companies as a function of company size. Table 3 shows the ratios of the 1997 statistical physics characterization parameters to their values in
1992 for large, intermediate, and small companies in the Los Angeles manufacturing sectors. Shown are the ratios for the number of companies in a size segment (#), the number of employees in the corresponding segment (E), the average number of employees per company (E/comp), the shipments of the sector (Sh), the shipments per employee (Sh/E), and a normalized “inverse temperature ($\beta$)

<table>
<thead>
<tr>
<th>Company size:</th>
<th>Large</th>
<th>Intermediate</th>
<th>Small</th>
</tr>
</thead>
<tbody>
<tr>
<td>IT rank</td>
<td>59</td>
<td>70</td>
<td>81</td>
</tr>
<tr>
<td>#</td>
<td>0.86</td>
<td>1.0</td>
<td>0.90</td>
</tr>
<tr>
<td>E(1000s)</td>
<td>0.78</td>
<td>0.98</td>
<td>1.08</td>
</tr>
<tr>
<td>E/company</td>
<td>0.91</td>
<td>1.0</td>
<td>1.21</td>
</tr>
<tr>
<td>Sh ($\text{million}$)</td>
<td>1.53</td>
<td>1.24</td>
<td>1.42</td>
</tr>
<tr>
<td>Sh/E ($\text{1000}$)</td>
<td>1.66</td>
<td>1.34</td>
<td>1.35</td>
</tr>
<tr>
<td>$\beta$</td>
<td>1.11</td>
<td>0.90</td>
<td>0.99</td>
</tr>
</tbody>
</table>

Table 3. Ratio (“97/’92) of the statistical physics characterization parameters (from Dozier & Chang, 2006b)

Without going into detail, it can be seen from Table 3 that the impact of information technology investments on the statistical physics characterization parameters does seem to be correlated with the company size. Whether or not the same type of size-dependency exists in the technology transfer realm bears analysis.

To summarize, Section 2 has shown that the application of the constrained optimization technique of statistical physics to (quasi) time-independent economic phenomena is supported by preliminary comparisons with U.S. Economic Census Data for the Los Angeles Metropolitan Statistical Area. The application of the technique appears to be justified both to systematically analyze the data and to provide a comprehensive and believable framework for presenting the results. For our immediate purpose of analyzing oscillatory phenomena in supply chains, the successes also provide confidence in the concept of an effective pseudo-thermodynamic-derived “information force”.

3. Normal mode oscillations in supply chains

In this section, we describe a simple statistical physics model of inventory oscillations in a supply chain. It is based on material presented in three conferences (Dozier & Chang, 2005, 2006a, 2007), and follows especially closely the presentation in Dozier & Chang (2006a)

Section 3a describes a simple time-dependent conservation equation for the production units flowing through the chain. This equation is the supply-chain equivalent of the Liouville or Vlasov equations of statistical physics.

Section 3b applies this equation to derive results for a chain in which each entity only exchanges information with the two entities immediately above and below it in the chain.

Section 3c derives the oscillation dispersion relation for a supply chain in which every entity in the chain exchanges information with all the other entities in the chain.

In order to focus on the bare essentials of the approach, only a linear supply chain will be considered in the following. In addition, end effects will be ignored. It should be quite apparent how to generalize the results to a situation in which the number of companies at each level of the supply chain varies, and in which effects associated with the finite length of a chain are included.
3.1 Conservation equation for supply chain
In order to replace difference equations with more familiar differential equations, instead of designating each level in the chain by a discrete label \( n \), the position in a chain will be designated by a continuum variable \( x \). (Dozier & Chang (2005) shows the modifications resulting from dealing with the discrete label \( n \) instead of the continuous variable \( x \), and demonstrates that for long supply chains, the basic features of both treatments are essentially the same.) The rate of flow of a production unit through each position \( x \) in the chain can be characterized by a velocity variable \( v \).

Introduce a distribution function \( f(x,v,t) \) that depends on position, production flow rate velocity, and time. In statistical physics parlance, \( x \) and \( v \) denote the phase space for the problem, and \( f(x,v,t)dxdv \) denotes the number of production units in the intervals \( dx \) and \( dv \) at a given \( x \) and \( v \) at the time \( t \).

From its definition, this distribution function satisfies a conservation equation in the phase space of \( x \) and \( v \), if it is assumed that no production units are destroyed at each level of the chain:

\[
\frac{\partial f}{\partial t} + \frac{\partial}{\partial x} \left[ f \frac{dx}{dt} \right] + \frac{\partial}{\partial v} \left[ f \frac{dv}{dt} \right] = 0 \tag{1}
\]

This equation simply states that the change of \( fdx dv \) is due only to the divergence of the flow into the phase space volume \( dx dv \). In a perfectly operating supply chain, we would expect that there would be no divergence in the flow. By permitting a divergence in the flow – i.e. permitting the flow into a volume element \( dx dv \) to be different than the flow out, the possible existence of local inventory fluctuations is allowed.

By introducing a force \( F \) that influences the velocity of the production rate \( v \), this equation can be rewritten

\[
\frac{\partial f}{\partial t} + \frac{\partial}{\partial x} \left[ f v \right] + \frac{\partial}{\partial v} \left[ f F \right] = 0 \tag{2}
\]

We assume that the force \( F \) is of the type discussed earlier in Section 2b(i), i.e. that it can be uniquely derived from a pseudo-thermodynamic partition function for the chain entities. Since position \( x \) and velocity of the production rate \( v \) are independent variables, \( \partial v / \partial x = 0 \). If, moreover, the force \( F \) does not depend on the flow rate \( v \), \( \partial F / \partial v = 0 \), so that Eq. (2) becomes

\[
\frac{\partial f}{\partial t} + v \frac{\partial f}{\partial x} + F \frac{\partial f}{\partial v} = 0 \tag{3}
\]

This is similar to the Liouville equation of classical mechanics, and has the familiar form of the Vlasov equation for collisionless plasmas (Spitzer, 1956).

3.2 Supply chain with local exchange of information
Now assume that \( F \) at the position \( x \) is determined only by the level of the inventories of the production units immediately above and below \( x \) in the chain. As explained below, it is reasonable to assume that the fractional change in the rate of change of velocity \( (1/v)dv/dt \) is proportional to the fractional change in the gradient of the density \( N(x,t) \):

\[
(1/v)dv/dt \propto - (1/N) dN/dx \tag{4}
\]

Where \( N(x,t) = \int dv f(x,v,t) \), and where the negative sign is explained below.

For local information exchange with the levels immediately above and below the level of interest, the change in the density is observed over only the interval \( dx = 2l \), where \( l \) is the spacing between levels in the supply chain. Thus, we can further write
The rationale for this expression is that when the inventory of the level below the level of interest is less than normal, the production rate \( v \) will be diminished because of the smaller number of production units being introduced to that level. At the same time, when the inventory of the level above the level of interest is larger than normal, the production rate will also be diminished because the upper level will demand less input so that it can “catch up” in its production through-put. Both effects give production rate changes proportional to the negative of the gradient of \( N \). It is reasonable also that the fractional changes are related rather than the changes themselves, since deviations are always made from the inventories at hand.

We note in passing that the quantity \( l \) is somewhat arbitrary, and reflects an equally arbitrary choice of a scale factor that relates the continuous variable \( x \) and the discrete level variable \( n \).

A time scale for the response is missing from Eq. (5). We know that a firm must make decisions on how to react to the flow of production units into the firm. Assume that the time scale of response \( \tau_{\text{response}} \) is given by \( \tau_{\text{response}} = (1/\xi)\tau_{\text{processing}} \), where \( \tau_{\text{processing}} \) is the processing time for a unit as it passes through the firm, and for simplification we are assuming \( \xi \) and \( \tau_{\text{processing}} \) are constant throughout the chain. Because of a natural inertia associated with cautious decision-making, it is likely that \( \xi \) will be less than unity, corresponding to response times being longer than processing times.

Then Eq. (5) becomes

\[
(1/v)dv/dt \propto -(2\xi l/\tau_{\text{processing}} N)dN/dx
\]

Since by definition, the steady state production rate velocity is given by \( V_0 \approx l/\tau_{\text{processing}} \), this gives finally for the effective internal force that changes production flow rates:

\[
F = dv/dt = -2\xi V_0^2(1/N)dN/dx
\]

Insertion of this expression into Eq. (3) then yields

\[
\partial f/\partial t + v\partial f/\partial x - 2\xi V_0^2(1/N)(dN/dx) \partial f/\partial v = 0
\]

In the steady state, the equation is satisfied by \( f(x,v,t) = f_0(v) \), i.e. by a distribution function that is independent of position and time: In this desired steady state, production units flow smoothly through the line without bottlenecks. For a smoothly operating supply chain, \( f_0(v) \) will be centered about the steady state flow velocity \( V_0 \), a fact that we shall make use of later.

Now suppose there is a (normal mode) perturbation of the form \( \exp[i(\omega t - kx)] \), i.e.

\[
f(x,v,t) = f_0(v) + f_1(v) \exp[-i(\omega t - kx)]
\]

On linearizing eq. (8) with this \( f(x,v,t) \), we find that \( f_1(v) \) satisfies

\[
-i(\omega-kv)f_1 - ik 2\xi V_0^2(1/N_0)N_1 \partial f_0/\partial v = 0
\]

Solving for \( f_1 \):

\[
f_1 = -2\xi(k/N_1/N_0) V_0^2 \partial f_0/\partial v(\omega-kv)^{-1}
\]
On integrating this equation with respect to v, we get the statistical physics dispersion relation relating \( \omega \) and \( k \):

\[
1 + 2\xi k V_0^2 (1/N_0) \int dv \frac{\partial f_0}{\partial v} (\omega - kv)^{-1} = 0
\]

This equation contains a singularity at \( \omega = kv \). This singularity occurs where the phase velocity \( \omega/k \) becomes equal to the velocity of flow \( v \). There are well-defined methods for the treatment of singularities: Following the Landau prescription (Landau, 1946; Stix, 1992)

\[
\int dv \frac{\partial f_0}{\partial v} (\omega - kv)^{-1} = \text{PP} \int dv \frac{\partial f_0}{\partial v} (\omega - kv)^{-1} - i\pi \frac{\partial f_0 (\omega/k)}{\partial v}
\]

where PP denotes the principal part of the integral, i.e. the value of the integral ignoring the contribution of the singularity.

To evaluate the principal part, assume that for most \( v \), \( \omega >> kv \). Then approximately

\[
\text{PP} \int dv \frac{\partial f_0}{\partial v} (\omega - kv)^{-1} \approx \int dv kv \frac{\partial f_0}{\partial v} (1/\omega^2) \approx -kN_0/\omega^2
\]

This gives the sound-wave-like dispersion relation

\[
\omega \approx (2\xi)^{1/2} kV_0
\]

Addition to this of the small contribution from the imaginary part yields

\[
\omega = (2\xi)^{1/2} kV_0 + i\pi/2 (k/N_0) (2\xi)^{3/2} V_0(2\xi^{1/2} V_0) / \partial v
\]

or, on using the approximate relationship of Equation [21] for the \( \omega \)'s in the second term on the RHS

\[
\omega = (2\xi)^{1/2} kV_0 + i(\pi/2)(k/N_0)(2\xi)^{3/2} V_0^3 \xi V_0 (2\xi^{1/2} V_0) / \partial v
\]

For the fast response times made possible by first order rapid information exchange, \( \xi = O(1) \). Thus, with \( f_0(\nu) \) peaked around \( V_0 \), \( \xi V_0 < 0 \).

Accordingly, the imaginary part of \( \omega \) is less than zero, and this corresponds to a damping of the normal mode oscillation. It is interesting to note that since \( (2\xi)^{1/2} V_0 >> V_0 \) (where the distribution is peaked), the derivative will be small, however, and the damping will be correspondingly small.

We note in passing that the discrete level variable is used instead of the continuous variable \( x \), the dispersion relation is the same as Eq. (10) for small \( k \), but when \( kl \rightarrow 1 \), the dispersion relation resembles that of an acoustic wave in a solid (Dozier & Chang, 2004, and Kittel, 1996).

To summarize, this sub-section has shown that when an entity in this linear supply chain exchanges information only with the two entities immediately above and below it in the chain, a slightly damped sound-wave-like normal mode results. Inventory disturbances in such a chain tend to propagate forwards and backwards in the chain at a constant flow velocity that is related to the desired steady-state production unit flow velocity through the chain.

### 3.3 Supply chain with universal exchange of information

Consider next what happens if the exchange of information is not just local. (Suppose that information is shared equally between all participants in a supply chain such as in the use of
grid computing.) In this case, the force $F$ in Eq. (3) is not just dependent on the levels above and below the level of interest, but on the $f(x,v,t)$ at all $x$.

Let us assume that the effect of $f(x,v,t)$ on a level is independent of what the value of $x$ is. This can be described by introducing a potential function $\Phi$ that depends on $f(x,v,t)$ by the relation

$$\partial^2 \Phi / \partial x^2 = - \left[ C / N_0 \right] \int dv f(x,v,t)$$

from which the force $F$ is obtained as $F = - \partial \Phi / \partial x$. (That this is so can be seen by the form of the 1-dimensional solution to Poisson’s equation for electrostatics: the corresponding field from a source is independent of the source position.)

The constant $C$ can be determined by having $F$ reduce approximately to the expression of Eq. (7) when $f(x,v,t)$ is non zero only for the levels immediately above and below the level $x_0$ of interest in the chain. For that case, take $N(x+l) = N(x_0) + dN/\partial x l$ and $N(x-l) = N(x_0) - dN/\partial x l$, and $N(x)$ zero elsewhere. Then

$$F = - \partial \Phi / \partial x = - \left[ C / N_0 \right] (dN/\partial x)^2$$

On comparing this with the $F$ of Eq. (7), $F = - 2\xi v^2 (1/N) dN/\partial x$, we find (since the distribution function is peaked at $V_0$) that we can write $C = \xi V_0^2 / l^2$.

Accordingly,

$$\partial^2 \Phi / \partial x^2 = - \left[ \xi V_0^2 / N_0 l^2 \right] \int dv f(x,v,t)$$

With these relations, $F$ from the same value of $f(x,v,t)$ at all $x$ above the level of interest is the same, and $F$ from the same value of $f(x,v,t)$ at all $x$ below the level of interest is the same but of opposite sign.

This is the desired generalization from local information exchange to universal information exchange.

It is interesting to see what change this makes in the dispersion relation. Eq. (3) now becomes

$$\partial f / \partial t + v \partial f / \partial x - \partial \Phi / \partial x \partial f / \partial v = 0$$

and again the dispersion relation can be obtained from this equation by introducing a perturbation of the form of Equation (15) and assuming that $\Phi$ is of first order in the perturbation. This gives

$$-i(\omega-kv)f_1 = ik\Phi_1 \partial f_0 / \partial v$$

i.e.,

$$f_1 = -k\Phi_1 \partial f_0 / \partial v (\omega-kv)^{-1}$$

Since Eq. (20) implies

$$\Phi_1 = (1/k^2) \left[ \xi V_0^2 / N_0 l^2 \right] \int dv f_1(v)$$

we get on integrating Eq. (23) over $v$:

$$1 + (1/k) \left[ \xi V_0^2 / N_0 l^2 \right] \int dv \partial f_0 / \partial v (\omega-kv)^{-1} = 0$$

Once again a singularity appears in the integral, so we write

$$\int dv \partial f_0 / \partial v (\omega-kv)^{-1} = PP \int dv \partial f_0 / \partial v (\omega-kv)^{-1} - i\pi(1/k) \partial f_0(\omega/k) / \partial v$$
Evaluate the principal part by moving into the frame of reference moving at $V_0$, and in that frame assume that $kv/\omega << 1$:

$$
PP[\text{d}v \partial f_0/\partial v (\omega-kv)] \approx [\text{d}v \partial f_0/\partial v (1/\omega)[1+(kv/\omega)]
= -kN_0/\omega^2
$$

(27)

Moving back into the frame where the supply chain is stationary,

$$
PP[\text{d}v \partial f_0/\partial v (\omega-kv)] \approx -kN_0/(\omega-kV_0)^2
$$

(28)

This gives the approximate dispersion relation

$$
1 - (1/k) \left[\xi V_0^2/N_0 l^2\right] kN_0/(\omega-kV_0)^2 \approx 0
$$

(29)

i.e.

$$
\omega = kV_0 + \xi^{1/2} V_0/l \quad \text{or} \quad \omega = kV_0 - \xi^{1/2} V_0/l
$$

(30)

To describe a forward moving disturbance, we take $\omega > 0$ as $k \to 0$, discarding the minus solution for this case.

Now add the small imaginary part to the integral:

$$
1 + (1/k) \left[\xi V_0^2/N_0 l^2\right] [-kN_0/(\omega-kV_0)^2 - i\pi(1/k)\partial f_0(\omega/k)/\partial v] = 0
$$

(31)

On iteration, this yields

$$
\omega \approx kV_0 + \xi^{1/2}(V_0/l) \left[1 + i \left[\pi\xi V_0^2/(2k^3N_0)\right]\partial f_0/\partial v\right]
$$

(32)

where $\partial f_0/\partial v$ is evaluated at $v = \omega/k \approx V_0 + (\xi^{1/2}V_0/k)l$.

Since for velocities greater than $V_0$, $\partial f_0/\partial v < 0$, we see that the oscillation is damped. Moreover, the derivative $\partial f_0/\partial v$ is evaluated at a velocity close to $V_0$, the flow velocity where the distribution is maximum. Since the distribution function is larger there, the damping can be large. (We note here that the expression of Eq. (32) differs a little from that in Dozier & Chang (2006a), due to an algebraic error in the latter.)

To summarize, Section 3 has shown that universal information exchange results both in changing the form of the supply chain oscillation to a plasma-like oscillation, and in the suppression of the resulting oscillation. Specifically, it has been shown that for universal information exchange, the dispersion relation resembles that for a plasma oscillation. Instead of the frequency being proportional to the wave number, as in the local information exchange case, the frequency now contains a component which is independent of wave number. The plasma-like oscillations for the universal information exchange case are always damped. As the wave number $k$ becomes large, the damping (which is proportional to $\partial f_0/\partial v$) can become large as the phase velocity approaches closer to the flow velocity $V_0$. This supports Sterman and Fiddaman’s conjecture that IT will have beneficial effects on supply chains.

### 4. External interventions that can increase supply chain production rates

In Section 3, we have seen that universal information exchange among all the entities in a supply chain can result in damping of the undesirable supply chain oscillations. In this
section, we change our focus to see if external interactions with the oscillations can be used to advantage to increase the average production rate of a supply chain.

A quasilinear approximation technique has been used in plasma physics to demonstrate that the damping of normal mode oscillations can result in changes in the steady state distribution function of a plasma. In this section, this same technique will be used to demonstrate that the resonant interactions of externally applied pseudo-thermodynamic forces with the supply chain oscillations also result in a change in the steady state distribution function describing the chain, with the consequence that production rates can be increased.

This approach will be demonstrated by using a simple fluid flow model of the supply chain, in which the passage of the production units through the supply chain will be regarded as fluid flowing through a pipe. This model also gives sound-like normal mode waves, and shows that the general approach is tolerant of variations in the specific features of the supply chain model used. A more detailed treatment of this problem is available at Dozier and Chang (2007).

4.1 Moment equations and normal modes

The starting point is again the conservation equation, Eq. (5), for the distribution function that was derived in Section 3a. To obtain a fluid flow model of the supply chain, it will be useful to take various moments of the distribution function:

Thus, the number of production units in the interval dx and x at time t, is given by the $v^0$ moment, $N(x, t) = \int dv f(x,v,t)$; and the average flow fluid flow velocity is given by the $v^1$ moment $V(x,t) = (1/N) \int v f(x,v,t)$.

By taking the $v^0$ and $v^1$ moments of Eq. (3) – see, e.g. Spitzer (2006) - we find

$$\frac{\partial N}{\partial t} + \frac{\partial [NV]}{\partial x} = 0$$

and

$$\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} = F_1 - \frac{\partial P}{\partial x}$$

where $F_1(x,t)$ is the total force $F$ acting per unit $dx$ and $P$ is a “pressure” defined by taking the second moment of the dispersion of the velocities $v$ about the average velocity $V$: $P(x,t) = \int dv (v-V)^2 f(x,v,t)$

We can write the pressure $P$ in the form

$$P(x,t) = \int dv (v-V)^2 f(x,v,t) = N(x,t) (\Delta v)^2$$

where

$$(\Delta v)^2 = \frac{\int dv (v-V)^2 f(x,v,t)}{N(x,t)}$$

This is a convenient form, since we shall assume for simplicity that the velocity dispersion $(\Delta v)^2$ is independent of level $x$ and time $t$. In that case, Eq. (34) can be rewritten as

$$\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} = F_1 - (\Delta v)^2 \frac{\partial N}{\partial x}$$

This implies the change in velocity flow is impacted by the primary forcing function and the gradients of the number density of production units. Equations (33) and (37) are the basic equations that we shall use in the remainder to describe temporal phenomena in this simple fluid-flow supply chain model.
Before considering the effect of externally applied pseudo-thermodynamic forces, we derive the normal modes for the fluid flow model. Accordingly, introduce the expansions $N(x,t) = N_0 + N_1(x,t)$ and $V(x,t) = V_0 + V_1(x,t)$ about the level- and time-independent steady state density $N_0$ and velocity $V_0$. (We can take the steady state quantities to be independent of the level in the supply chain, since again we are considering long supply chains in the approximation that end effects can be neglected.)

Upon substitution of these expressions for $N(x,t)$ and $V(x,t)$ into Eqs. (33) and (37), we see that the lowest order equations (for $N_0$ and $V_0$) are automatically satisfied, and that the first order quantities satisfy

$$\frac{\partial N_1}{\partial t} + V_0 \frac{\partial N_1}{\partial x} + N_0 \frac{\partial V_1}{\partial x} = 0$$ \hspace{0.5cm} (38)

and

$$\frac{\partial V_1}{\partial t} + V_0 \frac{\partial V_1}{\partial x} = F_1(x,t) - (\Delta v)^2 \frac{\partial N_1}{\partial x}$$ \hspace{0.5cm} (39)

where $F_1(x,t)$ is regarded as a first order quantity.

As before, the normal modes are propagating waves:

$$N_1(x,t) = N_1 \exp[i(\omega t - kx)]$$ \hspace{0.5cm} (40)

$$V_1(x,t) = V_1 \exp[i(\omega t - kx)]$$ \hspace{0.5cm} (41)

With these forms, Eqs. (38) and (39) become

$$i(\omega - kV_0)N_1 + N_0 ikV_1 = 0$$ \hspace{0.5cm} (42)

$$i N_0 (\omega - kV_0) V_1 = -ik (\Delta v)^2 N_1$$ \hspace{0.5cm} (43)

In order to have nonzero values for $N_1$ and $V_1$, these two equations require that

$$(\omega - kV_0)^2 = k^2(\Delta v)^2$$ \hspace{0.5cm} (44)

Equation (44) has two possible solutions

$$\omega_+ = k (V_0 + \Delta v)$$ \hspace{0.5cm} (44a)

$$\omega_- = k (V_0 - \Delta v)$$ \hspace{0.5cm} (44b)

The first corresponds to a propagating supply chain wave that has a propagation velocity equal to the sum of the steady state velocity $V_0$ plus the dispersion velocity width $\Delta v$. The second corresponds to a slower propagation velocity equal to the difference of the steady state velocity $V_0$ and the dispersion velocity width $\Delta v$. Both have the form of a sound wave: if there were no fluid flow ($V_0 = 0$), $\omega_+$ would describe a wave traveling up the chain, whereas $\omega_-$ would describe a wave traveling down the chain. When $V_0 \neq 0$, this is still true in the frame moving with $V_0$.

4.2 Resonant interactions resulting in an increased production rate

As indicated earlier, our focus in this section is on the effect of external interactions (such as government actions) on the rate at which an evolving product moves along the supply chain. This interaction occurs in the equations through an effective pseudo-thermodynamic...
force $F_1(x,t)$ that acts to accelerate the rate. From the discussion of Section 3, we expect that this force will be most effective when it has a component that coincides with the form of a normal mode, since then a resonant interaction can occur.

To see this resonance effect, it is useful to present the force $F$ in its Fourier decomposition

$$F_1(x,t) = (1/2\pi)\int\int d\omega dk F_1(\omega, k) \exp\{i(\omega t - kx)\} \quad (45)$$

where

$$F_1(\omega, k) = (1/2\pi)\int\int dx dt F_1(x,t) \exp\{-i(\omega t - kx)\} \quad (46)$$

With this Fourier decomposition, each component has the form of a propagating wave, and it would be expected that these propagating waves are the most appropriate quantities for interacting with the normal modes of the supply chain.

Our interest is in the change that $F_1$ can bring to $V_0$, the velocity of product flow that is independent of $x$. By contrast, $F_1$ changes $V_1$ directly, but each wave component causes an oscillatory change in $V_1$ both in time and with supply chain level, with no net (average) change.

To obtain a net change in $V$, we shall go to one higher order in the expansion of $V(x,t)$:

$$V(x,t) = V_0 + V_1(x,t) + V_2(x,t) \quad (47)$$

On substitution of this expression into Eq. (37), we find the equation for $V_2(x,t)$ to be

$$N_0(\partial V_2/\partial t + V_0 \partial V_2/\partial x) + N_1(\partial V_1/\partial t + V_0 \partial V_1/\partial x) + N_0 V_1 \partial V_1/\partial x = - (\Delta v)^2 \partial N_2/\partial x \quad (48)$$

This equation can be Fourier analyzed, using for the product terms the convolution expression:

$$\int\int dx dt \exp\{-i(\omega t - kx)\} f(x,t) g(x,t) = \int\int d\Omega dK f(-\Omega + \omega, -K + k) g(\Omega, K) \quad (49)$$

where

$$f(\Omega, K) = \int\int dx dt \exp\{-i(\Omega t - Kx)\} f(x,t) \quad (50a)$$

$$g(\Omega, K) = \int\int dx dt \exp\{-i(\Omega t - Kx)\} g(x,t) \quad (50b)$$

Since we are interested in the net changes in $V_2$—i.e. in the changes brought about by $F_1$ that do not oscillate to give a zero average, we need only look at the expression for the time rate of change of the $\omega=0, k=0$ component, $V_2(\omega=0, k=0)$.

From Eq. (48), we see that the equation for $\partial V_2(\omega=0, k=0)/\partial t$ requires knowing $N_1$ and $V_1$. When $F_1(\omega, k)$ is present, then Eqs. (42) and (43) for the normal modes are replaced by

$$i (\omega - kV_0) N_1(\omega, k) + N_0 i k V_1(\omega, k) = 0 \quad (51)$$

$$i N_0 (\omega - kV_0) V_1(\omega, k) = -i k (\Delta v)^2 N_1(\omega, k) + F_1(\omega, k) \quad (52)$$

These have the solutions

$$N_2(\omega, k) = -i k F_1(\omega, k) [((\omega - kV_0)^2 - k^2 (\Delta v)^2)^{-1}] \quad (53)$$

$$V_1(\omega, k) = -i [F_1(\omega, k)/N_0](\omega - kV_0) [((\omega - kV_0)^2 - k^2 (\Delta v)^2)^{-1}] \quad (54)$$
Substitution of these expressions into the $\omega=0$, $k=0$ component of the Fourier transform of Eq. (48) gives directly

$$\partial V_2(0,0)/\partial t = \int \int d\omega dk (ik/N_0^2) ((\omega-kV_0)^2 - k^2(\Delta v)^2)^2 F_1(-\omega,k) F_1(-\omega,k)$$

(55)

This resembles the quasilinear equation that has long been used in plasma physics to describe the evolution of a background distribution of electrons subjected to Landau acceleration [Drummond & Pines (1962)].

As anticipated, a resonance occurs at the normal mode frequencies of the supply chain, i.e. when

$$(\omega-kV_0)^2 - k^2(\Delta v)^2 = 0$$

(56)

First consider the integral over $\omega$ from $\omega = -\infty$ to $\omega = \infty$. The integration is uneventful except in the vicinity of the resonance condition where the integrand has a singularity. As before, the prescription of Eq. (13) can be used to evaluate the contribution of the singularity.

For Eq. (55), we find that when the bulk of the spectrum of $F_1(x,t)$ is distant from the singularities, the principal part of the integral is approximately zero, where the principal part is the portion of the integral when $\omega$ is not close to the singularities at $\omega = k(V_0 \pm \Delta v)$. This leaves only the singularities that contribute to $\partial V_2(0,0)/\partial t$.

The result is the simple expression:

$$\partial V_2(0,0)/\partial t = \pi/(N_0^2\Delta v) \int dk (1/k) [ F_1(-k(V_0-\Delta v),-k)F_1(k(V_0-\Delta v),k) - (-k(V_0+\Delta v),-k)F_1(k(V_0+\Delta v),k)]$$

(57)

Equation (57) suggests that any net change in the rate of production in the entire supply chain is due to the Fourier components of the effective statistical physics force describing the external interactions with the supply chain, that resonate with the normal modes of the supply chain. In a sense, the resonant interaction results in the conversion of the “energy” in the normal mode fluctuations to useful increased production flow rates. This is very similar to physical phenomena in which an effective way to cause growth of a system parameter is to apply an external force that is in resonance with the normal modes of the system.

To summarize, Section 4 has shown that the application of the quasilinear approximation of statistical physics to a simple fluid-flow model of a supply chain, demonstrates how external interactions with the normal modes of the chain can result in an increased production rate in the chain. The most effective form of external interaction is that which has Fourier components that strongly match the normally occurring propagating waves in the chain.

5. Discussion and possible extensions

In the foregoing, some simple applications of statistical physics techniques to supply chains have been described.

Section 2 briefly summarized the application of the constrained optimization technique of statistical physics to (quasi) time-independent economic phenomena. It showed some preliminary comparisons with U.S. Economic Census Data for the Los Angeles Metropolitan Statistical Area, that supported the approach as a good means of systemically analyzing the data and providing a comprehensive and believable framework for presenting the results. It also introduced the concept of an effective pseudo-thermodynamic-derived “information force” that was used later in the discussion of supply chain oscillations.
Section 3 discussed supply chain oscillations using a statistical physics normal modes approach. It was shown that the form of the dispersion relation for the normal mode depends on the extent of information exchange in the chain. For a chain in which each entity only interacts with the two entities immediately below and above it in the chain, the normal mode dispersion relation resembles that of a sound wave. For a chain in which each entity exchanges information with all of the other entities in the chain, the dispersion relation resembles that of a plasma oscillation. The Landau damping in the latter could be seen to be larger than in the limited information exchange case, pointing up the desirability of universal information exchange to reduce undesirable inventory fluctuations.

Section 4 applied the quasilinear approximation of statistical physics to a simple fluid-flow model of a supply chain, to demonstrate how external interactions with the normal modes of the chain can result in an increased production rate in the chain. The most effective external interactions are those with spectra that strongly match the normally occurring propagating waves in the chain.

The foregoing results are suggestive. Nevertheless, the supply chain models that were used in the foregoing were quite crude: Only a linear uniform chain was considered, and end effects were ignored.

There are several ways to improve the application of statistical physics techniques to increase our understanding of supply chains. Possibilities include (1) the allowance of a variable number of entities at each stage of the chain, (2) relaxation of the uniformity assumption in the chain, (3) a more comprehensive examination of the effects of the time scales of interventions, (4) a systematic treatment of normal mode interactions, (5) treatment of end effects for chains of finite length, (6) consideration of supply chains for services as well as manufactured goods, and (7) actual simulations of the predictions. We can briefly anticipate what each of these extensions would produce.

**Variable number of entities at each level** Equations similar to those in Sections 3 and 4 would be anticipated. However, in the equations, the produced units at each level would now refer to those produced by all the organizations at that particular level. The significance is that the inventory fluctuation amplitudes calculated in the foregoing refer to the contributions of all the organizations in a level, with the consequence that the fluctuations in the individual organization would be inversely proportional to the number of entities in that level. Thus, organizations in levels containing few producing organizations would be expected to experience larger inventory fluctuations.

**Nonuniform chains** In Sections 3 and 4, it was assumed that parameters characterizing the processing at each level (such as processing times) were uniform throughout the chain. This could very well be unrealistic: for example, some processing times at some stages could be substantially longer than those at other stages. And in addition, the organizations within a given stage could very well have different processing parameters. This would be expected both to introduce dispersion, and to cause a change in the form of the normal modes.

As a simple example, suppose the processing times in a change increased (or decreased) linearly with the level in the chain. The terms of the normal mode equation would now no longer have coefficients that were independent of the level variable \( x \). For a linear dependence on \( x \), the normal modes change from Fourier traveling waves to combinations of Bessel functions, i.e. the normal mode form for a traveling wave is now a Hankel function. The significance of this is that the inventory fluctuation amplitudes become level-
dependent: A disturbance introduced at one level in the chain could produce a much larger (smaller) fluctuation amplitude at another level.

**Time scales of interventions** Since inventory fluctuations in a supply chain are disruptive and wasteful of resources, some form of cybernetic control (intervention) to dampen the fluctuations would be desirable. In Section 4, it was suggested that interventions that resonate with the normal modes are most effective in damping the fluctuations and converting the “energy” in the fluctuations to useful increased production rates. Koehler (2001, 2002) has emphasized, however, that often the time scales of intervention are quite different from those of the system whose output it is desired to change.

A systematic means of analyzing the effects of interventions with time scales markedly different from those of the supply chain is available with standard statistical physics techniques:

For example, if the intervention occurs with a time scale much longer than the time scales of the chain’s normal modes, then the adiabatic approximation can be made in describing the interactions. The intervention can be regarded as resulting in slowly changing parameters (as a function of both level and time). Eikonal equations (Weinberg 1962) can then be constructed for the chain disturbances, which now can be regarded as the motion of “particles” comprising wave packets formed from the normal modes.

At the other extreme, suppose the intervention occurs with time scales much less than the time scales of the chain’s normal modes. When the intervention occurs at random times, the conservation equation (Eq. 3) can be modified by Fokker-Planck terms (Chandrasekhar, 1943). The resulting equation describes a noisy chain, in which a smooth production flow can be disrupted.

**Normal mode interactions** The beer distribution simulation (Sterman & Fiddaman, 1993) has shown that the amplitudes of the inventory oscillations in a supply chain can become quite large. The normal mode derivation in Sections 3 and 4 assumed that the amplitudes were small, so that only the first order terms in the fluctuation amplitudes needed to be kept in the equations. When higher order terms are kept, then the normal modes can be seen to interact with one another. This “wave-wave” interaction itself can be expected to result in temporal and spatial changes of the supply chain inventory fluctuation amplitudes.

**End effects of finite chains** The finite length of a supply chain has been ignored in the calculations of this chapter, i.e. end effects of the chain have been ignored. As in physical systems, the boundaries at the ends can be expected to introduce both reflections and absorption of the normal mode waves described. These can lead to standing waves, and the position and time focus of optimal means of intervention might be expected to be modified as a result.

**Supply chains for services as well as manufactured products** In the foregoing, we have been thinking in terms of a supply chain for a manufactured product. This supply chain can involve several different companies, or – in the case of a vertically integrated company – it could comprise several different organizations within the company itself. The service sector in the economy is growing ever bigger, and supply chains can also be identified, especially when the service performed is complex. The networks involved in service supply chains can have different architectures than those for manufacturing supply chains, and it will be interesting to examine the consequences of this difference. The same type of statistical physics approach should prove useful in this case as well.

**Numerical simulations** The statistical physics approach to understanding supply chain oscillations can lead to many types of predicted effects, ranging from the form and
frequencies of the inventory fluctuations to the control and conversion of the fluctuations. Computer simulations would be useful in developing an increased understanding of the predictions. This is especially true when the amplitudes of the oscillations are large, since then the predictions based on small-amplitude approximations would be suspect. The application of statistical physics techniques to understand and control supply chain fluctuations may prove to be very useful. The initial results reported here suggest that further efforts are justified.

6. References


With the ever-increasing levels of volatility in demand and more and more turbulent market conditions, there is a growing acceptance that individual businesses can no longer compete as stand-alone entities but rather as supply chains. Supply chain management (SCM) has been both an emergent field of practice and an academic domain to help firms satisfy customer needs more responsively with improved quality, reduction cost and higher flexibility. This book discusses some of the latest development and findings addressing a number of key areas of aspect of supply chain management, including the application and development ICT and the RFID technique in SCM, SCM modeling and control, and number of emerging trends and issues.

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