Adaptive Control of Piezoelectric Actuators with Unknown Hysteresis

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1. Introduction

Hysteresis phenomenon occurs in all smart material-based sensors and actuators, such as shape memory alloys, piezoceramics and magnetostrictive actuators (Bank & Smith, 2000; Tan & Baras, 2004). In order to study this phenomenon, different models were proposed (Brokate & Sprekels, 1996; Visintin, 1994). Normally, hysteresis models are classified into two categories, physics-based model such as Jiles-Atherton model (Jiles & Atherton, 1986) and phenomenology-based model such as Preisach operator (Brokate & Sprekels, 1996; Visintin, 1994) and Duhem model (Visintin, 1994). From control systems point of view, hysteresis is generally non-differentiable, nonlinear, and unknown. As a result, systems with hysteresis usually exhibit undesirable inaccuracies or oscillation and even instability. Mitigating the effect of hysteresis becomes necessary and important, thus it has received increasing attention in recent years (Tao & Kokotovic, 1995, Su, et al, 2000, Su, et al, 2005). Many of these studies are related to modeling of hysteresis and their control issues.

With the development of artificial intelligent (AI), AI is being applied to dealing with nonlinearities in systems (Ge & Wang, 2002). Only a few studies have been carried out by using NN to tackle hysteresis modeling and compensation (Makaveev, et al, 2002; Kuczmann & Ivanyi, 2002; Beuschel et al, 1998; Zhao & Tan, 2006). In the paper (Makaveev, et al, 2002), a NN model is used to describe the hysteresis behavior in different frequencies with the knowledge of some properties of magnetic materials, such as loss separation property to allow the separate treatment of quasi-static and dynamic hysteretic effects. Beuschel et al used (Beuschel et al, 1998) a modified Luenberger observer and NN are used to identify a general model of hysteresis. These researches demonstrate that NN can work as an unknown function approximator to describe the characteristics of hysteresis. Recently, two papers (Zhao & Tan, 2006; Lin et al 2006) applied the approximation property of NN to coping with the identification of Preisach-type hysteresis in piezoelectric actuator, and the hysteresis estimation problem for piezo-positioning mechanism based on hysteresis friction force function, respectively. It should be noted that the aforementioned results share a common assumption that the output of hysteresis is measurable.

In practical systems, smart actuators are integrated into the systems, which makes the measurement of output of hysteresis hard. Hence it is a challenge to design an observer for
the unavailable output of hysteresis. Due to the unavailability of the output of hysteresis, the major obstacle of pre-inversion compensator for hysteresis is the lack of effective observer design methods for piezoelectric actuators. Especially, the traditional “Luenberger-type” nonlinear observer design (Krener & Isidori, 1983) or the “high-gain” observer (Krener & Kang, 2003) cannot be applied directly, since the hysteresis is highly nonlinear. The sliding-mode observer was developed to estimate the internal friction states of LuGre model for the servo actuators with friction (Xie, 2007). This observer needs a low-pass filter to remove the high-frequency components in the estimated state variable, which is not applicable in this paper. Yang and Lin (Yang & Lin, 2004) proposed homogeneous observers design for a class of n-dimensional inherently nonlinear systems whose Jacobian linearization is neither controllable nor observable.

Inspired by NN’s universal approximation property, and the aforementioned facts in observer design, we propose an observer-based adaptive control of piezoelectric actuators with unknown hysteresis in this paper. The main contribution of this paper is the following: First, it applies the NN to on-line approximate complicated piecewise continuous unknown nonlinear functions in the explicit solution to Duhem model. Second, an observer is designed to estimate the output of hysteresis of piezoelectric actuator based on the system input and output. Third, the stability of the controlled piezoelectric actuator with the observer is guaranteed by using Lyapunov extension (Kuczmann & Ivanyi, 2002). The organization of the paper is as follows. In Section II, a Duhem model of hysteresis and the problem statement are given. The main results on NN-based compensator for hysteresis are presented in Section III. Section IV provides an example to show the feasibility of the proposed method. Conclusions are given in Section V.

2. Preliminaries

2.1 Duhem model of hysteresis

Many different mathematic models are built to describe the hysteresis behavior, such as Preisach model, Prandtl-Ishlinskii model and Duhem model (Coleman & Hodgdon, 1987; Macki et al, 1993). Considering its capability of providing a finite-dimensional differential model of hysteresis, we adopt classical Duhem model to develop the adaptive controller for the piezoelectric actuator.

The Duhem model is a rate independent operator, with input signal $v, \dot{v}$ and output signal $\tau$. The Duhem model describes hysteresis $H(t)$ by the following mathematical model (Coleman & Hodgdon, 1987; Macki et al, 1993).

$$\frac{d\tau}{dt} = \alpha \cdot \frac{dv}{dt} \cdot \left[ f(v) - \tau \right] + \frac{dv}{dt} \cdot g(v)$$

(1)

where $\alpha$ is a positive number, $f(v)$ and $g(v)$ are prescribed real-valued functions on $(-\infty, +\infty)$.

It can also be represented as (Coleman & Hodgdon, 1987; Macki et al, 1993):

$$\frac{d\tau}{dv} = \begin{cases} \alpha \cdot [f(v) - \tau] + g(v), & \dot{v} > 0 \\ -\alpha \cdot [f(v) - \tau] + g(v), & \dot{v} < 0 \end{cases}$$

(2)
where $\alpha$ is the same positive number in (1), $g(v)$ is the slope of the model, and $f(v)$ is the average value of the difference between upward side and downward side.

**Property 1** (Coleman & Hodgdon, 1987; Macki et al, 1993): $f(v)$ is a piecewise smooth, monotone increasing, odd function with a derivative $f'(v)$, which is not identically zero. For large value of input $v(t)$, there exists a finite limit $f'(\infty)$;

$$f(v) = -f(-v), \lim_{v \to \infty} f'(v) < \infty$$  \hspace{1cm} (3)

**Property 2** (Coleman & Hodgdon, 1987; Macki et al, 1993): $g(v)$ is a piecewise continuous, even function with

$$g(v) = g(-v), \lim_{v \to \infty} g'(v) < \infty$$  \hspace{1cm} (4)

It has been shown that Duhem model can describe a large class of hysteresis in various smart materials, such as ferromagnetically soft material, or piezoelectric actuator by appropriately choosing $f(v)$ and $g(v)$ (Coleman & Hodgdon, 1987; Macki et al, 1993). One widely used pair of functions of $f(v)$ and $g(v)$ are

$$f(v) = \begin{cases} a_1 \cdot v + a_2 (v - v_s) & \text{for } v > v_s \\ a_1 \cdot v & \text{for } |v| \leq v_s \\ -a_1 \cdot v + a_2 (v + v_s) & \text{for } v < -v_s \end{cases}$$  \hspace{1cm} (5)

$$g(v) = a_3$$  \hspace{1cm} (6)

where $v_s > 0$, $a_1 > 0$, $a_2 > 0$, $a_1 > a_2 > 0$, $a_1$ and $a_2$ satisfy $a_1, a_2 \in [a_{\min}, a_{\max}]$, $a_{\min}$ and $a_{\max}$ are known constants. Substituting the $f(v)$ and $g(v)$ into (2), we have

$$\dot{\tau} = \begin{cases} \alpha \cdot [a_1 \cdot v + a_2 (v - v_s) - \tau] + a_1 \cdot \dot{v} & v > v_s, \dot{v} > 0 \\ \alpha \cdot [a_1 \cdot v - \tau] + a_1 \cdot \dot{v} & 0 < v \leq v_s, \dot{v} > 0 \\ \alpha \cdot [\tau - a_1 \cdot v] + a_1 \cdot \dot{v} & -v_s \leq v < 0, \dot{v} < 0 \\ \alpha \cdot [a_1 \cdot v - a_2 (v + v_s) + \tau] + a_1 \cdot \dot{v} & v < -v_s, \dot{v} < 0 \end{cases}$$  \hspace{1cm} (7)

The above equation can be solved for $\tau$

$$\tau = \begin{cases} a_2 \cdot v - f_{21} & v > v_s, \dot{v} > 0 \\ a_1 \cdot v - f_{22} & 0 < v \leq v_s, \dot{v} > 0 \\ a_1 \cdot v - f_{23} & -v_s \leq v < 0, \dot{v} < 0 \\ a_2 \cdot v - f_{24} & v < -v_s, \dot{v} < 0 \end{cases}$$  \hspace{1cm} (8)
with
\[
\begin{align*}
    f_2 &= (a_0 - a_1 - a_2) e^{-a_1 x} - e^{-a_1 x} \int (a_1 - a_2) e^{a_2 x} d_2^x - (a_1 - a_2) \cdot v, \quad v > v_s, \dot{v} > 0 \\
    f_1 &= (a_0 - a_1 - a_2) e^{-a_1 x} - e^{-a_1 x} \int (a_1 - a_2) e^{a_2 x} d_2^x + (a_1 - a_2) \cdot v, \quad v < v_s, \dot{v} < 0 \\
    f_3 &= (a_0 - a_1 - a_2) e^{-a_1 x} - e^{-a_1 x} \int (a_1 - a_2) e^{a_2 x} d_2^x - (a_1 - a_2) \cdot v, \quad 0 < \dot{v} < v_s, \dot{v} > 0 \\
    f_4 &= (a_0 - a_1 - a_2) e^{-a_1 x} - e^{-a_1 x} \int (a_1 - a_2) e^{a_2 x} d_2^x + (a_1 - a_2) \cdot v, \quad v < v_s, \dot{v} < 0
\end{align*}
\]

In order to describe the piezoelectric actuator, we choose the same functions \( f(v) \) and \( g(v) \) as those in (Banning, et al, 2001), which is a special case of the foregoing choice of \( f(v) \) and \( g(v) \), i.e. \( a_1 = a \), \( a_2 = 0 \) and \( a_3 = \bar{b} \) when \( |v| \leq v_s \).

\[
f(v) = \begin{cases} 
    a \cdot v_s & \text{for } v > v_s \\
    a \cdot v & \text{for } |v| \leq v_s \\
    -a \cdot v_s & \text{for } v < -v_s
\end{cases}
\]

(10)

\[
g(v) = \begin{cases} 
    0 & \text{for } v > v_s \\
    \bar{b} & \text{for } |v| \leq v_s \\
    0 & \text{for } v < -v_s
\end{cases}
\]

(11)

where \( v_s > 0 \), \( a > 0 \), \( \bar{b} > 0 \) and \( a > \bar{b} \geq a/2 \). Suppose the parameter \( a \) satisfies \( a \in [a_{\min}, a_{\max}] \), \( a_{\min} \) and \( a_{\max} \) are known constants.

Substituting (5) and (6) into (1), we have

\[
\dot{\tau} = \begin{cases} 
    \alpha \cdot \dot{v} (a \cdot v_s - \tau) & v > v_s, \dot{v} > 0 \\
    \alpha \cdot \dot{v} (a \cdot v - \tau) + \bar{b} \cdot \dot{v} & 0 < v \leq v_s, \dot{v} > 0 \\
    \alpha \cdot \dot{v} (\tau - a \cdot v) + \bar{b} \cdot \dot{v} & -v_s \leq v < 0, \dot{v} < 0 \\
    \alpha \cdot \dot{v} (a \cdot v_s + \tau) & v < -v_s, \dot{v} < 0
\end{cases}
\]

(12)

Equation (7) can be solved for \( \tau \)

\[
\tau = \begin{cases} 
    -f_{21} & v > v_s, \dot{v} > 0 \\
    a \cdot v - f_{22} & 0 < v \leq v_s, \dot{v} > 0 \\
    a \cdot v - f_{23} & -v_s \leq v < 0, \dot{v} < 0 \\
    -f_{24} & v < -v_s, \dot{v} < 0
\end{cases}
\]

(13)
where

\[
\begin{aligned}
 f_{21} &= -\tau_0 e^{\alpha (v-v_0)} - a v \\
 f_{22} &= (a \cdot v_0 - \tau_0) \cdot e^{\alpha (v-v_0)} - e^{\alpha v} \int_0^v (b - a) \cdot e^{\alpha \xi} d\xi \\
 f_{23} &= (a \cdot v_0 - \tau_0) \cdot e^{\alpha (v-v_0)} - e^{\alpha v} \int_0^v (b - a) \cdot e^{\alpha \xi} d\xi \\
 f_{24} &= -\tau_0 e^{\alpha (v-v_0)} + a v
\end{aligned}
\]

(14)

Equation (8) can be also expressed as:

\[
\tau = a \cdot \chi \cdot \dot{v} - (f_{21} \dot{x}_2 \cdot \dot{x}_3 + f_{22} \dot{x}_1 \cdot \dot{x}_3 + f_{23} \dot{x}_1 \cdot \dot{x}_4 + f_{24} \dot{x}_2 \cdot \dot{x}_4)
\]

(15)

where \(\chi_i (i = 1, 2 \cdots 4)\) are indicator functions defined as:

\[
\chi_1 = \begin{cases} 1 & v \leq v_s \\ 0 & v > v_s \end{cases}, \quad \chi_2 = \begin{cases} 0 & v < v_s \\ 1 & v \geq v_s \end{cases}, \quad \chi_3 = \begin{cases} 1 & \dot{v} \geq 0 \\ 0 & \dot{v} < 0 \end{cases}, \quad \chi_4 = \begin{cases} 0 & \dot{v} \geq 0 \\ 1 & \dot{v} < 0 \end{cases}
\]

Following the definition of the indicator functions, we get

\[
\chi_1 \cdot \chi_2 = 0, \quad \chi_1 + \chi_2 = 1, \quad \chi_3 \cdot \chi_4 = 0, \quad \chi_1 + \chi_2 = 1, \quad \chi_k^2 = \chi_k, k = 1, 2, 3, 4
\]

By defining \(\dot{\chi}_1 = \dot{\chi}_2 = 0\), we have

\[
\dot{\tau} = a \cdot \chi_1 \cdot \ddot{v} - (f_{21} \dot{x}_2 \cdot \dot{x}_3 + f_{22} \dot{x}_1 \cdot \dot{x}_3 + f_{23} \dot{x}_1 \cdot \dot{x}_4 + f_{24} \dot{x}_2 \cdot \dot{x}_4)
\]

Let

\[
F_2 = f_{21} \dot{x}_2 \cdot \dot{x}_3 + f_{22} \dot{x}_1 \cdot \dot{x}_3 + f_{23} \dot{x}_1 \cdot \dot{x}_4 + f_{24} \dot{x}_2 \cdot \dot{x}_4
\]

and \(K_a = a \chi_1\). We can also write the derivative of \(\tau\) as

\[
\dot{\tau} = K_a \dot{v} - F_2
\]

(16)

### 2.2 Augmented Multilayer Perceptron (MLP) Neural Network

The MLP NN has been explored to approximate any function with arbitrary degree of accuracy (Hornik et al, 1989). However, it needs a large number of NN nodes and training iterations to approximate non-smooth functions (i.e. piecewise continuous), such as friction, hysteresis, backlash and other hard nonlinearities. For these piecewise continuous functions, the MLP needs to be augmented to work as a function approximator. Results for approximation of piecewise continuous functions or functions with jumps are given in the
paper (Selmic & Lewis, 2000). We use the augmented NN to approximate the piecewise continuous function in hysteresis model.

Let $S$ be a compact set of $\mathbb{R}^n$ and define $C^n(S)$ be the space such that the map $f(x) : S \rightarrow \mathbb{R}^n$ is piecewise continuous. The NN can approximate a function $f(x) \in C^n(S)$, $x \in \mathbb{R}^n$, which has a jump at $x = c$ and is continuous from the right as

$$f(x) = W^T \cdot \sigma(V^T \cdot x) + W_f^T \cdot \phi[V_f^T \cdot (x - c)] + \varepsilon(x)$$

(17)

where $\varepsilon(x)$ is a functional restructure error vector, $W^T$, $W_f^T$ and $V^T$, $V_f^T$ are nominal constant weight matrices. $\sigma(\cdot)$ and $\phi(\cdot)$ are activation functions for hidden neurons.

For the hysteresis model (16), the piecewise continuous function $F_2$ will be approximated by the augmented NN. In this paper, it is assumed that there exists weight matrix $W$ such that $\|\varepsilon(x)\| \leq \varepsilon_N$ with constant $\varepsilon_N > 0$, for all $x \in \mathbb{R}^n$, and the Frobenius norm of each matrix is bounded by a known constant $\|W\| \leq W_N$ with $W_N > 0$.

3. NN-based compensator and controller design

Given the augmented MLP NN and hysteresis model, a NN-based pre-inversion compensator for the hysteresis is designed to cancel out the effect of hysteresis. In this section, a novel approach is developed to compensate the hysteretic nonlinearity and to guarantee the stability of integrated piezoelectric actuator control system.

3.1 Problem statement

Consider a piezoelectric actuator subject to a hysteresis nonlinearities described by Duhem model. It can be identified as a second-order linear model preceded by hysteretic nonlinearity as follows:

$$m \cdot \ddot{y}(t) + b \cdot \dot{y}(t) + k \cdot y(t) = k \cdot c \cdot \tau_{pr}(t)$$

$$\tau_{pr}(t) = H[y(t)]$$

(18)

where $v(t)$ is the input to piezoelectric actuator, $y(t)$ denotes the position of piezoelectric actuator, $m$, $b$, $k$ denote the mass, damping and stiffness coefficients, respectively, $H(\cdot)$ represents the Duhem model (1).

In order to eliminate the effect of hysteresis on the piezoelectric actuator system, a NN-based hysteresis compensator is designed to make the output from hysteresis model $\tau_{pr}$ approach the designed control signal $\tau_{pd}$. After the hysteresis is compensated by the NN, an
adaptive control for piezoelectric actuator is to be designed to ensure the stability of the overall system and the boundedness of output tracking error of the piezoelectric actuator with unknown hysteresis. We consider the tracking problem, in which \( y(t) \) is to asymptotically track a reference signal \( y_d(t) \) having the properties that \( y_d(t) \) and its derivatives up to second derivative are bounded, and \( \dot{y}_d(t) \) is piecewise continuous, for all \( t \geq 0 \). The tracking error of the piezoelectric actuator is defined as

\[
e_p(t) = y_d(t) - y(t).
\]

A filtered error is defined as

\[
r_p(t) = \dot{e}_p(t) + \lambda_p \cdot e_p(t)
\]

where \( \lambda_p > 0 \) is a designed parameter.

Differentiating \( r_p(t) \) and combining it with the system dynamics Eq. (18), one may obtain:

\[
\frac{m}{k \cdot c} \ddot{r}_p = -\frac{b}{k \cdot c} \cdot \dot{r}_p - \tau_{pr} + \frac{m}{k \cdot c} \cdot (\dot{y}_d + \lambda_p \cdot \dot{e}_p) \\
= \frac{b}{k \cdot c} \cdot (\dot{y}_d + (\lambda_p \cdot \frac{k}{b}) \cdot e_p) + \frac{1}{c} \cdot y_d.
\]

The tracking error dynamics can be written as

\[
\frac{m}{k \cdot c} \ddot{r}_p = -\frac{b}{k \cdot c} \cdot \dot{r}_p - \tau_{pr} + Y_d^T \cdot \theta_p
\]

where \( Y_d = \begin{bmatrix} \dot{y}_d + (\Lambda_p - \frac{k}{b}) \cdot \dot{e}_p & \dot{y}_d + \Lambda_p \cdot e_p & y_d \end{bmatrix}^T \) is a regression vector and

\[
\theta_p = \begin{bmatrix} \frac{m}{k \cdot c} & \frac{b}{k \cdot c} & \frac{1}{c} \end{bmatrix} \in \mathbb{R}^3
\]

is a unknown parameter vector with

\[
\theta_{p_{\text{min}}} \leq \theta_{p_i} \leq \theta_{p_{\text{max}}} \quad i = 1, 2, 3
\]

where \( \theta_{p_{\text{min}}} \) and \( \theta_{p_{\text{max}}} \) are some known real numbers.

### 3.2 NN-based Compensator for Hysteresis

In presence of the unknown hysteresis nonlinearity, the desired control signal \( \tau_{pd} \) for the piezoelectric actuator is different from the real control signal \( \tau_{pr} \). Define the error as

\[
\tilde{\tau}_p = \tau_{pd} - \tau_{pr}
\]
Differentiating (23), yields

$$\dot{\hat{r}}_p = \dot{r}_{pd} - \dot{r}_{pr}$$

(24)

thus, we have

$$\ddot{\hat{r}}_p = \ddot{r}_{pd} - K_0 \ddot{v} + F_2$$

(25)

Here we utilize a second first-layer-fixed MLP to approximate the nonlinear function $F_2$.

$$F_2 = W_{21}^T \sigma(V_{21}^T \cdot h) + W_{22}^T \cdot \varphi_{21} \cdot [V_{f12}^T \cdot (h - c_{21})]$$

$$+ W_{23}^T \cdot \varphi_{23} \cdot [V_{f13}^T \cdot (h - c_{23})] + e_2(h)$$

(26)

where $h = \begin{bmatrix} r_{pd} & r_{p0} & v & \ddot{v} & 1 \end{bmatrix}^T$, $r_{p0}$ is the initial value of the control signal, $V_{21}^T$, $V_{f12}^T$, $V_{f13}^T$, and $V_{f13}^T$ are input-layer weight matrices, $W_{21}^T$, $W_{f12}^T$, $W_{f13}^T$, and $W_{f13}^T$ are output-layer weight matrices, $0$, $\nu_s$, and $-\nu_s$ are jump points on the output layer, and $\sigma(\cdot)$, $\varphi_{21}(\cdot)$, $\varphi_{23}(\cdot)$, and $\varphi_{23}(\cdot)$ are the activation functions, and $e_1(h)$ is the functional restructure error in which inversion error is included. Output-layer weight matrices $W_{21}^T$, $W_{f12}^T$, $W_{f13}^T$, and $W_{f13}^T$ are trained so that the output of NN approximates to the nonlinear function $F_2$.

Let

$$\Theta(h, \nu_s) = \{ \sigma(V_{22}^T \cdot h) \ \varphi_{24} \cdot [V_{f24}^T \cdot (h - c_{24})] \ \varphi_{24} \cdot [V_{f24}^T \cdot (h + c_{24})] \}$$

and $W_1^T = [W_{21}^T \ W_{f12}^T \ W_{f13}^T \ W_{f13}^T]$. The nonlinear function $F_2$ is expressed as:

$$F_2 = W_1^T \Theta(h, \nu_s) + e_1(h)$$

(27)

It is assumed that the Frobenius norm of weight matrix $W_1$ is bounded by a known constant $\|W_1\| \leq W_{1N}$ with $W_{1N} > 0$ and $\|e_1(h)\| \leq e_{1N}$ with constant $e_{1N} > 0$, for all $x \in \mathbb{R}^n$.

The estimated nonlinear function $\hat{F}_2$ is constructed by using the neural network with the weight matrix $\hat{W}_1$:

$$\hat{F}_2 = \hat{W}_1^T \Theta(h, \nu_s).$$

Hence the restructure error between the nonlinear functions $F_2$ and $\hat{F}_2$ is derived as:
\[ \tilde{F}_2 = F_2 - \hat{F}_2 = \tilde{W}_1^T \Theta(h, v_s) + \varepsilon_1(h). \]

“where \( \tilde{W}_1^T = W_1^T - \hat{W}_1^T \).”

**Remark 1** When the input changes its sign derivative (Beuschel et al, 1998), the augmented MLP can approximate the piecewise continuous functions. In the process, the “jump functions” leads to vertical segments in the feed-forward pre-inversion compensation, where the “functional restructure error” can be confronted by the adaptive controller in Section III.C (Selmic & Lewis, 2000).

A hysteresis pre-inversion compensator is designed:

\[
\dot{v} = \hat{\mu} \cdot \{ k_b \cdot \tilde{\tau}_p + \hat{\tau}_{pd} + \tilde{W}_1^T \cdot \Theta(h, v_s) + r_p \} \tag{28}
\]

where \( \hat{\mu} = \frac{a_{min}}{\hat{a}} \) is an estimated constant, which satisfies \( 0 < \hat{\mu} \leq 1 \) with the known boundary of \( a \in [a_{min} \quad a_{max}] \), \( k_b \) is a positive constant, \( \hat{a} \) is the estimated values of \( a \), and \( \tilde{W}_1^T = [\tilde{W}_1^T \quad \tilde{W}_{f21}^T \quad \tilde{W}_{f22}^T \quad \tilde{W}_{f23}^T ] \) is the estimated output-layer weight matrix \( W_1^T \).

Define error matrix as:

\[ \tilde{W}_1^T = W_1^T - \hat{W}_1^T. \]

Inserting (26), (28) into (25), we obtain

\[
\dot{\tilde{\tau}}_p = -k_b \cdot \hat{\mu} \cdot K_a \cdot \tilde{\tau}_p + (1 - \hat{\mu} \cdot K_a) \cdot \hat{\tau}_{pd} + (1 - \hat{\mu} \cdot K_a) \cdot \tilde{W}_1^T \cdot \Theta(h, v_s) - \hat{\mu} \cdot K_a \cdot r_p + \hat{\tau}_{pd} + \tilde{W}_1^T \cdot \Theta(h, v_s) + \varepsilon_1(h) \tag{29}
\]

We choose weight matrix update rule as

\[
\dot{\tilde{W}}_1 = \Gamma \Theta(h, v_s) \cdot \tilde{\tau}_p + k_{p1} \tilde{\tau}_p \cdot \tilde{\tau}_p \cdot \tilde{W}_1 \tag{30}
\]

where \( \Gamma \) is a positive adaptation gain diagonal matrix, and \( k_{p1} \) is a positive constant.

Design the update rule of parameter \( \hat{\mu} \) in pre-inversion compensator \( \dot{v} \) as

\[
\dot{\hat{\mu}} = \text{Proj}(\hat{\mu}, \eta \cdot \tilde{\tau}_p \cdot [\hat{\tau}_{pd} + \hat{\tau}_p]) \tag{31}
\]

where \( \eta \) is positive constant, Proj(.) is a projection operator, which is defined as follows:

\[
\dot{\hat{\mu}} = \text{Pr} of (\hat{\mu}, -\eta \cdot \tilde{\tau}_p \cdot \hat{\mu} \cdot [\hat{\tau}_{pd} + \hat{\Theta}(h, v_s)]) =
\]
The adaptive NN-based pre-inversion compensator $\hat{v}$ is developed to drive the adaptive control signal $\tau_{pd}$ to approach the output of hysteresis model $\tau_{pr}$ so that the hysteretic effect is counteracted.

### 3.3 Controller Design Using Estimated Hysteresis Output

It is noticed that the output of hysteresis is not normally measurable for the plant subject to unknown hysteresis. However, considering the whole system as a dynamic model preceded by Duhem model, we could design an observer to estimate the output of hysteresis based on the input and output of the plant.

The velocity of the actuator $\dot{y}(t)$ is assumed measurable. Define the error between the outputs of actuator and observer as

$$e_1 = y - \hat{y}$$

(33)

The observed output of hysteresis is denoted as $\hat{\tau}_{pr}$ and the error between the output of hysteresis $\tau_{pr}$ and the observed $\hat{\tau}_{pr}$ is defined as $e_2 = \tau_{pr} - \hat{\tau}_{pr}$. Then the observer is designed as:

$$\dot{\hat{y}} = \dot{y} + L_1 e_1$$

(34)

$$\dot{\hat{\tau}}_{pr} = \hat{K}_a \dot{\hat{y}} - \hat{F}_2 + L_2 e_1 - K_{pr} \hat{\tau}_{pr}$$

(35)

The error dynamics of the observer is obtained based on the actuator model and hysteresis model.

$$\dot{e}_1 = -L_1 e_1 = -L_1 e_1$$

$$\dot{e}_2 = \tilde{K}_a \dot{e}_1 - \tilde{F}_2 - L_2 e_1 + K_{pr} \hat{\tau}_{pr}$$

(36)

where the parameter error is defined as $\tilde{K}_a = K_a - \hat{K}_a$.
By using the observed hysteresis output $\hat{\tau}_{pr}$, we may define the signal error between the adaptive control signal $\tau_{pd}$ and the estimated hysteresis output as:

$$\tilde{\tau}_{pe} = \tau_{pd} - \hat{\tau}_{pr}$$  \hspace{1cm} (37)

The derivative of the signal error is:

$$\dot{\tilde{\tau}}_{pe} = \dot{\tau}_{pd} - \dot{K}_a \dot{\psi} + \dot{\hat{F}}_2 - L_2 \dot{e}_1 + K_{pr} \dot{\hat{\tau}}_{pr}.$$  \hspace{1cm} (38)

A hysteresis pre-inversion compensator is designed:

$$\dot{\psi} = \dot{\mu} \cdot \{ k_b \cdot \tilde{\tau}_{pe} + \dot{\tau}_{pd} + \dot{\hat{F}}_2 + r_p \}.$$  \hspace{1cm} (39)

By substituting the neural network output $\dot{\hat{F}}_2 = \dot{\hat{W}}_1^T \Theta(h, v_s)$ and pre-inversion compensator output into the derivative of the signal error, one obtains:

$$\dot{\tilde{\tau}}_{pe} = (1 - \dot{K}_a \dot{\mu}) \dot{e}_{pd} - \dot{K}_a \mu \cdot k_b \dot{\tau}_{pe} + (1 - \dot{K}_a \dot{\mu}) \dot{\hat{W}}_1^T \Theta(h, v_s) - \dot{K}_a \mu \cdot r_p - L_2 \dot{e}_1 + K_{pr} \dot{\hat{\tau}}_{pr}.$$  \hspace{1cm} (40)

The weight matrix update rule is chosen as:

$$\dot{\hat{W}}_1 = \Gamma \Theta(h, v_s) \cdot \tilde{\tau}_{pe} + k_p \left| \tilde{\tau}_{pe} \right| \cdot \tilde{\hat{W}}_1$$  \hspace{1cm} (41)

And the update rule of parameter $\dot{\mu}$ in pre-inversion compensator $\dot{\psi}$ is designed with the same projection operator as (32):

$$\dot{\mu} = \text{Proj} \dot{\hat{\mu}}, \quad \eta \cdot \tilde{\tau}_{pe} \cdot \left[ \dot{e}_{pd} + \dot{\hat{W}}_1^T \Theta(h, v_s) + r_p \right].$$  \hspace{1cm} (42)

The update rule of parameter $\dot{\dot{K}}_a$ in the observer (35) is designed with the same projection operator as (32):

$$\dot{\dot{K}}_a = \text{Proj} \dot{\hat{K}}_a, \quad \gamma \cdot \dot{\mu} \cdot \tilde{\tau}_{pe} \cdot \left[ \dot{e}_{pd} + \dot{\hat{W}}_1^T \Theta(h, v_s) + r_p \right] \cdot \dot{\tau}_{pe}.$$  \hspace{1cm} (43)

Hence we design the adaptive controller and update rule of control parameter as:

$$\tau_{pd} = k_{pd} \cdot r_p + Y_d^T \cdot \dot{\hat{\theta}}_p$$  \hspace{1cm} (44)

$$\dot{\hat{\theta}}_p = \text{Proj} \dot{\hat{\theta}}_p, \quad \beta \cdot Y_d \cdot r_p.$$  \hspace{1cm} (45)
where the projection operator is

$$\{\text{Proj}_{\hat{\theta}_p}(\hat{\theta}_p, \beta \cdot Y_d \cdot r_p)\}_i = \begin{cases} 
0 & \text{if } \hat{\theta}_pi = \theta_{p\max} \text{ and } \beta \cdot (Y_d \cdot r_p)_i < 0 \\
\text{if } \theta_{p\min} < \hat{\theta}_pi < \theta_{p\max} \\
\beta \cdot (Y_d \cdot r_p)_i \text{ or } \hat{\theta}_pi = \theta_{p\max} \text{ and } \beta \cdot (Y_d \cdot r_p)_i \geq 0 \\
\text{or } \hat{\theta}_pi = \theta_{p\min} \text{ and } \beta \cdot (Y_d \cdot r_p)_i \leq 0 \\
0 & \text{if } \hat{\theta}_pi = \theta_{p\min} \text{ and } \beta \cdot (Y_d \cdot r_p)_i > 0 
\end{cases}$$

With the adaptive robust controller, pre-inversion hysteresis compensator and hysteresis observer, the overall control system of integrated piezoelectric actuator is shown in Fig. 3. The stability and convergence of the above integrated control system are summarized in Theorem 1.

**Theorem 1** For a piezoelectric actuator system (18) with unknown hysteresis (1) and a desired trajectory $y_d(t)$, the adaptive robust controller (44), NN based compensator (39) and hysteresis observer (34) and (35) are designed to make the output of actuator to track the desired trajectory $y_d(t)$. The parameters of the adaptive robust controller and the NN based compensator are tuned by the updating rules (41)-(43) and (45). Then, the tracking error $e_p(t)$ between the output of actuator and the desired trajectory $y_d(t)$ converge to a small neighborhood around zero by appropriately choosing suitable control gains $k_{pd}, k_b$ and observer gains $L_1, L_2$ and $K_{pr}$.

**Proof:** Define a Lyapunov function

$$V_2 = \frac{1}{2} \frac{m}{k_c} r_p^2 + \frac{1}{2} \dot{\tau}_{pc}^2 + \frac{1}{2} \text{tr}(\ddot{W}_1^T \Gamma^{-1} \dot{\ddot{W}}_1) + \frac{1}{2} \frac{2}{\eta K_a}(1 - \dot{\mu} K_a)^2$$

$$+ \frac{1}{2} \frac{2}{\gamma}(K_a - \dot{K}_a)^2 + \frac{1}{2} \frac{2}{\beta} (\theta_{p} - \dot{\theta}_{p})^T (\theta_{p} - \dot{\theta}_{p}) + \frac{1}{2} \epsilon_1^2 + \epsilon_2^2 \frac{1}{2}$$

The derivative of Lyapunov function is obtained:

$$\dot{V}_2 = \frac{m}{k_c} \ddot{r}_p + \ddot{\tau}_{pe}^2 - \eta \text{tr}(\dddot{W}_1^T \Gamma^{-1} \dddot{W}_1) - \frac{1}{1 - \dot{\mu} K_a} \dot{\mu} - \frac{1}{\gamma} (K_a - \dot{K}_a)$$

$$- \frac{1}{\beta} (\theta_{p} - \dot{\theta}_{p})^T (\theta_{p} - \dot{\theta}_{p}) + e_1 \dot{e}_1 + e_2 \dot{e}_2$$

Introducing control strategies (39), (44) and the update rules (41)-(43), (45) into above equation, one obtains

$$\dot{V}_2 = \left( \frac{b}{k_c} + k_{pd} \right) \ddot{r}_p - k_b \ddot{\dot{r}}_p + \dddot{\dot{W}}_1^T \Gamma^{-1} \dddot{W}_1$$

$$- e_2^2 \ddot{\tau}_{pe} + K_{pr} \dddot{\tau}_{pe} - L_1 \epsilon_1^2 - (L_2 \epsilon_1 + \dot{\overline{F}_2}) \epsilon_2 + K_{pr} \epsilon_2$$

By using $\dot{\dot{\tau}}_{pr} = \dot{e}_2^2$, $|\dddot{F}| \leq \epsilon_{1N}$ and inequality: $\pm ab \leq \frac{1}{2} a^2 + \frac{1}{2} b^2$, one has:
\[ V_2 = -\left( \frac{b}{k \cdot c} + k_{pd} \right) \cdot \tau_{pr}^2 - k_b \cdot \hat{K}_a \cdot \hat{r}_{pe}^2 + \varepsilon_1(h) \hat{r}_{pe} \cdot - k_p \| \hat{\tilde{W}}_1 \| \cdot \tilde{W}_1 \]
\[ + \frac{1}{2} \varepsilon_2^2 + \frac{1}{2} \varepsilon_3^2 + \frac{1}{2} \tilde{L}_2^2 \varepsilon_1^2 + \frac{1}{2} \tilde{r}_{pr}^2 + \frac{1}{2} \tilde{r}_{pe}^2 + \frac{1}{2} K_{pr} \cdot \tilde{r}_{pr}^2 + \frac{1}{2} K_{pr} \cdot \tilde{r}_{pe}^2 + \frac{1}{2} e_1^2 \]
\[- L_1 \varepsilon_1^2 + \frac{1}{2} (L_2 \varepsilon_1 + \varepsilon_{1N})^2 + \frac{1}{2} e_1^2 + K_{pr} \cdot r_{pr} \cdot e_1 - K_{pr} \cdot e_1^2 \]

By using the inequality \( \frac{1}{2} (a + b)^2 \leq a^2 + b^2 \), we can derive the following inequality:
\[ \dot{V}_2 = -\left( \frac{b}{k \cdot c} + k_{pd} - \frac{1}{2} \right) \cdot \tau_{pr}^2 - (k_b \cdot \hat{K}_a - \frac{1}{2} K_{pr}^2 - 1) \cdot \tilde{r}_{pr}^2 + \| \hat{\tilde{W}}_1 \| \cdot \varepsilon_{1N} \]
\[- k_{pd} \cdot \| \hat{\tilde{W}}_1 \| \cdot \| W_{1N} - \| \hat{\tilde{W}}_1 \| - (L_1 - \frac{3}{2} L_2^2) \varepsilon_1^2 - (K_{pr} - 2) e_1^2 + \varepsilon_{1N}^2 + K_{pr} \cdot e_1^2 \]

From the Property 1 of Chapter 2 in the recent book (Ikhouane & Rodellar, 2007), we know \( \tau_{pr}^2 \) is bounded (say, \( \tau_{pr}^2 \leq M^2 \) where \( M \) is a constant), and then define a constant \( \delta = \varepsilon_{1N}^2 + K_{pr} \cdot M^2 > \varepsilon_{1N}^2 + K_{pr} \cdot \tau_{pr}^2 \) such that
\[ \dot{V}_2 = -\left( \frac{b}{k \cdot c} + k_{pd} \right) \cdot \tau_{pr}^2 - (k_b \cdot \hat{K}_a - \frac{1}{2} K_{pr}^2 - 1) \cdot \tilde{r}_{pr}^2 + \| \hat{\tilde{W}}_1 \| \cdot \varepsilon_{1N} \]
\[- k_{pd} \cdot \| \hat{\tilde{W}}_1 \| \cdot \| W_{1N} - \| \hat{\tilde{W}}_1 \| - (L_1 - \frac{3}{2} L_2^2) \varepsilon_1^2 - (K_{pr} - 2) e_1^2 + \varepsilon_{1N}^2 + K_{pr} \cdot e_1^2 + \delta \]

We select the control parameters \( k_{pd}, k_b \) and observer parameters \( L_1, L_2 \) and \( K_{pr} \) satisfying the following inequalities:
\[ \frac{b}{k \cdot c} + k_{pd} = \frac{1}{2} > 0 \]
\[ K_{pr} > 2 \]
\[ k_b \cdot a_{\text{max}} \cdot \hat{K}_a - \frac{1}{2} K_{pr}^2 - 1 > 0 \]
\[ L_1 > \frac{3}{2} L_2^2 \]

Let \( k_m = k_b \cdot a_{\text{max}} \cdot \hat{K}_a - \frac{1}{2} K_{pr}^2 - 1 \). If we have
\[ \| \tilde{r}_{pe} \| > - \frac{k_{pd} \cdot W_{1N}^2}{2 + 4 + \varepsilon_{1N}} \]
\[ \left\| \tilde{W}_1 \right\| > W_{1N}/2 + \sqrt{W_{1N}^2/4 - \varepsilon_{1N}/k_{pd}} \]

we can easily conclude that the closed-loop system is semi-globally bounded (Su & Stepanenko, 1998).
Hence, the following inequality holds
\[
- k_p \cdot W_{1N}^2 / 4 + \varepsilon_{1N} < b_r
\]
where \( b_r > 0 \) represents the radius of a ball inside the compact set \( C_r \) of the tracking error \( \tilde{\tau}_{pe}(t) \).

Thus, any trajectory \( \tilde{\tau}_{pe}(t) \) starting in compact set \( C_r = \{ \varphi | \| \varphi \| \leq b_r \} \) converges within \( C_r \) and is bounded. Then the filtered error of system \( r_p(t) \) and the tracking error of the hysteresis \( \tilde{\tau}_{pe}(t) \) converge to a small neighborhood around zero. According to the standard Lyapunov theorem extension (Kuczmann & Ivanyi, 2002), this demonstrates the UUB (uniformly ultimately bounded) of \( r_p(t) \), \( \tilde{\tau}_{pe}(t) \), \( \tilde{W}_1 \), \( e_1 \) and \( e_2 \).

**Remark 2** It is worth noting that our method is different from (Zhao & Tan, 2006; Lin et al. 2006) in terms of applying neural network to approximate hysteresis. The paper (Zhao & Tan, 2006) transformed multi-valued mapping of hysteresis into one-to-one mapping, whereas we sought the explicit solution to the Duhem model so that augmented MLP neural networks can be used to approximate the complicated piecewise continuous unknown nonlinear functions. Viewed from a wavelet radial structure perspective, the WNN in the paper (Lin et al. 2006) can be considered as radial basis function network. In our scheme, the unknown part of the solution was approximated by an augmented MLP neural network.

### 4. Simulation studies

In this section, the effectiveness of the NN-based adaptive controller is demonstrated on a piezoelectric actuator described by (18) with unknown hysteresis. The coefficients of the dynamic system and hysteresis model are \( m = 0.016 \text{kg} \), \( b = 1.2 \text{Ns/μm} \), \( k = 4500 \text{N/μm} \), \( c = 0.9 \text{μm/V} \), \( a = 6 \), \( b = 0.5 \), \( v_s = 6 \text{ μm/s} \), \( \beta = 0.1 \), \( k_{pu} = 50 \).

The input reference signal is chosen as the desired trajectory: \( y_d = 3 \cdot \sin(0.2\pi t) \). The control objective is to make the output signal \( y \) follow the given desired trajectory \( y_d \). From Fig. 1, one may notice that relatively large tracking error is observed in the output response due to the uncompensated hysteresis.

The Neural Network has 10 hidden neurons for the first part of neural network and 5 hidden neurons for the rest parts of neural network with three jumping points \( (0, v_s, -v_s) \).

The gains for updating output weight matrix are all set as \( \Gamma = \text{diag}(10)^{5} \). The activation function \( \sigma(x) \) is a sigmoid basis function and activation function \( \phi(x) \) has the definition \( \phi(x) = \frac{1 - e^{-\alpha x}}{1 + e^{-\alpha x}} \) \( x \geq 0 \), otherwise zero. The parameters for the observer are set as: \( K_a = 20 \), \( k_b = 100 \), \( \eta = 0.1 \), \( \gamma = 0.1 \), \( K_{pu} = 10 \), \( L_1 = 100 \), \( L_2 = 1 \) and initial
conditions are \( \hat{y}(0) = 0, \hat{\tau}(0) = 0 \). The system responses are shown in Fig. 2, from which it is observed that the tracking performance is much better than that of adaptive controlled piezoelectric actuator without hysteric compensator.

The input and output maps of NN-based pre-inversion hysteresis compensator and hysteresis are given in Fig. 3, respectively. The desired control signal and real control signal map (Fig. 3c) shows that the curve is approximate to a line which means the relationship between two signals is approximately linear with some deviations.

In order to show the effectiveness of the designed observer, we compare the observed hysteresis output \( \hat{\tau}_{pr} \) and the real hysteresis output \( \tau_{pr} \) in Fig. 4. The simulation results show that the observed hysteresis output signal can track the real hysteresis output. Furthermore, the output of adaptive hysteresis pre-inversion compensator \( v(t) \) is shown in Fig. 5. The signal is shown relatively small and bounded.

Fig. 1 Performance of NN controller without hysteretic compensator (a) The actual control signal (dashed line) with reference (solid) signal; (b) Error \( y_1 - y_d \)

Fig. 2. Performance of NN controller with hysteresis, its compensator and observer (a) The actual control signal (dashed line) with reference (solid) signal; (b) Error \( y_1 - y_d \)
Fig. 3. (a) Hysteresis’s input and output map $v(t)$ vs. $v$; (b) Pre-inversion compensator’s input and output map $v$ vs. $\tau_{pd}$; (c) Desired control signal and Observed control signal curve $\hat{v}_{pr}$ vs. $\tau_{pd}$.

Fig. 4. Observed Hysteresis Output $\hat{v}_{pr}$ and Real Hysteresis Output $v_{pr}$.

Fig. 5. Adaptive Hysteresis Pre-inversion Compensator $v(t)$. 

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5. Conclusion

In this paper, an observer-based controller for piezoelectric actuator with unknown hysteresis is proposed. An augmented feed-forward MLP is used to approximate a complicated piecewise continuous unknown nonlinear function in the explicit solution to the differential equation of Duhamel model. The adaptive compensation algorithm and the weight matrix update rules for NN are derived to cancel out the effect of hysteresis. An observer is designed to estimate the value of hysteresis output based on the input and output of the plant. With the designed pre-inversion compensator and observer, the stability of the integrated adaptive system and the boundedness of tracking error are proved. Future work includes the compensator design for the rate-dependent hysteresis.

6. References


Adaptive control has been a remarkable field for industrial and academic research since 1950s. Since more and more adaptive algorithms are applied in various control applications, it is becoming very important for practical implementation. As it can be confirmed from the increasing number of conferences and journals on adaptive control topics, it is certain that the adaptive control is a significant guidance for technology development. The authors the chapters in this book are professionals in their areas and their recent research results are presented in this book which will also provide new ideas for improved performance of various control application problems.

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