Velocity Observer for Mechanical Systems

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1. Introduction

Many controllers incorporate knowledge of both position and velocity, such as PD, PID and most robust controllers. In many applications direct access to actual velocity is not available. In certain scenarios the velocity signal may contain an excessive amount of noise and is not well suited for use in a control law; in the particular case of measuring robot joint velocities, direct measurement may even be undesirable (Arteaga and Kelly, 2004). Consequently it is necessary to estimate the velocity signal using an observer, and feed it into the controller.

Velocity observer design is a very important topic that continues to be studied, as in (Arteaga and Kelly, 2004; Berghuis and Nijmeijer, 1993; Canudas de Wit and Fixot, 1992). Discontinuous state observers for inexact nonlinear plants have been designed in (Choi et al., 1999; Xiong and Saif, 2001; Xian et al., 2004). However, little research has focused on the specific case of velocity observation for mechanical systems with friction working at low velocities. In these systems friction has been shown to cause mechanical difficulties which are usually unwanted phenomena (Armstrong-Hélouvry et al., 1994). Under such conditions, the use of the stated observers leads to high frequency oscillations in the estimated velocity signal, this can lead to accelerated degradation of system performance, which is why the observer in (Xian et al., 2004) is used as a basis for developing two new observers in (Guerra et al., 2007b) for the specific purpose of being used in mechanical systems with friction working at low velocities. Our objective is to mitigate the high frequency oscillations and increase the reliability of velocity observers.

The observers developed with this approach retain the stability qualities of their predecessor yet do not exhibit the oscillatory behaviour, as will be seen later. The observers presented in (Xian et al., 2004; Guerra et al., 2007b) are numerically compared before proceeding to an experimental verification. Afterwards, one of the observers from (Guerra et al., 2007b) is used as part of a control scheme for mechanical systems which includes a PD controller and an adaptive friction compensator that depends on knowledge of velocity.

2. Observer Design

2.1 Previous Work

Consider the class of mechanical systems described by (Xian et al., 2004):
\[ \dot{x} = h(x, \dot{x}) + G(x, \dot{x})u \]  

(1)

where \( x \in \mathbb{R} \) is the system output, \( u(t) \in \mathbb{R} \) is the control input, and \( h(x, \dot{x}) \in \mathbb{R} \) as well as \( G(x, \dot{x}) \in \mathbb{R} \) are nonlinear functions\(^1\). The system (1) satisfies the following assumptions (Xian et al., 2004):

**Assumption A1.** Both \( h(x, \dot{x}) \in \mathbb{R} \) and \( G(x, \dot{x}) \in \mathbb{R} \) are \( C^1 \) functions.

**Assumption A2.** The control input is a \( C^1 \) function and \( u(t), \dot{u}(t) \in L_\infty \).

**Assumption A3.** The system state is bounded for all time; i.e., \( x(t), \dot{x}(t) \in L_\infty \).

The velocity observer aims to estimate the inaccessible velocity signal \( \dot{x}(t) \) using only the position \( x(t) \) and assuming that \( h(x, x), G(x, \dot{x}) \) and \( u(t) \) are unknown (Xian et al., 2004). The objective is then to ensure that the estimation error tends to zero as time tends to infinity. Consider the velocity observer presented in (Xian et al., 2004):

\[
\begin{align*}
\dot{x} &= p + k_0 \tilde{x} \\
\dot{p} &= k_1 \text{sgn}(\tilde{x}) + k_2 \tilde{x}
\end{align*}
\]

(2)

where \( k_0, k_1, \) and \( k_2 \) are constant observer design parameters and \( \text{sgn}(\cdot) \) is the signum function, let:

\[ N_0(x, \dot{x}, t) = h(x, \dot{x}) + G(x, \dot{x})u(t) \]

(3)

**Theorem 1** (Xian et al., 2004). The observer (2) ensures that the velocity estimation error \( \dot{\tilde{x}}(t) \) tends to zero as time tends to infinity provided that \( k_1 \) satisfies:

\[ k_1 > \|N_0(x, \dot{x}, t)\|_\infty + \|N_0(x, \dot{x}, t)\|_e \]

(4)

For detailed proof see Theorem 2 in (Xian et al., 2004).

### 2.2 Proposed observers

Consider now the following observer (Guerra et al., 2007b):

\[
\begin{align*}
\dot{x} &= p + k_0 \tilde{x} \\
\dot{p} &= -k_1 \text{sgn}(\tilde{x}) + k_2 \tilde{x}
\end{align*}
\]

(5)

where \( k_0, k_1, \) and \( k_2 \) are constant positive observer design parameters.

**Theorem 2** (Guerra et al., 2007b). The observer (5) ensures that the velocity estimation error \( \dot{\tilde{x}}(t) \) tends to zero as time tends to infinity provided that \( k_1 \) satisfies the same restriction as in Theorem 1. The proof is identical.

A third option for estimating velocity is given by (Guerra et al., 2007b):

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\(^1\) Without loss of generality, we have assumed a one Degree-of-Freedom mechanical system.
Theorem 3 (Guerra et al., 2007b). The observer (6) also ensures that the velocity estimation error \( \dot{x}(t) \) tends to zero as time tends to infinity provided that \( k_1 \) satisfies the same restriction as in Theorem 1. The proof is the same as in Theorem 2.

The modifications implemented in these observers are: observer (5) proposes an inversion of the sign in the second term of the estimation dynamic which produces a filtering effect. Observer (6) introduces a change in the argument of the sign function to reduce the high frequency content present in observer (2).

3. Numerical Experiments

Consider a linear motion of unit mass:

\[
\dot{x} = u - f
\]

where \( f \) is the friction force and \( u \) is the control force acting on the mass. Assuming that there is no friction in the system (i.e. \( f = 0 \)) and that \( k_i = 10 \). The PID controller:

\[
u = -k_p (x - x_d) - k_i \int (x - x_d) dt - k_d \dot{x}
\]

makes the closed loop system asymptotically stable with \( k_d = 6, k_p = 3, k_i = 4 \) and the constant reference set at \( x_d = 1 \text{m} \), for full details consult (Canudas de Wit et al., 1995). Since friction is to be expected in mechanical systems, we include it in our simulations using the LuGre model with the parameters given in (Canudas de Wit et al., 1995), thus the friction force \( f \) is obtained as a non linear dynamic. The observer design is completed by setting \( k_0 = k_2 = 10 \). Figure 1 depicts the position and velocity of the system considering that the velocity is available for use in the PID controller, as shown in (Canudas de Wit et al., 1995).

We repeat the experiment in order to test the observers. At this point the actual velocity, not the observers, is used in the control law. The results obtained are shown in Figure 2, where it can clearly be seen that the observer (2) generates a small amplitude chattering (high frequency oscillation). In mechanical systems such signals are undesirable because they can cause damage and accelerate wear, as well as activate un-modelled dynamics. Since the premise of the observers is to be used in systems where velocity is not available, the previous experiment was repeated using instead of the actual velocity, the observed velocity \( \dot{x} \) employing Theorems 1, 2 and 3, the results are shown in Figures 3, 4 and 5, where the slight differences in reached position show that the observer influences system performance. Figure 6 shows the results of modifying the observer gains to \( k_1 = 5 \) and \( k_0 = k_2 = 1 \); Figures 7 and 8 show the results of using the estimated velocity from observers (5) and (6) respectively, in the PID controller (observer (2) becomes unstable).
4. Application to an Industrial Emulator

We proceed to evaluate the observers previously discussed on an experimental testbed.

Fig. 1. Positioning experiment presented in (Canudas de Wit et al., 1995).

Fig. 2. Simulated performance of velocity estimators for observation only.

4.1 Experimental Platform
The experimental evaluation was carried out on an ECP Model 220 industrial emulator which includes a PC-Based control platform and a DC brushless servo system (ECP, 1995). The system includes two motors, one as a servo actuator and one as a disturbance input (not used here), a power amplifier, and two encoders which provide accurate position measurements, i.e., 4000 lines per revolution with 4× hardware interpolation yielding 16000 counts per revolution to each encoder, 1 count (equivalent to 0.000392 radians or 0.0225 degrees) is the lowest measurable angular displacement (ECP, 1995). The system was set up to incorporate inertia and friction. The friction coefficients for the system were found to be
The drive and load disks were connected through a 4:1 speed reduction (Figure 9).

The output voltage signal generated by the system is in the range of ±5 V and is delivered to the motor drive through the DAC, the measurement feedback is a position signal (in counts or radians) measured at the shaft of the two disks by the optical rotary incremental position encoders, which is then read by the microcomputer by means of the counter board and delivered into the PC. A software interface has been built to easily transfer the data collected from the plant (using the ECP USR Executive program) to the Matlab workspace environment, in order to display the results. Four weights of 0.5 Kg each were placed on the
load disk at a radius of 10 cm, while the drive disk remained unweighted. It is worth mentioning that the mechanical system has encoders that provide accurate position measurements but not velocity sensors, i.e., there is no direct access to velocity (ECP, 1995). Under these circumstances we proceed to implement the aforementioned velocity observers, the results obtained are shown in Figures 10 through 12.

4.2 Experimental Results

The control law implemented in all three cases is (Guerra et al., 2007b):

\[ u = -k_p (x - x_d) - k_i \int (x - x_d) \, dt - k_d \ddot{x} \]  

\[ (9) \]

Fig. 7. Simulated performance of observer (5) with modified gains when used in the control law.
with \( k_d = 0.0011, \ k_p = 0.135, \ k_i = 0.4 \). The desired position for the load disk was set to \( x_d = 100 \) [counts] = 0.0392 [radians] = 2.25 [degrees]. It can be seen in Figure 10 that observer (2) produces oscillations of relatively large amplitude high frequency; as previously stated, this is undesirable in mechanical systems. The observer from Theorem 2 significantly reduces these unwanted behaviours after a transient, as seen in Figure 11. The observer (6) further reduces both the amplitude and the duration of the transient and it eliminates the chattering effect, as seen in Figure 12. It should be noted that the rectangular shaped limit cycles clearly visible in Figures 11 and 12 follow the behaviour presented in (Canudas de Wit et al., 1995) whereas in Figure 10 they are indistinguishable (Guerra et al., 2007b).

5. Inclusion in Friction Compensation Strategies

Having seen that observer (6) exhibits better performance in the mechanical system, it was decided to include it in friction compensators that rely on knowledge of the unavailable velocity. Four compensators were selected: two for positioning applications and two for trajectory tracking applications. The purpose of this experiment is to compare the performance of the friction compensators in a real system, this would not have been possible without an adequate velocity observer, as both the PD control law and the friction compensator depend on knowledge of the velocity. The stability analysis and full details for each compensator can be found in the mentioned references.

![Mechanical System](image-url)
5.1 Positioning Applications
The two positioning application friction compensators considered were presented in (Friedland & Park, 1992) and (Guerra et al., 2007a). These compensators are very similar and both have very good performance in simulated experiments. In our system they achieved the results shown in Figure 13.

5.2 Trajectory Tracking Applications
The trajectory tracking application friction compensators considered were presented in (Liao & Chien, 2000) and (Guerra & Acho, 2007). Again the structure of the compensators is very similar as are the simulated results that show very good performance. The experimental results obtained are shown in Figure 14.

![Fig. 10. Experimental results with the observer from Theorem 1.](image)

6. Conclusions
Two velocity observers are presented in this chapter that are based on the one presented in (Xian et al., 2004) but have practical advantages. It has been shown with both numerical and experimental results that the proposed observers can accurately estimate velocity and avoid chattering that is undesirable in mechanical systems, when there is only access to position measurements. The observers (5) and (6) are especially interesting for industrial applications, since it has been shown that velocity sensing hardware can be replaced with reliable inexpensive software without difficulty.

![Fig. 11. Experimental results with the observer from Theorem 2.](image)

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![Experimental results with the observer from Theorem 3.](image)

8. References


Fig. 13. Experimental results using observer (6) in a friction compensation scheme for positioning applications. Left: using the compensator from (Friedland & Park, 1992). Right: using the compensator from (Guerra et al., 2007a).

Fig. 14. Experimental results using observer (6) in a friction compensation scheme for trajectory tracking applications. Left: using the compensator from (Liao & Chien, 2000). Right: using the compensator from (Guerra & Acho, 2007).

This book represents the contributions of the top researchers in the field of robotics, automation and control and will serve as a valuable tool for professionals in these interdisciplinary fields. It consists of 25 chapter that introduce both basic research and advanced developments covering the topics such as kinematics, dynamic analysis, accuracy, optimization design, modelling, simulation and control. Without a doubt, the book covers a great deal of recent research, and as such it works as a valuable source for researchers interested in the involved subjects.

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