Worm-like Locomotion Systems (WLLS) – Theory, Control and Prototypes

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1. Introduction

Most of biologically inspired locomotion systems are dominated by walking machines - pedal locomotion. A lot of biological models (bipedal up to octopedal) are studied in the literature and their constructions were transferred by engineers in different forms of realization. Non-pedal forms of locomotion show their advantages in inspection techniques or in applications to medical technology for diagnostic systems and minimally invasive surgery (endoscopy). These areas of application were considered by (Choi et al., 2002), (Mangan et al., 2002), (Menciassi & Dario, 2003). Hence, this type of locomotion and its drive mechanisms are current topics of main focus.

In this chapter we discuss the problem of developing worm-like locomotion systems, which have the earthworm as a living prototype, from two points of view:

• modelling and controlling in various levels of abstraction,
• designing of prototypes with classical and non-classical forms of drive.

2. Motion and Control of WLLS

2.1 General Approach to WLLS

The following is taken as the basis of our theory:

i. A worm is a terrestrical locomotion system of one dominant linear dimension with no active legs nor wheels.

ii. Global displacement is achieved by (periodic) change of shape (such as local strain) and interaction with the environment.

iii. The model body of a worm is a 1-dimensional continuum that serves as the support of various fields.

The continuum in (iii) is just an interval of a body-fixed coordinate. Most important fields are: mass, continuously distributed (with a density function) or in discrete distribution (chain of point masses), actuators, i.e., devices which produce internal displacements or forces thus mimicking muscles, surface structure causing the interaction with the environment.

Observing the locomotion of worms one recognizes first a surface contact with the ground. It is well known, that, if there is contact between two bodies (worm and ground), there is some kind of friction, which depends on the physical properties of the surfaces of the bodies. In particular, the friction may be anisotropic (depends on the orientation of the relative displacement). This interaction (mentioned in (ii)) could emerge from a surface texture as asymmetric Coulomb friction or from a surface endowed with scales or bristles (we shall speak of spikes for short) preventing backward displacements. It is responsible for the conversion of (mostly periodic) internal and internally driven motions into a change of external position (undulatory locomotion (Ostrowski et al., 2006)), see (Steigenberger, 1999) and (Zimmermann et al., 2003).

One of the first works in the context of worms, snakes and scales is the paper of (Miller, 1988). The author considers, in a computer graphics context, mass-spring systems with scales aiming at modelling virtual worms and snakes and their animations. Summarizing, we consider a WLLS in form of a chain of point masses in a common straight line (a discrete straight worm), which are connected consecutively by linear visco-elastic elements, see (Behn, 2005), (Behn & Zimmermann, 2006), (Zimmermann et al., 2002), (Zimmermann et al., 2003) for instance and Fig. 1.

2.2 Kinematic Theory of WLLS
In this subsection we focus on interaction via spikes. The kinematics of this DOF = n + 1 system formulates as follows. The spikes entail the differential constraint
\[ \dot{x}_i(t) \geq 0, \quad i = 0, \ldots, n, \quad \forall t \] (1)
whence with \( S_i = x_0 - x_i \) there holds \( \dot{x}_0 \geq S_i \) for all \( i \) and thus
\[ \dot{x}_0 = V_0 + w, \quad V_0 := \max \{ S_i, \quad i = 0, \ldots, n \}, \quad w \geq 0. \] (2)
w is a common part of the velocities, it describes a rigid motion of the system. The value \( w \) remains undetermined in kinematics, it only follows from dynamics. The dynamics of the WLLS are governed by the momentum laws of the point masses,

\[
mx_i = g_i + \mu_i - \mu_{i+1} + \dot{\lambda}_i, \quad i = 0, \ldots, n,
\]

where the \( g_i \) are external physically given forces, \( \mu_i \) are internal forces (\( \mu_0 = \mu_{n+1} := 0 \)), whereas \( \lambda_i \) are the reaction forces corresponding to the one-sided constraint (1), so there hold the complementary slackness conditions

\[
\dot{x}_i \geq 0, \quad \dot{\lambda}_i \geq 0, \quad \dot{x}_i \cdot \dot{\lambda}_i = 0.
\]

Let \( g_i + \mu_i - \mu_{i+1} = f_i \), then for any motion (4) will be satisfied by the 'controller'

\[
\dot{\lambda}_i = -\frac{1}{2} (1 - \text{sign}(\dot{x}_i))(1 - \text{sign}(f_i))f_i.
\]

Now let us suppose a kinematic drive, i.e., the actuators are assumed to precisely prescribe the mutual distances \( l_j \) as functions of time, see Fig. 2.

![Fig. 2. WLLS with kinematic drive](image)

The kinematic drive implies the holonomic constraint

\[
x_{j-1} - x_j - l_j(t) = 0, \quad j = 1, \ldots, n
\]

with \( \mu_j \) as respective reactions. \( \dot{s}_i \) and \( V_0 \) are now given functions of time, the DOF of the system shrinks to 1.

We confine the external forces to \( g_i = -k\dot{x}_i - \Gamma \) then, summing up all the momentum laws (3) while observing \( \dot{x}_i = V_0 - \dot{s}_i + w \) there follows a bimodal ADE for \( w \) and \( \dot{\lambda} := \frac{1}{n+1} \sum_{0}^{n} \dot{\lambda}_i \),

\[
\begin{align*}
m \ddot{w} + k w + \sigma(t) &= \lambda, \\
\lambda &\geq 0, \quad w \geq 0, \quad w \dot{\lambda} = 0, \\
\sigma &= m\dot{W}_0 + kW_0 + \Gamma, \\
W_0 := V_0 &= -\frac{1}{n+1} \sum_{0}^{n} \dot{s}_i.
\end{align*}
\]
In mode 1 \( \{w > 0; \lambda = 0\} \) no point mass is at rest whereas in mode 2 \( \{w = 0, \lambda > 0\} \) at least one does not move (active spike). If we set \( w = 0 \) then all what follows is called kinematic theory.

It is easy to deduce: If \( \sigma(t) > 0 \) for all \( t \) then the motion is always in mode 2, at any time at least one spike is active, the motion is well-determined by kinematics.

We consider an example with \( n = 2 \). The actuators are chosen so as to generate \( l_1 \) and \( l_2 \) as T-periodic piecewise cosine functions. This given gait will be reconsidered in Subsection 2.3. Fig. 3 shows \( l_1, l_2 \) vs. \( t/T \) (left), and with some system data \( m, k, \Gamma \) chosen the corresponding worm motion (right, \( t \)-axis upward).

![Fig. 3. Gait and worm motion governed by kinematic theory](image)

Mind that one spike is active (resting point mass) at any time. So this gait might be suitable for motion ‘in the plane’. An uphill gait (two resting point masses an any time) could easily be constructed, too (Steigenberger, 2004). To ensure \( w = 0 \) and sufficiently small \( \lambda \) (to prevent the spikes from breakdown) certain restrictions for \( T \) and \( \Gamma \) must be obeyed, see (Steigenberger, 2004).

Two items deserve particular observation.

i. Using common actuators the proper control variable \( u \) is not \( l_i \) or \( \dot{l}_i \) but rather an electrical voltage or a pressure applied to some piezo or hydraulic device that in turn acts to the point masses via some visco-elastic coupling. In this case there remains

\[
\text{DOF} = n + 1 \quad \text{and the internal forces} \quad \mu_j \quad \text{follow a law of the form}
\]

\[
\mu_j = c(x_{j+1} - x_j) + d(x_{j+1} - x_j) + u_j(t)
\]

ii. By chance it could practically be promising to consider asymmetric dry friction instead of spikes (though the simple kinematical theory then is passé). In a rough terrain, unknown or changing friction coefficients lead to uncertain systems and require adaptive control to track a kinematic gait, that has been discovered as a favourable one. This objective will be addressed in the next section.
2.3 WLLS as a Dynamical Control System

In this subsection we model the ground interaction as an asymmetric Coulomb dry friction force $F$ (see above), which is taken to be different in the magnitude depending on the direction of each point mass motion:

$$
\dot{x} \mapsto F(x) = \begin{cases} 
-F^+, & x > 0 \\
F^0, & x = 0 \\
F^-, & x < 0 
\end{cases}
$$

(9)

where $F^+, F^- > 0$ are fixed with $F^- >> F^+$ and $F^0$ is arbitrary, $F^0 \in \{F^-, -F^+\}$.

For later simulation we restrict the number of point masses to $n = 2$, but we point out that our theory is valid for fixed but arbitrary $n \in \mathbb{N}$, see (Behn & Zimmermann, 2006).

Mathematical model

Firstly, we derive the differential equations of motion of the WLLS by using Newton’s second law:

$$
m_0 \ddot{x}_0 = -c_1 (x_0 - x_1) - d_1 (\dot{x}_0 - \dot{x}_1) + F_0(\dot{x}_0) + u_1(t)
$$

$$
m_1 \ddot{x}_1 = -c_1 (x_1 - x_2) + c_2 (x_0 - x_1) - d_1 (\dot{x}_1 - \dot{x}_2) + d_2 (\dot{x}_0 - \dot{x}_1) + F_1(\dot{x}_1) + u_2(t) - u_1(t)
$$

$$
m_2 \ddot{x}_2 = c_2 (x_1 - x_2) + d_2 (\dot{x}_1 - \dot{x}_2) + F_2(\dot{x}_2) - u_2(t)
$$

(10)

with $x_0(0) = x_{00}$, $x_1(0) = x_{10}$, $x_2(0) = x_{20}$, $\dot{x}_0(0) = x_{01}$, $\dot{x}_1(0) = x_{11}$, $\dot{x}_2(0) = x_{21}$ (all initial values are real numbers) and $g_i = 0$. Putting

$$
u_1 := c_1 l_1 \text{ and } u_2 := c_2 l_2
$$

(11)

then $u_{ij}$ is in fact a control of the original spring length. Therefore, we have internal inputs and no longer external force inputs, as in (Behn, 2005). New outputs of this system could be the actual distances of the point masses

$$y_1 := x_0 - x_1 \text{ and } y_2 := x_1 - x_2.
$$

(12)

Therefore, this system (10), (12) is described by a mathematical model that falls into the category of quadratic, nonlinearly perturbed, minimum phase, multi-input $u(\cdot)$, multi-output $y(\cdot)$ systems with strict relative degree two. In a normalized form (after elaborate transformations) the zero dynamics of the system are decoupled from the controlled part of the system.
Control Objective

For the further analysis of this system we suppose that the masses are all equal, but unknown, also the damping factors and spring stiffnesses, and the friction magnitudes as well (uncertain systems).

The consideration of uncertain systems leads to the use of adaptive control. The aim is to design universal adaptive controllers, which learn from the behavior of the system, so automatically adjust their parameters and achieve a pre-specified control objective. Simple adaptive mechanisms, which achieve tracking of a given reference signal within any pre-specified accuracy $\lambda$, will be introduced. $\lambda > 0$ denotes the size of the feasible tracking error (we do not focus on exact tracking).

Precisely, given an arbitrarily small $\lambda > 0$, a control strategy $y \mapsto u$ is sought which, when applied to the system, achieves $\lambda$-tracking ($\lambda$-tracking control objective) for every reference signal $y_{ref}(\cdot)$ (belonging to a certain function class, for instance a given favoured kinematic gait presented in the previous subsection), i.e., the following:

i. every solution of the closed-loop system is defined and bounded on $R_{\geq 0}$, and

ii. the output $y(\cdot)$ tracks $y_{ref}(\cdot)$ with asymptotic accuracy quantified by $\lambda > 0$ in the sense that $\max\left\{0, \left\| y(t) - y_{ref}(t) \right\| - \lambda \right\} \to 0$ as $t \to +\infty$.

The last condition means that the error $e(t) = y(t) - y_{ref}(t)$ is forced, via the adaptive feedback mechanism (controllers (13) and (14)), towards a ball around zero radius $\lambda > 0$, see Fig. 4.

![Fig. 4. The $\lambda$-radius around the reference signal](image)

Controllers

Let us consider the following two $\lambda$-trackers, see also (Behn, 2005).
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\[ e(t) := y(t) - y_{ref}(t) \]
\[ u(t) = -k(t) e(t) + \frac{d}{dt}[k(t) e(t)] \]
\[ k(t) = \max\{0, \|e(t)\| - \lambda\}^2 \]  

with \( k(0) = k_0 \in \mathbb{R}, \lambda > 0, y_{ref}(t) \in W^{2,\infty} \) (a Sobolev-Space), \( u(t), e(t) \in \mathbb{R}^2 \) and \( k(t) \in \mathbb{R} \).

The second one includes a dynamic compensator due to a controller of (Miller & Davison, 1991). This controller allows us to avoid the drawback of using the derivative of the output, mentioned above, in the following way:

\[ e(t) := y(t) - y_{ref}(t) \]
\[ u(t) = -k(t) e(t) + \frac{d}{dt}[k(t) \dot{e}(t)] \]
\[ \dot{e}(t) = -k(t)^2 \dot{e}(t) + k(t)^2 e(t) \]
\[ k(t) = \max\{0, \|e(t)\| - \lambda\}^2 \]  

with \( \dot{e}(0) = \dot{e}_0, k(0) = k_0 \in \mathbb{R}, \lambda > 0, y_{ref}(t) \in W^{2,\infty} \), \( u(t), e(t) \in \mathbb{R}^2 \) and \( k(t) \in \mathbb{R} \).

We stress, that the controller (14) does not invoke any derivatives.

The structure of the feedback law and the simple adaptation law of the controllers in this subsection already exist in the literature, but they were only applied to systems with relative degree one. The considered WLLS has relative degree two. Therefore, the novelty is the application of the controller to systems with relative degree two. There are only a few papers which focus the adaptive \( \lambda \)-tracking problem for system with relative degree two, but the feedback law here is simpler than the introduced ones in (Dragan & Halanay, 1999), (Ye, 1999), (Miller & Davison, 1991).

These controllers achieve \( \lambda \)-tracking (for the proofs the reader’s attention is invited to (Behn, 2005)) as presented in the following simulations. We apply both controllers (13) and (14) to system (10), (12) to track the given gaits.

2.4 Simulations

In this subsection we apply the presented simple adaptive \( \lambda \)-tracking control strategies to our WLLS (dynamical control system) in order to track given reference signals. Firstly, we try to track a time-shift \( \sin(\cdot) \)-signal, and, second, a kinematic gait developed in (Steigenberger, 2004) to achieve a certain movement of the WLLS. The numerical simulations will demonstrate and illustrate that the adaptive controllers work successfully and effectively. We point out, that the adaptive nature of the controllers is expressed by the arbitrary choice of the system parameters. It is obvious that for numerical simulation the system data have to be chosen fixed and known, but the controllers are able to adjust their gain parameter to each set of system data.

The tracking results when applying controller (13) to our WLLS are already presented in (Behn & Zimmermann, 2006). Therefore, we choose the same parameters (dimensionless) as there to obtain comparable simulations in using the second controller (14). Then we have:
• system: \[ m_0 = m_1 = m_2 = 1, \quad c_1 = c_2 = 10, \quad d_1 = d_2 = 5, \quad x_{00} = 0, \quad x_{01} = 0, \quad x_{11} = 0, \quad x_{21} = 0; \]

• Coulomb friction forces: \[ F^+ = 1, \quad F^- = 10, \] hence we have by an approximation \[ v_i \mapsto 5.5 \tanh(40 v_i) - 4.5, \] \[ i = 0, \ldots, 2; \]

• \( \lambda \)-tracker (controller): \( \lambda = 0.2, \ k_0 = 1, \ \varphi_1(0) = 1, \ \varphi_2(0) = 1. \)

In order to detect differences we present the simulation results with the \( \lambda \)-trackers (14) and (13), respectively, side by side.

Tracking of a Gait Presented in Fig. 3, left

This gait is derived from the theory of a chain of point masses with spikes and links of given \( t \)-dependent lengths. Such a theory is essentially kinematical and does not require dynamics from the very beginning, see Subsection 2.2. In (Steigenberger, 2004) two paradigmatic gaits for a three point system were derived such that at any time during motion the same (prescribed) number of spikes is active: one in a fast `in-plane gait', two in a slow `up-hill gait'. We try to track this fast gait in our dynamical theory, it is for \( t \in [0,1] \):

\[
t \mapsto y_{\text{ref}}(t) = \begin{cases} 
0 \lfloor 1 - \epsilon(1 + \cos(3\pi)) \rfloor, & t \in [0, 2/3), \\
0 \lfloor 1 - \epsilon(1 - \cos(3\pi)) \rfloor, & t \in [2/3, 1), \\
0 [1 - 2\epsilon], & t \in [1/3, 2/3), \\
0 \lfloor 1 + \epsilon(1 + \cos(3\pi)) \rfloor, & t \in [2/3, 1].
\end{cases}
\]

(15)

where \( l_0 \) is the original length (dimensionless chosen as 2 units) and \( 2\epsilon = 0.3 \) is the elongation. This gait is periodically repeated. We obtain the following simulations.

![Fig. 5. Outputs and \( \lambda \)-strips – left: for (14), right: for (13)](image_url)
Fig. 5 shows the outputs of the systems and the according \( \lambda \) -strips. The reference signal is tracked very quickly with controller (13) in comparison to controller (14). In Fig. 5, left, the...
outputs are not captured yet. The gain parameters, shown in Fig. 6, increase as long as the outputs are outside the $\lambda$-strips. Fig. 7 shows the necessary control inputs, and Fig. 8 the corresponding motions of the worm. It can be seen that controller (13) works more effectively than controller (14) because we feedback more information about the output derivative than (14), which has to estimate the derivative. Hence, in the simulation with controller (14), the outputs are not captured on the considered time interval and the gain parameter is still increasing. Fig. 6, right, clearly shows the convergence of the gain parameter in the simulation with controller (13).

### 2.5 Electromechanical Prototype

A prototype was designed in order to check the above-mentioned theory (Abaza, 2006). It consists of two stepping motors and a dummy to produce a three point mass worm system (Fig. 9). Each stepping motor can separately travel along a threaded rod in both directions with different controllable speeds to generate $l_i$.

![Fig. 9. Three point mass worm system](image)

Additionally, a special bristle-structure had been integrated (Fig. 10) to prevent the point masses from slipping backwards. Experiments justified the results of the theory.

![Fig. 10. The bristle-structure](image)

### 2.6 Open Problems

The following problems should be considered for future work:
- algorithms to construct preferable kinematic gaits;
• to include a suitable friction law allowing for stiction;
• revised control objectives
  - keep spikes (improved Coulomb friction with a big stiction coefficient for negative ve-
locities $F_0$);
  - prescribe $y_{ref}(\cdot)$ as a kinematic gait;
  - let $c_i$ and $d_i$ be random (possibly $t$-dependent).

3. WLLS Using Deformable Magnetizable Media

3.1 Introductional Remarks
The realization of locomotion systems using the deformation of magnetizable materials (a
magnetic fluid or a magnetizable polymer) in a magnetic field is an actual problem. The
initiator of motion in such devices is an alternate magnetic field, which forms to exterior
sources (electromagnetic system or motion permanent magnets). In (Zimmermann et al.,
2004a, b) the theory of a flow of layers of magnetizable fluids in a travelling magnetic field is
considered. It is shown, that the travelling magnetic field can create a flux in the fluid layers.
This effect can be applied for the realization of locomotion. A micro-robot with individual
cells corresponding to the earthworm’s segment and sealed with water-based magnetic fluid
is developed by (Saga & Nakamura, 2002, 2004).
In the present chapter theoretical and experimental possibilities of using deformable mag-
etizable media as actuators for mobile robots are investigated.
In the subsection 2.5 a classical electromechanical drive was used for controlling the distance
between the two masses of the WLLS. In (Zimmermann et al. 2003) we considered a system
of two masses connected by a piezo element. In the following second subsection we used a
magneto-elastic element as an internal drive. The undulatory locomotion of an biological
inspired artificial worm is based on travelling waves on its surface. Therefore the expression
for the magnetic field strength creating a sinusoidal wave on the surface of a viscous mag-
etic fluid as a function of the characteristics of the fluid (viscosity, surface tension, and
magnetic permeability) and the parameters of the wave are obtained in the third also theo-
retically oriented subsection of this section. In the fourth subsection the motions of three
samples of a magnetizable body (magnetizable worm) in an alternate magnetic field are
studied experimentally for large diapason of the electromagnetic system frequency. The
prolate bodies of the magnetizable composites (an elastic polymer and solid magnetizable
particles) and a capsule with a magnetic fluid are used. The analytical estimation and nu-
merical calculation of the deformation of the bodies in applied magnetic field and of the
velocity of the bodies are done. A deformable magnetizable worm in a magnetic field is a
prototype of a mobile crawling robot. Such devices have some characteristics, which allow
to use them in medicine and biology. For example, it does not contain solid details contact-
ing with a surrounding medium and it moves autonomously.
The fives subsection deals with fundamental investigations necessary for the design and the
application of segmented artificial worms, which have the earthworm as living prototype,
and of new passive locomotion systems. A important question in this interconnection is the
estimation of the pressure distribution in the magnetic fluid under the influence of a con-
trolled magnetic field.
3.2 Modelling of WLLS with Magneto-Elastic Elements

In (Turkov, 2002) a deformation of the elastic composite body, when a magnetic field is applied, is considered. A formula, which allows to calculate the deformation of a parallelepiped in noninductive approximation was obtained. If the Lame coefficients $\lambda$ and $\eta$ satisfy the condition $\lambda \gg \eta$ then relative lengthening in the direction of the axis $OX$ (for small strains), when magnetic field $\mathbf{H} = H_0$ in orthogonal direction is applied, is given by expression $u = \frac{p}{E_{y}^{\text{eff}}} + u_0$ (Fig. 11). Here $p$ is the density of surface force, $u_0$ is the deformation of the sample, produced by the action of the magnetic field only in absence of force ($p = 0$), $E_{y}^{\text{eff}}$ is the effective value of the Young modulus

$$u_0 = \frac{\mu_0}{12 \eta} \left[ \frac{2(\mu^0 - 1) - a_1}{1 + \frac{2}{3} G} \right] H_0^2 > 0, \quad E_{y}^{\text{eff}} = 3 \eta \left[ \frac{1 + \frac{2}{3} G}{1 + \frac{4}{3} G} \right], \quad G = \frac{\mu_0 a_1 H_0^2}{2 \eta}.$$  

Fig. 11. Model of a WLLS with two mass points, non-symmetric Coulomb friction and a periodic internal force $G(t)$ produced by an external magnetic field $H$

The value $\mu_0 = 4 \pi \times 10^{-7} \text{N/A}^2$ is the permeability of free space, $\mu^0$ is the permeability of the unstrained material, $a_1 = -\frac{1}{2} (\mu^0 - 1)^2$. By the action of a magnetic field the sample of the composite with a drop of a magnetic fluid lengthens, and the value modulus $E_{y}^{\text{eff}}$ in a magnetic field has to become less than without a magnetic field. The experimental dependencies between the stress and the strain without influence of field ($H = 0$) and with influence of a field ($H = 1.5 \times 10^5 \text{A/m}$) for a sample with the length $l_0 = 29 \text{mm}$ and the area of cross section $S_0 = 5.5 \text{mm}^2$ is shown in Fig. 12.
The size of the ferromagnetic particles in the magnetic fluid is about 110 Å. The experiment allowed us to define $\frac{u}{u_0} = 0.071$.

The motion of a system of two material points $x_1$ and $x_2$ with the masses $m$, connected by a spring of stiffness $c = E_{eff} \cdot S_0 / l_0$ ($\omega = c/m$) along an axis $X$ is considered (Fig. 11). It is supposed that the points are under the action of a small non-symmetric Coulomb dry frictional force $F(x) = F(x)$, depending on velocities $\dot{x} = \dot{x}_i$ ($i = 1, 2$), where $F(x) = F^+$ if $\dot{x} > 0$, $F(x) = -F^-$ if $\dot{x} < 0$, $F(x) = F_0$ if $\dot{x} = 0$; $F^+ < F_0 < F^+$, $F^+ > F^-$ $\geq 0$. By influence of an external harmonic magnetic field a small harmonic internal forces is produced:

$$G(t) = \frac{1}{2} u_0 c/l_0 (1 + \cos \psi), \psi = \nu \tau.$$  

We introduce the dimensionless variables (the dimensional variables are denoted here with the asterisk):

$$x_1, x_2 = \frac{x_1^*, x_2^*}{l_0}, \tau = \frac{\tau^*}{\omega}, \nu = \frac{\nu^*}{\omega}, F = F^* \left( \frac{1}{2} u_0 c l_0 \right)^{\frac{1}{2}}, \epsilon = \frac{1}{2} u_0 << 1.$$  

The dimensionless equations of the motion are:

$$\begin{align*}
\dot{x}_1 + x_1 - x_2 &= -\epsilon [F(\dot{x}_1) + (1 + \cos \psi)], \\
\dot{x}_2 + x_2 - x_1 &= -\epsilon [F(\dot{x}_2) - (1 + \cos \psi)].
\end{align*}$$

To the system (16) we apply the procedure of averaging. For this purpose we introduce new variables: the velocity of the center of mass $\dot{V} = (\dot{x}_1 + \dot{x}_2)/2$ and the deviation relatively to the center of mass $\dot{Z} = (x_2 - x_1)/2$. Replacing $\dot{z} = a \cos \phi$, $\dot{z} = -a \Omega \sin \phi$, $\phi = \Omega \tau + \theta$, $\Omega = \sqrt{2}$ where $V, a, \theta$ - slow variables, we receive system (16) in a standard form:

$$\begin{align*}
\dot{V} &= -\frac{\epsilon}{2} [F(V + a \Omega \sin \phi) + F(V - a \Omega \sin \phi)], \\
\dot{a} &= -\frac{\epsilon}{2 \Omega} \sin \phi [F(V + a \Omega \sin \phi) - F(V - a \Omega \sin \phi) + 2(1 + \cos \psi)], \\
\dot{\phi} &= \Omega - \frac{\epsilon}{2a \Omega} \cos \phi [F(V + a \Omega \sin \phi) - F(V - a \Omega \sin \phi) + 2(1 + \cos \psi)], \\
\dot{\psi} &= \nu.
\end{align*}$$

Fig. 12. Stress p vs. strain u
We investigate the system (17) in a vicinity of the main resonance $\nu = \Omega + \varepsilon \Delta$ ($\Delta \neq 0$), introducing a new slow variable $\xi = \psi - \phi$. After averaging on a fast variable $\phi$ we obtain:

$$
\dot{V} = \begin{cases} 
-\varepsilon \left( \frac{F^+ - F^+}{\pi} \arcsin \frac{V}{a\Omega} - \frac{F^- - F^+}{2} \right) & \text{if } 0 \leq V < a\Omega, \\
-\varepsilon F^+ & \text{if } V \geq a\Omega, 
\end{cases}
$$

$$
\dot{a} = \begin{cases} 
-\varepsilon \frac{F^+ + F^+}{\Omega} \sqrt{1 - \frac{\nu^2}{a^2\Omega^2} + \frac{a}{2} \sin \xi} & \text{if } 0 \leq V < a\Omega, \\
-\varepsilon \frac{a}{2\Omega} \sin \xi & \text{if } V \geq a\Omega, 
\end{cases}
$$

$$
\xi = \varepsilon \left( -\frac{1}{2\Omega} \cos \xi + \Delta \right).
$$

(18)

We are interested in an approximately steady motion as a single whole, therefore we seek for the solution $\dot{V} = 0$. Then from system (18) we have $\dot{a} = \xi = 0$ and

$$
V = \frac{\sin \Phi}{|A|} \sqrt{\frac{1}{4} - E^2 \cos^2 \Phi}, \quad a = \frac{1}{\Omega|A|} \sqrt{\frac{1}{4} - E^2 \cos^2 \Phi}, \quad E \cos \Phi \leq \frac{1}{2},
$$

$$
\xi = \arccos \left[ -\frac{2A}{|A|} \sqrt{\frac{1}{4} - E^2 \cos^2 \Phi} \right], \quad E = \frac{F^+ + F^+}{\pi}, \quad \Phi = \frac{\pi}{2} \frac{F^+ - F^+}{F^- + F^+}.
$$

(19)

The result of the numerical integration of the exact system (1) is given in Figure 13.

![Fig. 13. Velocity V vs. time t](image)

The following values of parameters for a magneto-elastic sample as described in Section 1 are taken: $m = 3 \times 10^{-3} \text{kg}$, $E_{\text{eff}} = 1.6 \times 10^4 \text{N/m}^2$, $u_0 = 0.071$, $l_0 = 29 \text{mm}$, $S_0 = 5.5 \text{mm}^2$, $\omega = 32 \text{s}^{-1}$. So, the values of the parameters are: $\varepsilon = 0.036$, $F_+ = 0.5$, $F_- = 1$, $\Delta = 0.5$. The
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formula (4) gives the value for the dimension velocity of center of mass \( V = 0.29 \text{ m/s} \). The average velocity of such locomotion systems depends on the difference of the friction coefficients between the system and the substrate, which depends on the directions of motion.

3.3 Locomotion Using a Magnetic Fluid in a Traveling Magnetic Field

We consider a plane flow of an incompressible viscous magnetic fluid layer along a horizontal surface in a non-uniform magnetic field (see Fig. 14). The magnetic permeability of the fluid \( \mu \) is assumed to be constant. The pressure on the free fluid surface is constant. In the case of a constant magnetic permeability, the body magnetic force is absent and the magnetic field manifests itself in a surface force acting on the free surface (Rosensweig, 1985).

![Fig. 14. Magnetic fluid layer](image)

The gravity is not taken into account. In this case, the system of equations consists of the continuity and Navier-Stokes equations:

\[
\text{div} \ V = 0, \quad \frac{\partial V}{\partial t} + (V \cdot \nabla) V = -\frac{1}{\rho} \nabla p + \frac{\eta}{\rho} \Delta V.
\]

Here, \( V \) is the velocity vector (\( u \) and \( w \) are the horizontal and vertical coordinates), \( p \) is the fluid pressure, \( \eta \) is the dynamic fluid viscosity coefficient, \( \rho \) is the fluid density, and \( t \) is time.

On the rigid substrate \( z=0 \), the no-slip condition is satisfied:

\[ V(z=0)=0 \]

On the free surface \( z=h(x,t) \), conditions of two types, kinematic and dynamic ones, should be satisfied. The kinematic condition is of the form:

\[
\frac{dh}{dt} = \frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} = w
\]

The dynamic conditions of continuity of the normal and tangential stresses on the free surface \( z=h(x,t) \) take the form (neglecting the influence of the environment)

\[
\left[ -p + \frac{\gamma}{R} \frac{1}{\delta \eta (\mu)} + \frac{H^2}{8 \eta} |\mu - 1| \right] \mathbf{n} + \tau_p \mathbf{n} \delta \eta \mathbf{e} = 0.
\]
Here, $\tau_{ij}$ are the viscous stress tensor components, $R$ is the radius of curvature of the surface $z=h(x,t)$, $n$ is the vector of outward normal to the surface, $e^j$ are the basis vectors, $\gamma$ is the film surface tension coefficient, $B_n = \mu H_n$ is the normal component of the magnetic field strength vector. The magnetic field $H$ is assumed to be fixed, since the non-inductive approximation implies $\mu - 1 \ll 1$.

We will assume that the magnetic field creates the periodic travelling wave on the surface of a sufficiently thin layer (we denote the dimensional variables with the asterisk $^*$):

$$h^*(x,t) = d + a \cos\left(\omega t^* - kx^*\right), \quad \varepsilon = d \cdot k \ll 1.$$  \hspace{1cm} (24)

We introduce the dimensionless variables

$$x = x^* \cdot k, \quad z = z^*/d, \quad h = h^*/d, \quad \delta = a/d, \quad u = u^* / U_c, \quad w = w^* / U_c / \varepsilon, \quad U_c = \omega / k,$$

$$t = t^* \cdot \omega, \quad p = p^* / P, \quad P = \eta \omega / \varepsilon^2, \quad H^2 = H^*/P, \quad Re = \rho U_c d / \eta, \quad W = \gamma d k^2 / P.$$  \hspace{1cm} (25)

For $\varepsilon \ll 1$, we will seek for a solution in the form of a power series in $\varepsilon$ ($A = h, u, w, p$):

$$A(x,z,t) = A_0(x,z,t) + \varepsilon \cdot A_1(x,z,t) + \ldots.$$  \hspace{1cm} (26)

In the zeroth approximation in $\varepsilon$, for $Re < 1$ and $W = O(1)$, using the equations (20) and the condition (21) – (23), we obtain an expression for the velocity components (subscript “0” is omitted):

$$u(x,z,t) = F(x,t) \left(\frac{z^2}{2} - h^2\right), \quad w(x,z,t) = F(x,t) \frac{\partial h}{\partial x} z^2 + \frac{\partial F}{\partial x} \left(\frac{h}{2} - \frac{z^2}{2} - \frac{z^3}{6}\right),$$  \hspace{1cm} (27)

where $F(x,t) = \frac{\partial p}{\partial x}$.

The mass conservation law (20), condition (22), expression (24), and the assumption that $h(x,t) = h(\xi)$, $\xi = t - x$ imply the following equation for the flow rate $Q$:

$$\frac{\partial h}{\partial \tau} + \frac{\partial Q}{\partial x} = 0, \quad Q(x,t) = -F h^3 / 3 = h + C.$$  \hspace{1cm} (28)

If the flow is $T$-periodic, we can introduce the average flow rate: $\overline{Q}(x) = \frac{1}{T} \int_0^T Q(x,t) \, dt$. For $h(\xi) = 1 + \delta \cos(\xi)$, the dimensionless average flow rate is $\overline{Q} = 1 + C$.

Relation (23), with respect to (24), implies

$$\frac{\partial^3 h}{\partial \xi^3} + (\mu - 1) \frac{\partial H^2}{\partial \xi} = \frac{3}{\mu} \frac{3C}{h^2}.$$  \hspace{1cm} (29)

Using equation (29) it is possible to find the magnetic field creating the prescribed film shape.
The requirement for the magnetic field magnitude to change periodically leads to the expression
\[ Q = 1 + C \] on the surface oscillation amplitude \( \delta \). The dependence of the average volume flow rate \( \bar{Q} \) on the surface oscillation amplitude \( \delta \) is shown in Fig. 15. The expression for the magnetic field (26) takes the form
\[ H^2 = H_0^2 - D, \quad D = \frac{8\pi}{\mu - 1} \left( -W \delta \cos \zeta + \frac{3\delta \sin (\zeta)(2 + \delta \cos \zeta)}{2 + \delta^2 (1 + \delta \cos \zeta)^2} \right). \] (31)

The constant \( H_0^2 \) can be chosen arbitrarily. However, this constant must not be less than the maximum value, \( D_{\text{max}} > 0 \), of the function \( D \) in order not to violate the condition \( H^2 \geq 0 \). Thus we can assume \( H_0^2 = D_{\text{max}} \).

Fig. 15. Average volume flow rate \( \bar{Q} \) vs. surface oscillation amplitude \( \delta \)

### 3.4 Magnetizable Bodies in an Alternate Magnetic Field

In the experiments we use cylindrically-shaped bodies located in a cylindrical channel. The channel diameter \( d \) exceeds that \( d_w \) of the body. We denote the length of the body as \( l_w \). The magnetic field is created by coils. The axes of the coils are in the horizontal plane, \( L \) is the distance between the axes of the coils and \( I \) is the current in the coils (see Fig. 16). The coils are placed at the left and right hand sides of the channel. Magnetic field is created by three coils simultaneity (for example, coils numbers 6-8 in Fig. 16), the axis of the middle coil is the symmetry axis of the magnetic field.
Periodically, the left coil is switched off and the next coil is switched on, \( n \) is the number of the coil switches per second (the frequency), so \( T = \frac{1}{n} \) is the period between change-over of the coils. Currents flowing through the coils are unidirectional. Such an electromagnetic system forms a travelling magnetic field \( H \), which is a complex function of \( x, y, z, t \) (\( x \) is the coordinate along the channel, \( z \) is parallel to axis of the coils, \( y \) is orthogonal to \( x \) and \( z \)).

It is shown experimentally that in such periodic magnetic field the cylindrical magnetizable bodies move along the channel. The direction of the body motion is opposite to the direction of the travelling magnetic field.

The Bodies from a Magnetizable Polymer

In the first experiment \( I = 4.6A, L = 10\text{mm}, d = 11\text{mm} \) and the parameters of the “worm” (sample 1) are as follows: Young's modulus \( E_y = 50000\text{Pa} \), length \( l_w = 48\text{mm} \), diameter \( d_w = 4\text{mm} \). The frequency \( n \) changes from \( 5s^{-1} \) to \( 1000s^{-1} \) in this experiment. In the second experiment the parameters are \( I = 4.6A, L = 10\text{mm}, d = 10\text{mm}, l_w = 75\text{mm}, d_w = 4.5\text{mm} \) and the sample 2 consists of a polymer with Young's modulus \( E_y = 22000\text{Pa} \). The frequency \( n \) changes from \( 5s^{-1} \) to \( 1000s^{-1} \). A cycle of body deformation by the travelling magnetic field is the process when the travelling magnetic field covers the body (see Fig. 17). At the end and beginning of this process the body is not deformed.
In a third experiment an elastic cylindrical capsule filled with a magnetic fluid is inside a cylindrical channel. The channel and capsule diameters ($d$, $d_c$) are 10 mm and 4 mm. The length of the capsule filled with a magnetic fluid $l_c$ is 75 mm.

In our experiments the frequency $n$ changes from $5 \text{s}^{-1}$ to $1000 \text{s}^{-1}$. The phases of deformation of the capsule are shown in Fig. 18. The direction of the worm motion is opposite to the direction of the travelling magnetic field.

The body velocity depends on the geometrical shape of the deformed body and that of the channel. If $n$ is small enough and the body inertia does not affect the body velocity, the following formula is valid:
Here $l_s$ is the segment length (a segment is a part of deformed body between two neighbouring coils), $k_s$ is number of the segments, The symbol $[\cdot]$ denotes the integral part of the number, $t_c$ is the time of a cycle. The length of the segment may be determined under assumption about it's form. A segment form is determined by the elastic and magnetic properties of the body material, and the value of the magnetic field. The problem of determination of the body form is very complex and here we consider three assumptions about the segment form.

**Sinusoidal Form**

Let us assume that the segment of the body between two coils has sinusoidal form. In this case the equation of the central line of the segment is as follows:

$$v = k_s(l_s - L)/t_c, \quad t_c = (k_s + 3)T, \quad k_s = \left[\frac{lt}{4}\right].$$  \hspace{1cm} (32)

For parameters $L = 10\text{mm}$, $d = 11\text{mm}$, $l_w = 48\text{mm}$, $d_w = 4\text{mm}$ the length of the sinusoidal segment is $l_s = 12.6\text{mm}$, $k_s = 4$. The analytical estimation of the body velocity is determined as $v = 1.46 \cdot n \text{ mms}^{-1}$.

**Body Form is Determined by the Model of Elastic Beam**

Let us assume that the form of the segment of the body between two coils is determined by the model of the elastic beam without extension (the bending moment is due to the magnetic forces, assuming that magnetic forces act on the ends of the segment). In this case the equation of the central line of the segment is as follows:

$$y_E = ax^3 + bx^2 + d_w/2,$$  \hspace{1cm} (34)
where $a = -2(d - d_w)/L^3$, $b = 3(d - d_w)/L^2$. For this assumption and for parameters as above the length of the segment is equal to 12.5 mm, $k_s = 4$ and the analytical estimation of the body velocity is $v = 1.43 \cdot n$ mm/s.

**Body Form is a Broken Line**

Let us assume that the form of the body segment between two coils is a straight line. The equation of the central line of the segment is as follows:

$$y_R = (d - d_w)x/L.$$  \hspace{1cm} (35)

In this case for parameters as above the length of the segment is 12.2 mm, $k_s = 4$ and the analytical estimation of the body velocity is $v = 1.26 \cdot n$ mm/s. From Fig. 19 we can see that for $n < 100$ s$^{-1}$ the theoretical result (the body form is determined by the model of an elastic beam) matches with the experimental data for the sample 1 for the first experiment. The maximal obtained body velocity is $v = 7.89$ cm/s$^{-1}$ for $n = 100$ s$^{-1}$. For $n > 950$ s$^{-1}$ in the first experiment sample 1 does not move.

From the second experiment it follows that the segment form of the capsule is a straight line. The length of the segment is determined by the formula

$$l_s = \sqrt{L^2 + (d - d_f)^2} = 11.66 \text{ mm}, \quad k_s = 6.$$  \hspace{1cm} (36)

From (20) we find dependency of the velocity of the body on $n$ $v = 1.1 \cdot n$ mm/s. The theoretical dependency of the velocity of the body $v$ on $n$ and experimental data are shown in Fig. 20.

![Body velocity $v = v(n)$ (a capsule with a magnetic fluid)](image-url)
For the frequency $n < 50s^{-1}$ the theoretical estimation of the velocity of the capsule matches with the experiments. In our experiment for $n > 700s^{-1}$ the capsule does not move. The maximal obtained capsule velocity is $v = 5.56\,\text{cm}\,s^{-1}$ for $n = 50s^{-1}$. The body velocity depends on the geometrical shape of the deformed body and that of the channel. Only if $n$ is small enough the body inertia does not affect the body velocity and the formula (32) is valid. A simulation of the dynamic behavior of the elastic body was made by Finite-Element-Method (Fig. 21). For $n < 100\,\text{Hz}$ the numerical results coincide with experimental data.

![Fig. 21. Analysis of the locomotion (sample 1) for $n < 100\,\text{Hz}$ using Finite-Element-Method](image1)

The Finite-Element-Method is also a useful tool to define optimal control frequencies for the cascaded system of the coils (switching frequencies). As it is shown in Fig. 22 there exists a correlation between the measured velocity of the worm, the switching frequencies of the coils and the eigenfrequencies of the worm respectively.

![Fig. 22. The velocity of the worm vs. eigenfrequencies (switching frequencies of the coils)](image2)
Finally, we should remark that the type of locomotion realized with the magnetic elastomer or the elastic capsule filled with ferrofluid is a snake-like motion called *concertina motion*.

### 3.5 Design of Active and Passive Locomotion Systems and the Interaction between a Controlled Magnetic Field and a Magnetic Fluid

A moving magnetic field can generate a travelling wave on the surface of magnetic fluids. This travelling wave can be useful as a drive for locomotion systems. Therefore, peristaltically moving active locomotion systems could be realized with an integrated electromagnetic drive (see Fig. 23, left (A)). Also passive locomotion systems can be taken into account. Objects, which are on the surface of the fluid or are lying in the fluid, could be carried floating and/or shifting (see Fig. 23, left (B) and Fig. 26, 27).

![Fig. 23. Schema of possible locomotion systems (left), and the experimental setup (right)](image)

The following properties are important for the locomotion: (i) mass and geometry of the moving or moved object, (ii) the change of the shape and the position of the magnetic fluid, and (iii) the pressure distribution of the magnetic fluid with respect to the action of the moving magnetic field.

To analyse the behavior of the magnetic fluids (under the described action of the magnetic field) and such locomotion systems, the experimental setup consists of 20 consecutively arranged cascaded electromagnets (1 coil generates 3000 ampere turns). The measurement system to detect the pressure of the fluid and the optical system to analyse the shape of the fluid are connected to a 3 axis-positioning unit (see Fig. 23, right). Fig. 24 shows a travelling wave in a magnetic fluid.

![Fig. 24. Travelling wave generated by a moving magnetic field](image)
Fig. 25 shows schematically the magnetic field, which emerges from an electromagnet, the shape of the fluid and the pressure distribution.

Fig. 25. Schematical presentation of the electric induction density of an excited coil (left top), the emerged shape of the magnetic fluid surface (left bottom), and the pressure distribution of the magnetic fluid (right)

Fig. 26. Example of a passive locomotion by means of travelling waves in a magnetic fluid

Fig. 27. Functional principal of a passive locomotion system (form of the magnetic field (l.) and the corresponding video sequences (r.))
In the experimental setup using a water–based ferrofluid a maximal change of the fluid pressure about 2200 MPa was measured in the origin (see Fig. 25, right) after applying the magnetic field. Thus, it could be a realistic scenario to construct a cascaded structure of cylindrical membranes filled with a magnetic fluid (“worm”) and to get the necessary interaction between “worm” and the environment for peristaltic locomotion.

3.6 Conclusional Remarks

The expression for the magnetic field strength creating a sinusoidal wave on the surface of a viscous magnetic fluid as a function of the characteristics of the fluid (viscosity, surface tension, and magnetic permeability) and the parameters of the wave are obtained. It is experimentally shown that in a specially structured periodic travelling magnetic field a cylindrical magnetizable elastic body moves along the channel. The direction of the body motion is opposite to the direction of the travelling magnetic field. The maximal obtained body velocity is $v = 10 \text{ cm s}^{-1}$ for $n = 250 \text{s}^{-1}$. For the frequency $n < 100 \text{s}^{-1}$ (samples 1) and for $n < 50 \text{s}^{-1}$ (the capsule with the magnetic fluid) the theoretical (analytical and numerical) estimations of the velocity of the elastic body (the capsule with the magnetic fluid) coincide with the experimental data.

The creation of active biologically inspired locomotion systems and new principle for a passive motion is possible using the deformation deformable magnetizable media in controlled magnetic fields.

4. Summary and Outlook

At the beginning of the chapter it was mentioned that the motion of an earthworm was the inspiration for a technical solution of an artificial worm. A theory is developed for the peristaltic motion of such systems, which to a large extent allows to characterize these motions already on a kinematic level. The advantage of adaptive control for the dynamical realization of these motions is shown. Experiments using a simple prototype checked the results of the theory.

Using magnetizable materials in compliant structures rather snake-like motion (concertina movement) has been realized until now. Since the peristaltic crawling of the earthworm has many advantages for the locomotion in difficult environments the realization of such a motion remains a challenge in theory and control as well as in experiments (Fig. 28).

![Fig. 28. From the snake-like concertina motion to worm-like peristaltic crawling](image)

This also applies to the technological realization of an enveloping membrane structure for the artificial worm.
Here two problems (and actually opposite demands) are to be solved:

- membrane thickness as small as possible, to achieve a big force extraction and a very flexible worm structure and
- membrane thickness as big as possible, to avoid diffusion processes of the ferrofluid through the membrane and to keep environmental influences away from the ferrofluid to improve the long-term stability of the worm system.

The objective is to find optimal parameters and to verify these experimentally.

Another challenge for future research is to realize two-dimensional (planar) motions using ferrofluids.

5. References


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With the advancement of technology, new exciting approaches enable us to render mobile robotic systems more versatile, robust and cost-efficient. Some researchers combine climbing and walking techniques with a modular approach, a reconfigurable approach, or a swarm approach to realize novel prototypes as flexible mobile robotic platforms featuring all necessary locomotion capabilities. The purpose of this book is to provide an overview of the latest wide-range achievements in climbing and walking robotic technology to researchers, scientists, and engineers throughout the world. Different aspects including control simulation, locomotion realization, methodology, and system integration are presented from the scientific and from the technical point of view. This book consists of two main parts, one dealing with walking robots, the second with climbing robots. The content is also grouped by theoretical research and applicative realization. Every chapter offers a considerable amount of interesting and useful information.

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