1. Introduction

In optimization problems, the constraints play a fundamental role as restrictive clauses that limit the search process and the quality of solutions to be found. To propose efficient solutions, it is necessary to satisfy stakeholders’ constraints. However, stakeholders often require multiple and complex constraints, which are difficult to satisfy and constrain the task of optimization. In addition, some constraints require complex treatments, which are incompatible with a strategy of optimization and complicate the search for the best solution.

Indeed, in a constraint satisfaction process, a traditional problem is to verify if the set of constraints is coherent. As this problem is largely treated in literature, we consider that the set of constraints is coherent and the following problem consists in determining if the problem has a solution.

We deal with the framework of constraints satisfaction and optimization problems. An optimisation approach consists in exploring the search space to find the best solution according to an objective function [1][15]. This framework is then adapted to problems with multiple solutions. However, some real-life optimization applications are over-constrained and no solutions can be found. Therefore, the application of an optimization algorithm is not pertinent. The problem is how to act in order to face such a situation and how to take into account the constraints and their adequacy with the optimisation task?

Works about constraints satisfaction and optimisation problems often orient themselves toward the framework of optimisation or of the constraints satisfaction. We think that the two tasks of optimization and of constraints satisfaction must be treated simultaneously to find adequate solutions.

We seek in this work to develop a general approach of constraints satisfaction and optimization to ensure simultaneously the tasks of optimization and of constraints satisfaction. This approach is based on heuristics in order to guide the search for solutions.

We illustrate our approach with examples from an application of optimization of the spatial distribution of crops in an agricultural territory.

This chapter is organized as follows. Section 2 presents an analysis of the problem and the need for an approach of optimization under constraints. We present in section 3 our satisfaction and optimization approach. Sections 4 and 5 develop a spatial optimization
problem related to the optimization of crops distribution on agricultural parcels as well as results of experiments in Seine Maritime, France. Section 6 presents an experimental model based on Tabu Search. We discuss and we conclude this work in section 7.

2. Constraint satisfaction or optimization process?

In real-life optimization applications, the treatments of users are in general simple and do not reflect the complex treatments necessary to the satisfaction of their constraints. The real complexity of constraints on the system level can penalize the search for optimal solutions. Then, the tasks of constraints satisfaction and optimization of the required objective can be incompatible. A solution largely used in the literature consists in reducing the problem by favoring the satisfaction of the most relevant constraints to the application according to an order of preference \[6\][7][18][19][20][22][23]. In addition, some methods have been developed, such as Branch and Bound [13]. This method uses heuristics to guide the search of good solutions. The solution quality depends on the treated problem and on the defined heuristics. Pre-processing techniques have also been developed to reduce the search space of constraints satisfaction problems and can be used in combination with other search techniques \[4\][12][16][17][21]. These methods allow us to avoid an enumeration of all potential solutions but they are very expensive in computation times.

2.1 Analysis of the search process

We analyse in this part the process of constraints satisfaction and of optimisation in order to understand the role of variables, by their respective values, in the improvement of the objective and in the satisfaction of the constraints. We thus propose an intelligent heuristic to orient the search of satisfactory solutions in constraints satisfaction and optimisation problems.

In practice, optimization problems can reach a high complexity and thus require high computing times because of the great number of potential solutions. Exact methods cannot generally treat this type of problem and the use of an approximate optimization method is the adequate way. However, an optimization process based on an approximate method requires a flexibility to explore the search space. It is generally not adapted to problems handling hard constraints.

Thus, the question is how to apply an optimization algorithm and to ensure at the same time the satisfaction of constraints, essential to propose satisfactory solutions to stakeholders? To provide a solution to this problem, we analyze the search process regarding separately the tasks of optimization and of constraints satisfaction.

An optimization approach affects values to variables in order to find the best combination of values which improves the objective function (cf., the neighbourhood methods [2]). The strategy of searching for solutions is based on a mechanism of neighbourhood exploration\(^1\) guided by the evolution of the objective function.

A satisfaction approach affects values to variables in order to satisfy constraints on these variables. In this approach, the strategy of affectation is guided by the structure of the

\(^{1}\) That depends on the applied algorithm.
constraints and their state of satisfaction during the resolution (cf., the backtrack methods [5]).

The processes of constraints satisfaction and of optimization are based on an affectation of values to variables, according to two different strategies. The variables have an important role since their respective values influence the quality of the solution according to the objective of optimisation and of constraints satisfaction. An interesting way consists in exploiting the role of variables to solve simultaneously the tasks of constraints satisfaction and optimization.

2.2 SOS-Heuristic: An intelligent heuristic to search solutions

A constraint can be defined as a condition to satisfy among a number of variables. It restricts the set of values that can be affected simultaneously to the implied variables [21]. A classic process of constraints satisfaction re-affects only the variables implied in constraints to satisfy. In the case of optimization, all variables likely to improve the objective value are re-affect, regardless of their implication in constraints. This re-affectation of variables can cause the dissatisfaction of some constraints that imply these variables. To remedy this problem, we define a heuristic ‘SOS-Heuristic’ (Heuristic for Satisfactory and Optimized Solutions) to guide the re-affectation of variables with an intelligent manner. We thus choose to re-affect variables that don’t satisfy constraints instead of re-affecting all variables in order to improve the objective function. The defined heuristic consists in affecting a penalty to each variable that violates a constraint. The variables that satisfy all constraints are not penalized. The penalized variables must be re-affect to satisfy the constraints which imply them. The objective of the SOS-Heuristic is to improve the value of the objective function without damaging the satisfaction of constraints.

Example. Let’s consider the constraint of slope: “The parcels with a slope higher than 15° must not contain the corn”.

All parcels with a slope higher than 15° are implied in this constraint. The implied parcels, which have a crop other than corn satisfy the constraint and are considered satisfactory. The implied parcels that are affected with the corn are unsatisfactory. To satisfy the constraint, we must affect the corn to all implied parcels. Thus, only the unsatisfactory parcels must be re-affect.

Moreover, in constraints satisfaction and optimisation problems, the constraints to satisfy do not generally have the same importance. Works about constraints satisfaction problems distinguish in particular hard constraints that should absolutely be satisfied and soft constraints that are to be satisfied as far as possible [20]. Thus, the value of penalty for each unsatisfactory variable can be adapted according to the level of preference of violated constraints (hard or soft).

3. The proposed approach

We consider here the framework of optimization while taking into account the violated constraints. We make abstraction of the optimization techniques and we not rely on a specific algorithm of resolution. However, we base our approach on the concepts of an approximate method that is compatible with big-size problems.
At the system initialization, a phase of constraints analysis identifies the variables which are implied in each one. The verification of values of these variables allows identifying the unsatisfactory variables\(^2\). The values of these variables must be modified during the optimization process in order to satisfy the constraints not yet satisfied. The search process remains oriented toward the improvement of the objective function but the re-affectation process is limited to the unsatisfactory variables.

### 3.1 Resolution strategy

The strategy of constraints satisfaction and optimization alternates a phase of optimization of the objective and a phase of satisfaction of the required constraints.

#### 3.1.1 Optimization phase

The variables penalized during the initialization phase represent the input of the optimization phase: we limit the set of the optimisation variables to those violating constraints. These variables are re-affected during the optimization while encouraging the re-affectation of the most penalized variables\(^3\). In order to not deteriorate the initial satisfaction for the constraints, the variables that satisfy these constraints are not penalized and are not re-affectated during the optimization. The re-affectation of penalized variables orients the search toward the satisfaction of the constraints initially violated and therefore the reduction of the number of penalized variables (which become satisfactory).

#### 3.1.2 Satisfaction phase

After the optimization phase, the phase of satisfaction re-evaluates the satisfaction of constraints according to new values of the re-affectated variables. This re-evaluation of the constraints satisfaction allows updating the set of unsatisfactory variables and thus the value of penalty for each one. The variables that have become satisfactory are removed from the initial set of unsatisfactory variables. The algorithm reiterates a new phase of optimization to improve the objective function by re-affecting a new set of penalized variables. This process is repeated until satisfying all constraints or until finding a good solution in relation with the value of the objective function and with the satisfaction of constraints.

In our approach, the variables have an important role in the exploration of solutions (during the optimisation phase) and in the evaluation of the constraints satisfaction (during the satisfaction phase).

### 3.2 Coding of the proposed approach

The proposed constraint satisfaction and optimization approach is based on a multi-phase mechanism that integrates the improvement of the objective at the level of the optimization phase and the satisfaction of the violated constraints at the level of the satisfaction phase.

The algorithm is based on the following steps:

\[\text{Algorithm 1:}\]

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\(^2\) The variables that violate at least one constraint.

\(^3\) The variables that violate several constraints or hard constraints.
Begin

/* Initialization phase */

- Evaluate the initial satisfaction of the constraints
- Extract the set of unsatisfactory variables /* the algorithm can re-affect only the variables that violate the hard constraints or can also re-affect all unsatisfactory variables */
- Attribute a penalty value for each unsatisfactory variable
- Repeat /* Do n simulations */

/* Optimization phase */

- Apply an approximate optimization method /* the algorithm is independent of a specific optimization method */.
- Re-affect the penalized variables
- Evaluate the objective function

/* Satisfaction phase */

- Evaluate the satisfaction of the constraints
- Update the penalized variables as well as their penalty value

Until a stop condition /* a global satisfaction state */

End

Although it is guided by the evolution of the objective value, the exploration of solutions is based on the re-affectation of only the penalized variables. The choice of the variable to re-affect depends on the value of penalty in order to treat in first the most penalized variables.

4. Spatial optimization problem

The problem we are trying to solve consists in affecting crops on agricultural parcels in order to reduce runoff risks. Runoff risk is the consequence of the assignment of crops on the parcels. Runoff is the accumulation of outflows on the parcels, due to a weak infiltration of water in the soil. Above a certain rate of rain, the soil doesn't absorb water that flows according to the natural slope and causes losses of land (erosion) and a significant deterioration of the agricultural soil’s surface. Important damage can be caused to the habitat and to the plantations, notably in the downstream areas [9][14].

To solve the runoff risk problem, we must optimize the distribution of crops on the parcels while respecting to the maximum the required constraints [10]. A configuration corresponds to a distribution of crops on the parcels. We evaluate the performance of each configuration according to the value of runoff risk and the average rate of satisfaction for the constraints. The satisfaction rate for each constraint is evaluated from the Database using SQL (Structured Query Language) requests. The value of runoff risk is evaluated by a spatial hydrological model that calculates values of sensitivity to the runoff risk for each parcel. It also simulates the runoff risk due to a distribution of crops on all parcels [9].

4.1 A hybrid optimization method

As mentioned above, the developed approach is independent of a specific algorithm of resolution. To experiment our approach, we elaborated a hybrid method based on the evolutionary strategy [3] and the simulated annealing [11].

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4 The database is implemented in Relational Database Management System.
We consider the real distribution of crops already available, as a start configuration. We manipulate only one configuration (an individual in analogy with the evolutionary strategy) at a time.

From the initial configuration, the optimization process of runoff risk repeats a procedure of neighbourhood exploration based on the simulated annealing method. The mechanism of neighbourhood exploration is based on a permutation of crops between two parcels according to their sensitivity to the risk. The value of risk represents the function of cost. The objective consists in seeking configurations of weaker cost while accepting with a controlled manner the configurations that damage the function of cost. The new configuration is accepted systematically if the value of runoff risk decreases. Otherwise, the acceptance of a new configuration with a higher cost is determined with a probabilistic manner: a real $0 \leq \theta < 1$ is randomly chosen and then compared with a probability of acceptance $p(\Delta E, T)$. This probability is expressed according to the value of cost increase and the current temperature. The temperature is controlled by a decreasing function that defines a cooling schedule. The system starts with an elevated temperature that progressively tends toward 0 along the advancement in the process in order to lead the system toward a stable state. If $\theta \leq p(\Delta E, T)$, then the new configuration is accepted and replaces the current configuration. Otherwise, the current configuration is preserved and is reused to generate another one. The acceptance of increases allows us to leave the stable states of non-decrease of the risk. It also advantages the exploration of a large search space in order to avoid a premature convergence toward a local minimum. The probability of acceptance of risk increases is weak in order to avoid a random dispersal in the search space.

4.2 The satisfaction strategy

We based the resolution of the spatial optimization problem related to the runoff risk on the algorithm presented above (algorithm 1) and we favour the satisfaction of the hard constraints.

Indeed, the parcels can be implied simultaneously in hard and soft constraints. The treatment of all unsatisfactory parcels (that violate hard and soft constraints) can sometimes improve the satisfaction of the soft constraints at the expanse of the hard constraints. To avoid such situations, we have chosen to re-affected crops only to the parcels that violate hard constraints. We treat by permutations only the parcels whose crop doesn't satisfies at least a hard constraint. The parcels that satisfy all hard constraints remain invariant and maintain their initial crop during the optimisation phase.

4.3 The hybrid algorithm

The algorithm used to reduce the runoff risk is based on the following steps:

Algorithm 2:

Begin

• Start from the initial distribution $x_i$ of crops on the parcels.
  /* Initialization phase */

• Evaluate the initial satisfaction of the constraints

• Extract the set $P$ of parcels that violate hard constraints /* If all constraints are satisfied, extract the parcels that violate soft constraints and that are not implied in hard constraints */

  /* Do n simulations */

End
• **Repeat**  
  /* Optimization phase */  
• Choose a high initial temperature $T$ and a function $\alpha$ of temperature reduction.  
• **For** each parcel $p_i$ from $P$ ($p_i$ is the most penalized parcel).  
  • Generate a neighbourhood $x$ (by permuting $p_i$ with another penalized parcel).  
  • If the risk value degrades then $x_i = x$.  
  • Else, accept the new configuration with a random probability  
• Reduce the temperature $T = \alpha(T)$  
/* Satisfaction phase */  
• Evaluate the satisfaction rate of the constraints  
• Update the set $P$ of penalized parcels.  
• Accept the solution if it satisfies the requirements of the application.  
• **Until** a stop condition.  

End  

The algorithm seeks to reduce the risk value at the level of the optimization phase and to improve the satisfaction of hard constraints at the level of the satisfaction phase.

**5. Experiments**

We present in this part an experimentation that aims to reduce the runoff risk in the watershed of Haute-Durdent in Seine Maritime - France, while satisfying the constraints required by the farmers.  

The watershed of Haute-Durdent (16 km²) has 450 parcels shared between a dozen of farms. Thirteen types of occupations: beet, wheat, wood, rape, fodder crops, escourgeon, fallow, linen, corn, potato, pea, prairie and village are placed on the parcels. The wood and village are considered as invariable and are not taken into account in the re-affectation process.  
We consider 21 constraints: 16 hard and 5 soft, required by several farmers in their farms. These constraints express the requirements of slope, size and type of sol compatible with the placement of each crop as well as the production quantity of crops in each farm.  

A solution to a constraints satisfaction and optimization problem is an affectation of values to the variables, which improves the objective value while satisfying the constraints. In our problem, we define a solution as a configuration that reduces the risk of runoff and maintains the initial rate of satisfaction for the hard constraints. The parcels are the variables of the system. Each parcel can be affected by a type of crop. The multiple possible affectations of crops on the parcels represent the search space.

**5.1 Reduction of the risk value (the function of cost)**

Several tests have been done in order to follow the evolution of the risk value (to minimize) during the optimization phase and at the end of each simulation. Fig. 1 shows the evolution of the risk value after each permutation during the optimization phase (inside the first simulation). Fig. 2 shows the evolution of the risk value after each simulation.  

The results show a reduction of the risk value calculated by the spatial hydrological model at the outlet of the watershed. The developed algorithm converges after some simulations toward a stable state. Some deteriorations of the risk value (Fig. 1) are accepted during the optimisation phase in order to avoid local minima.
To evaluate the efficiency of the proposed algorithm and to explain the reduction of the risk value, we present in Fig. 3 the number of permutations and of the penalized parcels during simulations.

Fig. 1. The value of runoff risk after each permutation during the first simulation.

Fig. 2. The value of risk after simulations.

Fig. 3. The number of permutations (thick feature) and of penalized parcels (thin feature) through simulations.
The number of crops permutations between the parcels tends toward 0 and explains the convergence of the algorithm after some simulations toward a stable state. The number of penalized parcels decreases by 19.42% (from 139 to 112) after 10 simulations (Fig. 3). Although the number of treated parcels is restricted since we treated only the parcels that violate hard constraints, the value of risk decreased by 56% after 10 simulations (from 9967 to 4377 mm, Fig. 2). This reduction obtained by the treatment of 30% of the parcels (139 among 450 parcels, Fig. 3) proves that the aggravation of the risk is essentially due to elementary parcels that are badly managed by farmers. Consequently, the affectation of appropriate values to variables which cause the dissatisfaction of constraints allows solving efficiently an optimization and constraints satisfaction problem.

5.2 Evaluation of the constraints satisfaction

We evaluate the average rate of satisfaction for the hard and the soft constraints (Fig. 4). The treated variables can be implied simultaneously in several constraints. A complex constraint that implies a big number of variables cannot often be satisfied by all implied variables and only a subset of variables satisfies the constraint. To this effect, we consider that a constraint is satisfied with a certain degree\(^5\) rather than satisfied (100%) or dissatisfied (0%). The satisfaction of a constraint is evaluated according to the ratio: the number of satisfactory variables / the total number of variables implied in the constraint. For example, the satisfaction of the constraint of slope defined above is evaluated according to the ratio: the number of satisfactory parcels (that have a slope higher than 15° and contain a crop different from corn) / the total number of parcels implied in the constraint (that have a slope higher than 15°).

![Fig. 4. The average rate of satisfaction for the constraints through simulations.](image)

The defined SOS-Heuristic allowed us to maintain (and improve) the initial rate of satisfaction for the hard constraints (Fig. 4). However, we note some deteriorations of the satisfaction rate without descending beyond the initial rate of satisfaction (at the simulation 3 and 5, Fig. 4). Indeed, at each simulation, the algorithm accepts a deterioration of the satisfaction rate to overtake local maxima.

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\(^5\) We find the principles of the Fuzzy Constraints Satisfaction Problems [7].
5.3 Spatial consistency
We visualize on the following maps the initial distribution of crops on the parcels of Haute-Durdent (Table 1 and Fig. 5) as well as the distribution proposed by the developed approach (Fig. 6).

Table 1. The colour assigned for each type of crop is darker as the crop is pro-runoff.

Fig. 5. The initial distribution of crops on the parcels of Haute-Durdent.
Fig. 6. The distribution of crops proposed by the developed approach.

The optimization process dissociates the regrouping of pro-runoff crops in the sensitive zones to minimize the risk (Fig. 5 and 6, circled parts).

The high complexity of the required constraints as well as the few number of treated parcels (we treated only 30% of the parcels) explain the small, but efficient, number of modifications made on the initial distribution of crops.

6. An experimental model based on Tabu search

To better analyse the contribution of the SOS-Heuristic, we present in this part an experimental optimization model based on Tabu search. This experimentation aims to redistribute crops between the parcels of Haute-Durdent without taking into account all stakeholders’ constraints. Thus, we do not use the SOS-Heuristic and our primary objective is to reduce the runoff risk.

6.1 Tabu search optimization

To optimize the distribution of crops on the parcels, we used an approximate method based on Tabu search [8]. Tabu search is an optimization metaheuristic, belonging to the class of local search techniques [1]. A local search technique is an iterative process based on two essential elements: a neighbourhood and a procedure exploiting this neighbourhood.
Starting from an initial configuration (solution), a typical local search method explores the search space to replace the current configuration by one of its neighbours in order to minimize a function of cost (or to maximize a function of benefit).

We consider the real distribution of crops as an initial configuration. We manipulate one configuration at a time. A configuration is a distribution of crops on all parcels of the territory. The process of crops permutation between two parcels acts as an explorer of the search space. It generates, after each permutation, a neighbouring configuration that minimizes the risk value. From the initial configuration, the optimization process permutes the crop of the most sensitive parcel with that of another parcel in the territory. The optimization algorithm doesn’t choose the first permutation that improves (minimizes) the risk value. All possible permutations are enumerated to discover the most profitable configurations. This solution is certainly expensive, but the discovered neighbouring configuration will be of a smaller risk value. The configuration generated after a permutation is accepted systematically if the run-off risk decreases. Otherwise, the last least expensive permutation is chosen, according to a probability of acceptance, in order to escape the stable states of non-reduction of risk. The acceptance of increases of the risk value encourages the exploration of a vast search space in order to avoid a premature convergence of the optimization algorithm.

However, to prevent cycles, the last explored configurations are kept in a short-term memory (Tabu list). This list memorizes information about the history of the last generated configurations in order to prohibit permutations that lead to one of them. It thus allows the algorithm to explore other parts of the search space. In addition, to avoid a memorization of expensive configurations, we memorize only the permutations that generate them.

### 6.2 Optimization strategy

The optimization algorithm is based on the following steps:

**Algorithm 3:**

**Begin**

1. Start from an initial empty Tabu list
2. Select $p$, the most sensitive of the untreated parcels.
3. Select $C$, the set of candidate parcels that can exchange their crops with that of $p$ while satisfying the constraints.
4. Eliminate parcels that take permutations already explored.
5. Classify the retained parcels according to their contribution to the runoff risk.
6. Permute the most sensitive parcel with another parcel.
7. Update the Tabu list

**End While.**

**End**

The treatment of a parcel consists in searching a permutation of its crop with that of another less sensitive in the watershed, in order to minimize the risk value. A simulation consists in

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6 When no permutation minimizes the risk after some permutations.
treat all parcels of the watershed by searching for possible permutations among the occupying crops.

![Graph showing the value of risk after simulations.](image)

**Fig. 7.** The value of risk after simulations.

The experimental model has reduced the value of the risk calculated at the watershed outlet (Fig. 7). The runoff value decreased by 63% after 20 simulations (from 9967 to 3640 mm, Fig. 7). This reduction is obtained by the treatment of all 450 parcels. However, only some restricted constraints are taken into account. In addition, we represent on the following map the distribution of crops proposed by the experimental optimization model (Fig. 9).

As shown in Fig. 9, to reduce the risk, the experimental optimization model re-distributes crops randomly since it does not take into account the spatial constraints related to the placement of crops. Consequently, the distribution of crops proposed by this model is not adapted to the reality of land and does not respect farmers’ requirements:

- The satisfaction of the production objectives for crops is not satisfied: farmers require that a quantity of production must be reached for each type of crop. This quantity is expressed in this problem by a total surface to cover by each crop in the territory.
- The spatial distribution of crops on the parcels is incoherent and the map obtained in Fig. 9 is not accepted by farmers.
7. Discussion and conclusion

We have proposed in this work a constraints satisfaction and optimization approach to orient the search process in optimization problems. We have developed a generic approach that allows to solve simultaneously the task of optimization and of constraints satisfaction based on an intelligent search heuristic: the SOS-Heuristic. The experiments concern a spatial optimization problem related to a distribution of crops on agricultural parcels to minimize the runoff risk. The application of a hybrid method based on the simulated annealing and the evolutionary strategy provided interesting results and proves the efficiency of the proposed approach regarding the minimization of the risk value as well as the satisfaction of the farmers’ constraints.

We have also presented an experimental model based on Tabu Search. This model, modelised into the framework of combinatorial optimization, is not oriented toward the constraints satisfaction since no global analysis has been conducted at this level. Only a limited set of constraints, like the requirements of slope and type of soil, is taken into

Fig. 9. The distribution of crops proposed by the optimization model.
account. The satisfaction of these constraints is explicitly provided by eliminating the assignments that do not verify them in order to limit the search space. The simplified model of the problem allowed a certain degree of flexibility to the system thanks to its little number of constraints. This explains the multitude of changes which occurred on the distribution of crops in the territory. However, this resolution is not realistic since it does not take into account all farmers’ constraints and it considers only some soft constraints. The solutions proposed by this experimental model are not accepted by the farmers. Thus, the optimization and satisfaction approach based on the SOS-Heuristic is more efficient than the experimental model and provide more realistic solutions. It could be used to solve other constraints satisfaction and optimization problems.

8. References


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7 We find the concepts of pre-processing techniques.


The goal of this book is to report original researches on algorithms and applications of Tabu Search to real-world problems as well as recent improvements and extensions on its concepts and algorithms. The book's Chapters identify useful new implementations and ways to integrate and apply the principles of Tabu Search, to hybrid it with others optimization methods, to prove new theoretical results, and to describe the successful application of optimization methods to real world problems. Chapters were selected after a careful review process by reviewers, based on the originality, relevance and their contribution to local search techniques and more precisely to Tabu Search.

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