Homogeneous Approach for Output Feedback Tracking Control of Robot Manipulators

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1. Introduction

The problem of robust tracking control of electromechanical systems has been studied and solved by many different approaches within the robot control community (see e.g. Sage et al., 1999; Kelly et al., 2005 and references therein) in order to ensure accurate motion in typical industrial tasks (painting, navigation, cutting, etc). In the last decade, the homogeneity approach attracted considerable interest from the research and engineering communities (see e.g. Lebustard et al., 2006; Ferrara et al., 2006; Bartolini et al., 2006) because it was demonstrated that homogeneous systems with homogeneity degree $\eta < 0$ exhibit robustness and finite-time convergence properties (Bhat & Bernstein, 1997; Hong et al., 2001; Orlov, 2005).

Control laws based on the homogeneity approach (Bhat & Bernstein, 1997; Hermes, 1995; Orlov, 2003a; Rosier, 1992) are attractive in robotic applications because they can cope with many mechanical perturbations, including external vibrations, contact forces, and nonlinear internal phenomena such as Coulomb and viscous friction, dead zone and backlash, while it is possible to ensure exact tracking to continuously differentiable desired trajectories.

Several homogeneous controllers and studies have been proposed in the literature. For example, Rosier (1992) constructed a homogeneous Lyapunov function associated with homogeneous dynamic systems. Hermes (1995) addressed the homogeneous stabilization control problem for homogeneous systems. Bhat and Bernstein (1997) examined the finite time stability of homogeneous systems. Levant (2005a, 2005b) developed robust output-feedback high-order sliding mode controllers that demonstrate finite-time convergence (see also Fridman & Levant, 1996; Fridman & Levant, 2002) where the controller design is based on homogeneity reasoning while the accuracy is improved in the presence of switching delay, and the chattering effect is treated by increasing the relative degree. Orlov et al. (2003a, 2003b) proposed applying homogeneous controller to solve the set-point problem dealing with mechanical imperfections such as Coulomb friction, viscous friction, and backlash. Orlov et al., (2005) extended the finite time stability analysis to nonlinear nonautonomous switched systems.
Motivated by the above-mentioned features, and taking into account that only incomplete and imperfect state measurements are available, the main objective of this paper is to introduce an output feedback homogeneous controller for tracking the trajectories of robot manipulators. The control design proposed here is inspired by the passivity-based approach, which consists of an observer part, a precomputed reference trajectory, and a controller part, but is augmented with a relay part in both the controller and the observer, which yields a certain degree of robustness under disturbances and finite-time stability of the closed-loop system. We note that the passivity-based approach, which is represented by the Slotine and Li observer controller, allows semi-global stability and the addition of the relay terms ensures robustness, despite the presence of the observer without destroying the closed-loop stability.

This paper is organized as follows: In Section 2 homogeneous systems are defined. Section 3 states the problem and introduces the Euler-Lagrange representation of the robot manipulator, along with some fundamental properties of the dynamic model. Section 4 presents the homogeneous controller and its stability analysis. Section 5 provides a simulation study for a 2-DOF robot manipulator using the controller described in Section 4 as well as performance of the controller for external perturbations. Section 6 establishes final conclusions. The following notation will be used throughout the paper. The norm $\|x\|_2$, with $x \in \mathbb{R}^n$, denotes the Euclidean norm and $\|x\| = |x_1| + \ldots + |x_n|$ stands for the sum norm. The minimum and maximum eigenvalues of a matrix $A \in \mathbb{R}^{n \times n}$ are denoted by $\lambda_{\text{min}} \{A\}$ and $\lambda_{\text{max}} \{A\}$, respectively. The vector $\text{sign}(x)$ is given by $\text{sign}(x) = [\text{sign}(x_1), \ldots, \text{sign}(x_n)]^T$ where the signum function is defined as

$$
\text{sign}(y) = \begin{cases} 
1 & \text{if } y > 0, \\
[-1,1] & \text{if } y = 0, \forall y \in \mathbb{R}.
\end{cases} \quad (1)
$$

2. Basic definitions

Let us begin by recalling some definitions of homogeneity for nonautonomous nonlinear systems governed by

$$
\dot{x} = f(x,t)
$$

where $x = (x_1, \ldots, x_n)^T$ is the state vector, $t$ is the time, and $f = (f_1, \ldots, f_n)^T$ is a piece-wise continuous function (Orlov, 2005). The function $f : \mathbb{R}^{n+1} \mapsto \mathbb{R}^n$ is piece-wise continuous if and only if $\mathbb{R}^{n+1}$ is partitioned into a finite number of
domains $G_j \subset \mathbb{R}^{n+1}, \ j = 1, \ldots, N$, with disjoints interiors and boundaries $\partial G_j$ of measure zero such that $f$ is continuous within each of these domains and for all $j = 1, \ldots, N$ it has a finite limit $f^j(x, t)$ as the argument $(x^j, t^j) \in G_j$ approaches a boundary point $(x, t) \in \partial G_j$. Throughout, the precise meaning of the differential equation (2) with a piece-wise continuous right-hand side is defined in the sense of Filippov (Filippov, 1988). An absolutely continuous function $x(\cdot)$ defined on an interval $I$, is said to be a solution of (2) if it satisfies the differential inclusion

$$\dot{x} \in F(x, t)$$

almost everywhere on $I$.

**Definition 1 (Orlov, 2005):** A piece-wise continuous function $f : \mathbb{R}^{n+1} \to \mathbb{R}^n$ is said to be homogeneous of degree $\eta \in \mathbb{R}$ with respect to dilation $(r_1, \ldots, r_n)$ where $r_i > 0, i = 1, \ldots, n$ if there exists a constant $c > 0$ and a ball $B_0 \subset \mathbb{R}^n$ such that

$$f_i(c^\eta x_1, \ldots, c^\eta x_n, c^{-\eta} t) = c^{\eta+\gamma} f_i(x)$$

for all $c > c_0$ and almost all $(x, t) \in B_0 \times \mathbb{R}$.

When continuous, a globally homogeneous time-invariant vector field $f(x)$ of degree $\eta < 0$ with respect to dilation $(r_1, \ldots, r_n)$ is known to be globally finite time stable whenever it is globally asymptotically stable (Orlov 2005, Thm 3.1) and an upper estimate of the settling time is given by

$$T(t_0, x^0) \leq \tau(x^0, E_L) + \frac{1}{1-2^n} (\delta L^{-1})^n s(\delta)$$

where

$$x(t_0) = x^0$$

and

$$\tau(x^0, E_L) = \sup_{x(0)} \inf_{x(t_0, x^0)} \{T \geq 0 : x(t, t_0, x^0) \in E_L \forall t \in \mathbb{R}, t \geq t_0 + T\}$$

and

$$s(\delta) = \sup_{x(0)} \tau(x^0, E_L, \frac{\tau}{2^\delta})$$
where $E_L$ denotes an ellipsoid of the form

$$E_L = \left\{ x \in \mathbb{R}^n : \sqrt{\sum_{i=1}^{n} \left( \frac{x_i}{L_i} \right)^2} \leq 1 \right\},$$

$E_L$ is located within a homogeneity ball, $\delta \geq c_0 L$, and $c_0 > 0$ is a lower estimate of the homogeneity parameter.

3. Dynamic model and problem statement

We here present a homogeneous tracking control for an $n$-degrees-of-freedom rigid serial links robot manipulator governed by the following equation of motion (Spong, 1989):

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = U + w$$

(4)

where $q$ is the $n \times 1$ vector of joint positions and is considered to be the only information available for feedback; $U$ is the $n \times 1$ vector of applied joint torques; $w$ is the $n \times 1$ unknown perturbation vector; $M(q)$ is the $n \times n$ symmetric positive-definite inertia matrix; $C(q,\dot{q})\dot{q}$ is the $n \times 1$ vector of centripetal and Coriolis forces; and $g(q)$ is the $n \times 1$ vector of gravitational torques. The dynamic equation (4) has the following properties, which will be used in the closed-loop stability analysis (Kelly et al., 2005):

- The inertia matrix $M(q)$ is bounded above and below for all $q \in \mathbb{R}^n$; that is, $m_1 I \leq M(q) \leq m_2 I$ where $m_1$ and $m_2$ are positive scalars and $I$ is the identity matrix.
- The matrix $C(q,\dot{q})$ is chosen such that the relation $\dot{q}^T [M(q) - 2C(q,\dot{q})]q = 0$ holds for all $q, \dot{q} \in \mathbb{R}^n$.
- The vector $C(q,x)y$ satisfies $C(q,x)y = C(q,y)x$ and $C(q,z + \alpha y) = C(q,z)y + \alpha C(q,x)y$ for all $q, x, y, z \in \mathbb{R}^n$ and $\alpha \in \mathbb{R}$.
- The matrix $C(q,x)$ satisfies $\|C(q,x)\| \leq k_1 \|x\|$ for all $x, q \in \mathbb{R}^n$ and $k_1$ is a positive constant.

We further assume

- A known constant, $W > 0$, is the upper boundary for the perturbation vector, $w$, that is, $|w| \leq W$. 
The control objective is established formally as follows: given bounded and continuously differentiable desired joint trajectories \( q_d(t) \in \mathbb{R}^n \), we must design a discontinuous control law \( U \) such that the joint positions \( q(t) \) reach the desired trajectories \( q_d(t) \) asymptotically; that is,

\[
\lim_{t \to \infty} |q_d(t) - q(t)| = 0. \tag{5}
\]

### 4. Homogeneous controller

In this section, we present the nonadaptive Slotine-and-Li controller (Slotine & Li, 1987) augmented with a homogeneous part to achieve asymptotic stability of the closed-loop system equilibrium point.

**Proposition 1.** Consider the equation of the robot \((2)\) along with the following control law:

\[
\begin{align*}
    U &= M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) - K_D(\dot{q}_o - \dot{q}_r) \\
    - K_P e - K_a \text{sign}(e) - K_b \text{sign}(\dot{q}_o - \dot{q}_r) \\
    \dot{\hat{q}}_r &= \hat{q}_d - \Lambda(\hat{q} - \hat{q}_d) \\
    \dot{\hat{q}}_o &= \dot{\hat{q}} - \Lambda z
\end{align*}
\tag{6}
\]

and the homogeneous observer,

\[
\begin{align*}
    \dot{\hat{q}} &= p + (\Lambda + l_d I)z \\
    \dot{p} &= \ddot{\hat{q}}_r + l_d \Lambda z + M^{-1}(q)K_p [z - e + \gamma \text{sign}(z)] \\
    & \quad - M^{-1}(q) [K_a \text{sign}(e) + 2K_b \text{sign}(\dot{q}_o - \dot{q}_r)]
\end{align*}
\tag{7}
\]

where \( e = q - q_d \) is the \( n \times 1 \) tracking error vector; \( \dot{\hat{q}} \) is the \( n \times 1 \) estimated velocity vector; \( z = q - \hat{q} \) is the \( n \times 1 \) observation error vector; \( \Lambda, K_P, K_D, K_a, K_b, \gamma \) are \( n \times n \) diagonal positive definite matrices and \( l_d \) is a positive constant. Then, for any initial condition some sufficiently large gains of the controller \((6), (7)\) always exist such that \((5)\) holds.

**Proof.** First, the equations of the closed-loop system \([4], (6), (7)\) must be introduced in terms of the tracking and observation errors, which are given, respectively, by
where $s = \dot{q} - \dot{q}_o = \dot{e} + \Lambda (e - z)$ and $r = \dot{q} - \dot{q}_o = \dot{z} - \Lambda z$.

It should be noted that the nonlinear nonautonomous closed-loop system (8) is a differential equation with a right-hand discontinuous side. Thus, the precise meaning of solutions of the differential equation with the discontinuous functions is defined in the Filippov sense (Filippov, 1988), as for the solutions of a certain differential inclusion with a multi-valued right-hand side. In fact, the control law (6)-(7) can be seen as a second-order sliding mode.

To conclude that the origin is asymptotically stable, consider the following Lyapunov function candidate for the closed-loop system (8):

$$V(x,t) = \frac{1}{2} e^T K_e e + \frac{1}{2} z^T M(q) s + \frac{1}{2} r^T M(q) r + \sum_{i=1}^n K_{a_i} |e_i| + \sum_{i=1}^n \gamma_i |z_i|$$

where $K_{a_i}$ and $\gamma_i$ are the elements of the main diagonal of $K_a$ and $\gamma$, respectively. The time derivative of $V(x,t)$ along the solution of (8) yields

$$\dot{V}(x,t) = e^T K_e \dot{e} + s^T M(q) \dot{s} + \frac{1}{2} \dot{s}^T M(q) s + z^T K_r \dot{z}^2 + r^T M(q) \dot{r} + \frac{1}{2} r^T \dot{M}(q) r$$

$$+ e^T K_a \text{sign}(e) + z^T \gamma \text{sign}(z).$$

Substituting equations (8) in $\dot{V}(x,t)$ and employing properties H2 and H3, it follows that

$$\dot{V}(x,t) = -e^T \Lambda K_e e + e^T \Lambda K_r z - z^T \Lambda K_r z - s^T K_r \dot{s} - r^T [M(q) I_d - K_d] r + s^T C(q,r)[s - \dot{q}]$$

$$+ r^T C(q,r)[r - \dot{q}] - e^T \Lambda K_a \text{sign}(e) - (s - r)^T K_b \text{sign}(s - r) - z^T \Lambda \gamma \text{sign}(z)$$

$$+ z^T \Lambda K_a \text{sign}(e) + (s + r)^T w.$$

Using properties H1, H2, and H5; and employing the well-known inequality the following boundary is obtained
\[2\|s\|_2 \leq \|s\|^2 + \|r\|^2, \quad g, h \in \mathbb{R}^n, \quad (11)\]

\[
\dot{V}(x, t) = -\frac{1}{2} \begin{bmatrix} \lambda_{\text{min}} \{\Lambda K_p\} & \Lambda_{\text{min}} \{\Lambda K_p\} \end{bmatrix} \begin{bmatrix} 0 \\ \lambda_{\text{min}} \{\Lambda K_p\} \end{bmatrix} \begin{bmatrix} \|s\| \\ \|r\| \end{bmatrix} - \left[ \begin{array}{cc} \lambda_{\text{min}} \{\Lambda \gamma\} - \lambda_{\text{max}} \{\Lambda K_u\} & \|z\| \\ \|z\| \end{array} \right] \\
- \left[ \begin{array}{c} \lambda_{\text{min}} \{K_D\} - k_r \|r\| \\ k_r \|z\| \end{array} \right] \left[ \begin{array}{c} \|s - r\| + \lambda_{\text{max}} \{W\}\|s + r\| \\ \lambda_{\text{max}} \{K_D\} - k_r \|z\| \end{array} \right] \begin{bmatrix} \|s\| \\ \|r\| \end{bmatrix} + \left[ \begin{array}{cc} \lambda_{\text{min}} \{\Lambda K_u\} \|s\| & \lambda_{\text{max}} \{\Lambda K_u\} \|r\| \\ \lambda_{\text{max}} \{\Lambda K_u\} \|s\| & \lambda_{\text{min}} \{\Lambda K_u\} \|r\| \end{array} \right] \begin{bmatrix} \|s\| \\ \|r\| \end{bmatrix} \right] \quad (12)\]

We now derive sufficient conditions for \(\dot{V}(x, t)\) to be locally negative definite. Firstly, \(Q_1\) will be positive definite by selecting positive definite matrices \(\Lambda\) and \(K_P\). Notice that \(Q_2\) will be positive definite if

\[\lambda_{\text{min}} \{\Lambda \gamma\} > \lambda_{\text{max}} \{\Lambda K_u\}.\]

Finally, \(Q_3\) is positive definite if

\[\frac{1}{k_c} \lambda_{\text{min}} \{K_D\} > \|z\|\]

and

\[m_i l_d > \lambda_{\text{max}} \{K_D\}.\]

Thus it is always possible to find some controller gains to ensure that all the above inequalities hold. Therefore, (12) is locally negative definite almost everywhere and the equilibrium point is exponentially stable.

Finally, we define the domain of attraction and prove that it can be enlarged by increasing the controller gains. For this, we first find some positive constant \(\alpha_1, \alpha_2\) such that

\[\alpha_1 \|s\|_2^2 \leq V(x, t) \leq \alpha_2 \|x\|_2^2. \quad (13)\]

Notice from (8) that

\[V > \frac{1}{2} \left[ \lambda_{\text{min}} \{K_p\} \|e\|^2 + m_i \|\|_2^2 + \lambda_{\text{min}} \{K_p\} \|z\|^2 + m_i \|\|^2 \right] - \lambda_{\text{max}} \{\Lambda K_u\} \|s + r\| \|

= \frac{1}{2} \left[ \lambda_{\text{min}} \{K_p\} \|e\|^2 + m_i \|\|_2^2 + \lambda_{\text{min}} \{K_p\} \|z\|^2 + m_i \|\|^2 \right] + \lambda_{\text{max}} \{\Lambda K_u\} \|s + r\| \|
so we define $\alpha_1$ as

$$\alpha_1 = \frac{1}{2} \min \{ \lambda_{\min} \{ K_p \}, m_1 \}$$

![Figure 1. Schematic diagram of the 2-DOF robot manipulator.](image)

In a similar manner, an upper bound on (8) is

$$V \leq \frac{1}{2} \left[ \lambda_{max} \{ K_p \} + 2n\lambda_{max} \{ K_a \} \right] \| z \|^2 + m_1 \| q_x \|^2 + m_2 \| q_y \|^2 + (\lambda_{max} \{ K_p \} + 2n\lambda_{max} \{ \gamma \}) \| z \|^2$$

so we define

$$\alpha_2 = \max \{ (\lambda_{max} \{ K_p \} + 2n\lambda_{max} \{ K_a \}), m_2 (\lambda_{max} \{ K_p \} + 2n\lambda_{max} \{ \gamma \}) \}.$$  

From (12), (13) we conclude that the domain of attraction contains the set

$$\| x \| \leq \frac{1}{k_c} \lambda_{min} \{ K_D \} \sqrt{\frac{\alpha_1}{\alpha_2}}. \quad (14)$$
5. Simulation results

To study the performance of the controllers, we generate simulations in which a two-link manipulator was required to follow the joint trajectory

\[ q_{di} = \pi + \frac{\pi}{2} \cos(2\pi ft), \quad i = 1, 2 \]

where \( f=10 \) Hz. The position and velocity initial conditions were set to \( q(0) = \dot{q}(0) = 0 \in \mathbb{R}^2 \). The motion of the 2-DOF manipulator with rotational joints, depicted in Figure 1, was governed by (4) where

\[
M(q) = \begin{bmatrix}
8.77 + 2.02 \cos(q_2) & 0.76 + 0.51 \cos(q_2) \\
0.76 + 0.51 \cos(q_1) & 0.62
\end{bmatrix},
\]

\[
C(q, \dot{q}) = \begin{bmatrix}
-0.51 \sin(q_2) \dot{q}_2 & -0.51 \sin(q_2)(q_1 + q_2) \\
0.51 \sin(q_2) \dot{q}_1 & 0
\end{bmatrix},
\]

And

\[
g(q) = 9.8 \begin{bmatrix}
7.6 \sin(q_1) + 0.63 \sin(q_1 + q_2) \\
0.63 \sin(q_1 + q_2)
\end{bmatrix}
\]

were taken from (Berghuis & Nijmeijer, 1993a). In the simulation, the control gains were selected as follows:

\[
K_P = \text{diag}\{10,10\} \quad K_D = \text{diag}\{5,5\}
\]

\[
K_a = \text{diag}\{5,5\} \quad K_\beta = \text{diag}\{2,2\}
\]

\[
\gamma = \text{diag}\{12,12\}.
\]

The resulting position and observation errors of the closed-loop system (8) for the unperturbed and perturbed case are depicted in Figure 2. This figure also shows the control input. The figure demonstrates that the homogeneous controller asymptotically stabilizes the manipulator in the desired trajectory, thus satisfying the control objective (5). Figure 3 shows chattering due to the relay part of the control law (last two terms of (6)):

\[
u_h = -K_a \text{sign}(e) - K_\beta \text{sign}(\dot{q}_0 - \dot{q}_r).
\]

For the sake of comparison, both controllers (6)-(7) and the controller without a
relay part \((K_a=K_b=\gamma=0)\) that were proposed by Berghuis and Nijmeijer (1993b) were simulated assuming permanent disturbances \((w=10)\). In contrast to the proposed controller, the simulations results depicted in Figure 4 show that the continuous controller drives position of each joint of the manipulator to a steady-stable error of 0.3 \([\text{rad}]\). We repeated the last simulations for a reference trajectory consisting of a circle in task space with radius \(r=0.25\) \([\text{m}]\) and center \((1,1/2)\) which is given by

\[
\begin{align*}
\begin{cases}
x_d &= 1 + \frac{1}{4} \cos(0.1t) \\
y_d &= \frac{1}{2} + \frac{1}{4} \sin(0.1t);
\end{cases}
\end{align*}
\]

where \(x_d\) and \(y_d\) are coordinates in the Cartesian space and the initial position in task space is fixed at \(x(0) = 2\) \([\text{m}]\) and \(y(0) = 0\) \([\text{m}]\). The corresponding trajectory in the joint space is

\[
q_{d1} = \tan^{-1}\left(\frac{x_d}{y_d}\right) - \tan^{-1}\left(\frac{l_1 \sin(q_{d2})}{l_1 + l_2 \cos(q_{d2})}\right), \quad q_{d2} = \cos^{-1}\left(\frac{x_d^2 + y_d^2 - l_1^2 - l_2^2}{2l_1l_2}\right)
\]

where \(l_1\) and \(l_2\) are the lengths of links 1 and 2 respectively. Figure 5 illustrates the effectiveness of the controller.
Figure 2. Joint position errors, observation errors, and control input for the unperturbed case (*left column*) and perturbed case (*right column*).
Figure 3. Chattering signal due to discontinuous terms in the input control.
Figure 4. Joint position errors for the continuous controller (i.e., $K_\alpha = K_\beta = \gamma = 0$): The perturbed case.
6. Conclusions

We developed a controller that exploits the advantages of homogeneity theory and that is applicable to the tracking control problem for \( n \)-degrees of freedom robot manipulators, assuming that position information is the only available feedback data. The basis of this work is the passivity-based approach, which consists of a tracking controller plus an observer complemented with a relay part that yields a certain degree of robustness in comparison with its continuous counterpart. Stability analysis was developed within the nonsmooth Lyapunov function framework (Baccioti & Rosier, 2005) where semi-global asymptotical stability has been concluded. The effectiveness of the controller was supported by simulations made for a two degrees-of-freedom robot manipulator taking into account the unperturbed and perturbed cases.
7. References


This book covers a wide range of topics relating to advanced industrial robotics, sensors and automation technologies. Although being highly technical and complex in nature, the papers presented in this book represent some of the latest cutting edge technologies and advancements in industrial robotics technology. This book covers topics such as networking, properties of manipulators, forward and inverse robot arm kinematics, motion path-planning, machine vision and many other practical topics too numerous to list here. The authors and editor of this book wish to inspire people, especially young ones, to get involved with robotic and mechatronic engineering technology and to develop new and exciting practical applications, perhaps using the ideas and concepts presented herein.

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