Tracking Control in an Upper Arm Exoskeleton with Differential Flatness

E. Y. Veslin¹, M. Dutra², J. Slama², O. Lengerke²,³ and M. J. M. Tavera²
¹Universidad de Boyacá,
²Universidade Federal do Rio de Janeiro,
³Universidad Autónoma de Bucaramanga
 wildfires Colombia
 Brazil

1. Introduction

The Exoskeleton is classified inside of a wider technology known as Wearable Robots, that group describes a robotic field who studies the interaction between the human body and the robotics. In those systems, a mechatronics structure is attached to different members of the human body, and while the wearer commands the mechanical system using physical signals, like the electronic stimulation produced by the orders of the brain, or the movements of the body generated by the muscles; the mechanical system do the hard work, like carrying heavier objects or helping the movement of handicaps members of the body. Since its conception in the sixties (Croshaw, 1969) the bibliography speaks about two models of wearable robots: the prosthetics and the orthotics systems (Fig. 1). The first group replaces the lost body members, while the second one assists to the body movements or, in other cases, extends the body capabilities.

Fig. 1. (Left), orthotic robot of upper parts (Perry et al., 2007). (Right), prosthetic robot of upper parts (Rehabilitation Institute of Chicago, 2006)

The exoskeleton take part of the orthotics wearable robots field, it’s a robotic structure (actually could be considered a mechatronics structure due to the integration of different
electronics equipments like sensors, actuators and an intelligent controller that are integrated into the mechanical system) fixed to the human body that follows along the movements of the wearer. At the present time are presented several models of exoskeletons: those that are attached to the legs was called lower part exoskeletons (Kazerooni, 2006) and also exists those that are attached to the arms, that are denominated upper part exoskeletons (Perry et al., 2007). In both structures, its applications goes from the medical camp to military applications; with functions like help handicap patients assisting on the body member’s movement recuperation (Gopura & Kiguchi, 2007) or helping to the soldiers to carry heavier loads (Kazerooni, 2005). Also, exists prototypes that integrates both systems (Sankai, 2006), those mechanisms like Kazerooni’s structure, used to help the wearer to extends his body capabilities in order to carry weights that are beyond his normal possibilities, or help to patients to made normal movements that a sane body could develop. Our work are focussed into the study of the upper arm exoskeleton and a application of differential flatness in order to control the system that moves in a determinate path (in this case a path generated by a polynomial). This document will describe its consecution, initiating with a morphological analysis that leads to a mathematical model and posterior to a cinematic simulation. This model is useful to the consecution of the dynamic model, this study was based on the mechanical structure and the arm movements that were selected to made. This dynamic model will be applied into the problem of trajectory tracking, in this part, the arm is controlled using differential flatness, this part was considered our contribution, therefore, we will discuss about its applications, benefits and problems founded. All the analysis was supported by a code made in Matlab® and gives the necessary simulation to this work, in this way, graphics of the model behaviours was characterized and captured in order to detail the different parts of this work.

2. Forward kinematics

The first problem to solve is the characterization of a human arm model than could be implemented for an exoskeleton design. In this part, forward kinematics plays an important role; offering a mathematical model that allows the prediction of the system’s behaviour, giving the necessary preliminary information about the structure and movement capabilities. Case of modelling the human arm for the study of the human motion (Maurel & Thalman, 1999), (Klopčar & Lenarčič, 2005), or in a implementation study of a rehabilitation system in an arm (Culmer et al., 2005), or the design of a master arm for a man-machine interface (Lee et al., 1999), All needs of the forward kinematical analysis in order to initiate a system’s modelling.

The forward kinematic study initiate with a morphological analysis in the part of the body that will be attached to the mechanical structure. At is says, the exoskeleton is a system that accompany the arm on their movements; therefore the design of the structure must not had interference with it. In order to do this, the morphological analysis gives the information about movements, lengths of the members, number of segments and its respective masses. Another justification is that the biological inspirations permits the creation of mechanical systems that are more compacted and reliable and also, more energy-efficient (Kazerooni, 2006).

In the nature, the biological designs of the living organisms allows the adaptation with the environment; the evolution mechanisms have made transformation into the body as the circumstances rules, for this reason the diversity of organic designs and also, the extinction
of those species that didn’t could adapt to its environment. In conclusion is possibly to say that exist optimizations mechanisms that transforms our structure in function of a goal: survival.

The human arm is a result of this optimization, following the bones that conforms it (Fig. 2) its seen that the member are divided into three segments: the arm, the forearm and the hand, united among them by three joints: the shoulder that unites the upper limb to the rest of the body, the elbow that unites the arm with the forearm and finally the wrist that unites the forearm with the hand. The mechanical model of the exoskeleton responds to this configuration, conserving the systems general characteristics, this process of make a copy of the nature design, is called bioimitation (Pons, 2008).

Fig. 2. Human arm structure (Pons, 2008)

Initially, for this study the hand and fingers movements will not be considered, so, it could be reduced to a single segment (Pons, 2008), in this way the kinematics is constructed with three segments, the arm, the forearm and the hand, also exists other considerations that could help into the study of the system (Rocon & Pons, 2005):

1. The mechanical behaviour of the arm is independent of the rest of the body.
2. All the components of each segment, including the bones and the soft parts (muscles, skin, etc.) take part of the same rigid body.
3. The deformations of the soft part will not affect significantly the mechanical properties of the entire segment.

The lengths of the segments are considered constants and depend of the height of the individual ($H$), this numerical relationship are detailed into Table 1, who also related another parameters of the human arm, like the position of the centre of mass of each segment.
<table>
<thead>
<tr>
<th>Body Segment</th>
<th>Length, L</th>
<th>Centre of Mass (% of L)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper Arm</td>
<td>0.186 H</td>
<td>0.436  0.564</td>
</tr>
<tr>
<td>Forearm</td>
<td>0.146 H</td>
<td>0.43   0.57</td>
</tr>
<tr>
<td>Hand</td>
<td>0.108 H</td>
<td>0.506  0.494</td>
</tr>
</tbody>
</table>

Table 1. Human arm Anthropometric data (Pons, 2008)

Each joint has its proper movements, which depending of the movement could it be: uni, bi or multiaxial (Gowitzke & Milner, 2000), it means that the joints allows movements in one, two or three degrees of freedom and planes generated by the rotation of the segments. For this study all the joints could be considered ideals (Rocon & Pons, 2005).

The movements of the arm are showed into Fig. 3. The first joint, the elbow, is compared with a spherical joint that allows movements in three degrees of freedom (multiaxial), its movements are defined as: flexion-extension, abduction-adduction, and rotation. The second joint, the elbow is an uniaxial joint, its movement is compared with a hinge, nevertheless, the bone of the forearm generates a movement of rotation around its own axis, that not affects the position of the three spatial coordinates, but if it rotation. This rotation could be located on the elbow joint or into the wrist joint. On early studies this rotation was considered on the elbow but later in order to make a simplification in the numerical operations it was transported to the wrist (Veslin, 2010).

Some research studies the movement of the arm in the plane (Gerald et al., 1997); others articles analyse the movement of the forearm for applications of trembling reduction (Rocon & Pons, 2007). The kinematic analysis on this work will be focused on the movement of the three joints of the elbow, and specifically to the posterior control of the hand position using a mechanical structure supported by the arm, for this reason is necessary to know its capabilities based in an analysis of the workspace.

Knowing the structure of the arm, the distance of the segments and the degrees of freedom of its joints, the mathematical model could be developed, in this part the concepts of morphology previously studied are combined with the classical robotic concepts, in this case using the model of homogenous transformation matrix (Siciliano et al., 2009). Equations (1-3) describe the rotations \( R \) around the three axis X, Y and Z, while (4) describe the translations \( T \) in whatever axis.

\[
R_z(\beta) = \begin{bmatrix}
\cos \beta & -\sin \beta & 0 & 0 \\
\sin \beta & \cos \beta & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

(1)

\[
R_y(\alpha) = \begin{bmatrix}
\cos \alpha & 0 & \sin \alpha & 0 \\
0 & 1 & 0 & 0 \\
-\sin \alpha & 0 & \cos \alpha & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

(2)
In (4) is necessary to define in which, or what axis the translation movement will be effectuated. The cinematic model obeys to a kinematics open chain, because only exists one sequence of joints together, connecting the two endings of the chain (Siciliano et al., 2009). Fig. 3 shows the arm structure from the shoulder to the hand, each movement is associated to a Latin symbol and an orientation respects to the inertial axis. In this way, all the movements of flexion-extension occur into the Y axis, the rotation movements on the bone into the Z axis, and the others into the X axis. Its seen that in the shoulder exists a special case where three rotations intercept in of one point without the existence of any translation, this kind of configuration it’s called spherical wrist. All the translations referents to the segments lengths (l1,l2 and l3) happens in the Z axis.

![Diagram of arm transformations](image-url)
the path from the origin to the point P1, T02 the path from P0 to P2 and T03 the path from P0 to P3.

\[
T_0^1 (\beta_1, \alpha_2, \gamma_3) = R_y (\alpha_2) R_z (\gamma_3) R_x (\beta_1) T_z (l_1) \\
T_0^2 (\beta_1, \alpha_2, \gamma_3, \alpha_4, \beta_2) = T_0^1 R_y (\alpha_4) R_z (\beta_2) T_z (l_2) \\
T_0^3 (\beta_1, \alpha_2, \gamma_3, \alpha_4, \beta_2, \alpha_6, \gamma_7) = T_0^2 R_y (\alpha_6) R_z (\gamma_7) 
\]  

(5)

The order of multiplication of each matrix is given according to an observation analysis; the orientation due to the consecutive operations on the rotation matrix could make variations into the final response, affecting both the visualization and the position of a point of the system. It’s important to be clear in what will be the order of the final equation (5), this gives an analysis of what happened into the arm with the variation of each joint. This analysis is compatible to both arms, only changes the direction of the rotation, remembering that according to the established parameters of the homogeneous transformation, every rotation made counter the clock is considered positive.

Fig. 4. Different positions of the arm
This mathematical model was implemented in MATLAB®, and the results graphics Fig. 4, describes different positions of the arm structure, as the anatomical position with all the rotations in 0°. The three segments represent the arm divisions, the triangle the body and the sphere the human head. All the graphics was made for an individual of 1.75 m of height.

The model obtained of the kinematical analysis give to the researcher a map of the system behaviour, the restrictions on the joint movements and the extensions of the segments of the arm can detail the workspace (Fig. 5), a graphical analysis that gives an understanding of the system capabilities. Analysing the workspace of the human arm, is possible to obtain information for the design and control of the mechanism, and also, gives an understanding of the movement properties. In the case of the human arm, this study allows the planning and programming of rehabilitation process comparing the workspace of a normal patient, with a person with pathology deficient, this differences could be used for the diagnosis study (Lenarčič & Umek, 1994).

![Fig. 5. Arm Workspace](image)

In an exoskeleton analysis, exist two workspaces in interaction: the human arm movement and the mechanical system movement (Pons, 2008), both generates a range of positions in the space that intercepts, this interception will be the plausible movement generated by the wearer of the exoskeleton. In the case of the human arm, the graphics was obtained using the three movements of the shoulder and the flections of the elbow. The arm was moved with movements of flexion-extension and, finally, the rotation of the elbow. The Fig. 5 shows the path made by the right arm around the shoulder, the figure describes a form like a circumference with radio equal to the distance of the arm (l1) and forearm (l2), all the dots points the wrist position for each movement. Exists a space that the arm cannot reach, this space correspond to the sector near to the back, where is impossible to reach due to the physics restrictions for the body. Is important to say that in ideal conditions, the workspace in both arms is the same.
The workspace is defined by the mobility of the articulations, these being from the human arm or the mechanical system that support the arm. Therefore, the workspace of a wearer that carries an exoskeleton could be affected by the mechanical restrictions that the system imposes, for this reason, better mechanical designs allows the performance of movements more obedient and with a wider rank.

3. Dynamics

While the kinematics analysis gives an approach of the system, the dynamic model generates the necessary information about the forces needed to unbalance the segments an initiate the movement, and posteriorly the behavior of the system across the different possible positions that the arm could reach.

Refers to other authors (Rosen et al. 2005), the conception of upper arm exoskeleton approach to the behavior of an human arm due to the morphological design of the mechanism. In some designs, the weight of the system doesn’t influence in the dynamical analysis, and all the inertial forces are related to the human arm physiology (Carignan et al., 2005), in this case the center of mass is related to the extension of the arm, and its mass depends of the organical structure (Table 1 and Table 2). In other cases, the mechanical system is more heavier than the human arm (Croshaw, 1969), in these models the structure must be included into the analysis, this cause that parameters like inertial center and structure change drastically. Nevertheless, with an adequate code this changes could be easily modified, because this information could be parametrized or previously changed to the simulation code.

Figure 6 illustrated the model of the arm, C1 and C2 are the position of the center of mass on the arm segments, the rotations consdired into the study are the shoulder movements and the flexion-extension of the forearm. The hand movements doesn’t be considered into this part of the study.

The initial evaluation are focused into the minimal forces necessaries to move an human arm, in order to do that, is necessary to known its composition. In this way, parameters like mass, length and position of the inertial center that depends of the wearer stature, are introduced into the model and simulated to obtain a further analysis.

<table>
<thead>
<tr>
<th>Body Segment</th>
<th>Percent respect to the body weight m(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper Arm</td>
<td>2.70</td>
</tr>
<tr>
<td>Forearm</td>
<td>1.60</td>
</tr>
<tr>
<td>Hand</td>
<td>0.66</td>
</tr>
</tbody>
</table>

Table 2. Mass of the segments body (Tözerem 2000)

To obtain the body mass, exists a relation with its height, this relation is called Body Mass Index (BMI) established by Adolphe Quelet in the XIX century, that evaluates the health conditions of the person using a number, a discussion of its validation it’s not an interest in this work, but this model gives a mathematical relation (6) that is useful to introduce into the code of the model, high values of BMI evidences complication in the health, a BDI of 24 considerate that the person have good health conditions, and this number will be used to obtain the body mass that finally gives the segments mass due to Table 2.
To do the simulation, the arm is modeled using a Lagrangian conception, where the human arm is considered a robotic manipulator. With the kinematics model, the information of the position of the ends of segments and inertial centers could be added into the model. The final result is a nonlinear mechanical model that simulates the arm behavior in different movements.

For the evaluation of the forces that involves the movement of the system, a PD controller is used to move the arm from an initial position to a desired position. Movements like lift the arm while the forearm is static or move only the forearm could be simulated. Initiating in different positions and with various wearer statures, the code, gives the necessary information about the maximal and minimal forces involved, the influence of the Coriolis and centrifugal forces and the gravitational forces into the arm, and also plausible reductions into the model to reduce the computational time in the simulation.

In previously works (Veslín 2010), the dynamical model was obtained and finally condensed into a form described in (7), in this equation, the joints are considered ideals so the mathematical model ignore the friction forces, also, the perturbations that could exist into the movement doesn’t be introduced because we don’t have a way to quantify it, and finally is considered that the system is fully actuated, its means that each movement have its proper force actuating into it.

\[ M(q)\ddot{q} + C(q,\dot{q}) + K(q) = \tau \]  

Fig. 6. Human arm model
The mass of the segments was concentrated in its geometrical center, the form of the arm was approximated into a cylinder (Admiral et al. 2004, Soechting et al., 1995) this gives a mathematical relationship and evades the use of tables or interpolations to find the inertial center of the arm. Was demonstrated (Veslin, 2010) that the presence of the inertial tensor doesn’t influence significantly the system behaviour, this consideration gives a reduction of the model equation and converts the tensor on a single equation.

Each movement was controlled using a feedback rule (8), it gives to the system the force needed to move the segments to the desired position, given by \( q_{nd} \), its equivalent to a PD controller, its applications are seen in industrial robots and lower parts exoskeletons (Carignan, et al. 2005).

\[
\tau_n = -\beta \dot{q}_n - \gamma (q_n - q_{nd}) + W_n(q_n)
\]  

(8)

The Gain \( \beta \) involve the arm velocities and the energy dissipation, in oscillatory movements it controls its amplitude and the duration of the movement, \( \gamma \) gives a force that depends of the distance between the actual position of the segment \( q_n \) and the desired position, in this way the equation reduce the force while the arm is reaching the goal. \( W_n \) are the forces that the actuators have to support when the segments are in static equilibrium. The implementation of these controllers adds a problem called overshoot (a high input that occurs in a small portion of time) into the beginning of the simulation (Figure 6).

![Fig. 7. Presence of overshoots at the beginning of the simulation](image)

In order to resolve this problem, a trajectory generator is added to the system control, this generator increments the position of the segment considering the initial and finally positions, in this way is possible to create trajectory that descript a desired behavior applying interpolation methods, like cubical, quadratic or another equation. Considering that the model initiate and finish with zero speed, the form that could be implemented into the model is a cubical equation (Figure 7), which is implemented into (8) replacing the term \( q_{nd} \).
Fig. 8. Variation of the angular position in a cubical way

The movement is generated around the movement determined by the trajectory equation, the Figures 8 and 9, shows a common displacement between two points in the shoulder and elbow respectively, this movement is called plane movement because only actuate in one degree of freedom of the shoulder. The dot line represents the movement interpolated by the cubic equation, and the solid line is the movement generated by the degrees of freedom of the system that are actuated, moving according by the path and controlled by equation 8.

Fig. 9. Comparison between the movement generated by the controller into the shoulder and the elbow respects the desired path
The absolute error (Fig. 10) between the movements executed and the path is evaluated, the mayor displacement happens in the middle of the movement around the 8 seconds, being major in the shoulder, presenting a displacement of 2.5 degrees and 1.4 degrees into the elbow. The final position also shows a displacement, the error in this position is near to 0.5 degrees in both movements.

![Error vs Time](image1.png)

![Displacement Variation](image2.png)

Fig. 10. Difference between the shoulder and elbow movement with the desired path

To evaluate the controller, different movements were generated and the result is showed in Fig. 11, the forearm final position is evaluated because it gives the location of the wrist, which is the object of control. The graphic evaluates the final position of the forearm initiating its movement from 0° to 140° during a elevation movement of the arm (that moves from 0° to 180°), the mayor displacements happens when the final position of the forearm is 90°. The internal forces between the segments cause this variations, the forearm in this position generates the biggest moment to the arm in movement. The controller assure that them doesn’t influencing moving the arm to another position, the moments caused by the mass segments and the energy induced by the velocity is controlled by the model proposed, with a minimal error as is illustrated that doesn’t overcome the 0.3° in the worst case.

Therefore, this error could be reduced according with the variation of the parameter $\gamma$ in the controller, Fig. 12 and 13 evaluates different errors in the final position with different values in $\gamma$ in a range of 100 Nm/rad to 600 Nm/rad, into ascending (Fig. 12) and descending (Fig. 13) movements. Bigger values into the constants reduces the difference in the displacement generated by the internal forces of the arm, the controller supplies the necessary force and consecutively the error of displacements change according to the variation of the constant.
For this graphics the forearm was conserved into a position of $0^\circ$ while the arm was moving in a range of $0^\circ$ to $180^\circ$.

The maximal forces generated by the controller are illustrated in Fig. 14. In this graphics two movements are compared, in the red line the movements begins from the $0^\circ$ and moves into different positions in a range of $0^\circ$ to $180^\circ$ with increments of $10^\circ$, always executed in a time of 20 seconds. The blue line effectuates the same movement in a time that is proportional with the space travelled, in other words, faster than the first move. This movement generates bigger forces because effectuate the same movement with less time, the augmentation of the velocities into the generalized coordinate generates an augmentation of the Coriolis forces between the segments, generating in the controller a bigger force to counteract the impulse generated by the movement and the mass of the arm.

![Graph](image-url)

**Fig. 11.** Deviation of the final position of the wrist depending of the position of the forearm while the arm is rotating into flexion movement

Also this forces changes depending of the height in the individual, as is show in (6) and Table 2, the mass on the arms differ depending of the stature of the individual and his BMI, in this case, the maximal forces in the arm (which actuator have to support the weight of the complete system) are illustrated in Fig 16, where in a range of heights from 1.5 m to 2m, the forces necessaries to arise the arm increases in almost 10 Nm.

Movements out of the plane, with a rotation of two degrees of freedom in the shoulder are illustrated in Fig 16, the shoulder was commanded to move the arm in flexion and abduction, while the forearm rotates from the rest position to $90^\circ$, (1) illustrates the movement according with the generalized coordinates, while (2), (3) y (4) illustrates the behaviour on the system in the space.
Fig. 12. Reduction of the error according to different values of $\gamma$ into the controller in a ascending movement

Fig. 13. Reduction of the error according to different values of $\gamma$ into the controller in a descending movement
Fig. 14. Variation into the maximal forces for movements in the arm with different velocities

Fig. 15. Variation into the maximal forces for movements in the forearm arm with different velocities
Fig. 16a. Variation into the maximal forces with different individual heights

Fig. 16b. Movement in the generalized coordinates generated by the controller (1), trajectory in the space described by the arm (2), projection in the transversal plane (3) and the sagittal plane (4)
The influence of the gravitational forces in the arm model was evident when was founded an increase in the maximal forces effectuated by each generalized coordinate, this is caused by the increasing of the mass in the segments and also to the position of the forearm respects to the arm, meanwhile and augmentation on the velocities also increase the value on the applied forces to effectuate the movement. The implementation of the PD controller with a compensation of the gravitational forces in the arm position allows to made the previously forces analysis, the constants have an important role in the system behaviour, adding stabilities and reducing the error in the positioning of the arm wrist.

4. Differential flatness

Introduced by Fliess (Fliess et al., 1994), the differential flatness is a concept of the dynamic control, where a system is assumed to be flat, if it have a number of outputs known as flat outputs (9), such that them and its derivatives, could describe the system inputs (that have the same number of outputs) and the states of the system (10-11).

\[ y = h(x, u, \dot{u}, ..., u^{(r)}) \]  
\[ x = f(y, y, ..., u^{(q)}) \]  
\[ u = g(y, y, ..., u^{(q)}) \]

This property called is called by Fliess diffeomorphism, and is very useful in the control of systems that are needed to achieve a desired behaviour. In order to do that, the flat outputs are parameterized into functions, and posterior introduced into the system; the result is a non linear equation system that is transformed into a polynomial, so, the solution of the problem is reduced significantly because is not necessary the use of integration or numerical solutions.

The focus of this analysis is to assume than a system is differential flat, work that isn’t easy to achieve, in fact, not all the systems have this property, and is considered that be flat it’s a characteristic of the system itself. Many systems are defined as flat and some authors have studied its implications (Murray et al., 1995). Like Murray’s text, these works are focused into the demonstration of the flat property and its application, in systems like mobile robots and cars with n trailers.

The manipulator trajectory tracking, consist in determinate the necessary inputs (forces) in each system’s generalized coordinate, in order to accomplish that the model moves between one point to another across a defined path. In this kind of problems the first step consist in the specification of a sequel of points in the manipulator space work, this points will become desires positions across the path, and posterior are integrated using an interpolation function (which is typically in a polynomial form). Actually, different techniques exist for the trajectory planning: (i) when is given each final and ending position, talk about a point to point motion, (ii) when is working with a finite point sequence, talk about motion through a sequence of points. Both techniques give a time function that describes the desired behavior (Siciliano, 2009, van Nieuwstadt, 1997a).

The second step is the implementation of this function into the system’s dynamical model and verifies that effectively is given a trajectory tracking. In this part, is necessary to take a decision, if the problem is going to be worked into the operational space, or in each joint.
Due to his generality, the first solution, could give singularity problems and redundancy caused by the number of degrees of freedom in each joint, and the nonlinear effects could make it difficult to predict, meanwhile, a joint position path parameterization shows to be furthermore suitable, because this study allows to work in each element, considering factors like the restrictions movements and degrees of freedom on each joint.

Everything joint parameterized trajectory is defined by a function \( q(t) \). This function is the source of a joint desired position collections for the manipulators movement across the time. Then, using different control techniques: like the PD control with gravity compensation, IDC (Inverse Dynamic Control) or adaptive control, is possible accomplish a manipulator movement on the defined path with the minimal deviation (Marguitu, 2009, Spong, 1992, Wang, 2009).

It was demonstrated that all robotic manipulator (if the upper arm exoskeleton is asumed of this form) could be flat if the system is fully actuated (Lévine, 2009); it means, that all its generalized coordinates have an actuator. In this case, from the model of the system (7), assuming that all the outputs are the position of the generalized coordinates \( q \), and the inputs are the forces that moves each coordinate \( \tau \), the systems is differential flat if the position of each generalized coordinate and its derivatives (the velocity and the acceleration) are the flat outputs. It must be realized that all the systems outputs didn’t are flat, so it’s important that those be differentiated from the normal ones by the letter \( Z \) (Van Nieuwstadt, 1997b). Thus the equation (7), remembering that the joints are considered ideals, can be represented as:

\[
M(z)\ddot{z} + C(z, \dot{z}) = \tau
\]  

(12)

The tracking trajectories problem doesn’t present complications in a flat system; as the inputs and its states are defined in function of the flat outputs, is possible to create paths that vary in the time in order to define an behavior on its outputs, and through them and its respective derivatives, determine which are the inputs needed by the system to generate an answer that approach to the desired one by the path. This path could be polynomial or a function of \( C^\infty \) type as the trigonometric or the exponentials (Rotella & Zambettakis, 2008). The application of the flat theories allows the transformation of the differential equation system (12) into a polynomial of order \( n \), making a simplification into the problem solution and transforming it to a completely algebraic system.

The next step is to applying the differential flat theories into the model; the general objective is control the behavior of the system in function of a desired action. This behavior will be parameterized into a trajectory. With knowledge of the flat outputs of the system, these trajectories could be easily implemented. The model in (12) shows that is possible to interpret the systems inputs knowing its flat outputs behaviors until the second derivative, therefore, is necessary add this functions and its derivatives into the model. By example, if is desired that the arm that is working in a plane movement rise starting from the origin, with a gradual variation of the positioning, it could be commanded a behavior using a cubic function in the generalized coordinates of the shoulder and the elbow, this is:

\[
z_1(t) = 0,0094t^2 - 0,0006t^3
\]  

(13)

\[
z_4(t) = 0,0157t^2 - 0,001t^3
\]  

(14)
The equation (13) leads the arm from the origin positioned in 0° to a position of 18°, on equation (14) the forearm moves from 0° to 30° (Fig. 4), both segments present null velocities at the beginning and at the end, executing the translation in a time of 10 seconds. The equations (13) and (14) and its respective derivatives are introduced in the dynamic model (12). To determine the inputs $\tau$ only is necessary to solve the system in a given time. The response provided by the Fig. 17 is a set of outputs that are applied into the system and must allow to displace it according to the behavior implemented in equations (13) and (14), for each segment.

This response is evaluated into the dynamic model (8), the system response is verified in Fig. 18, where the trajectory made by the generalized coordinates is superposed on the desired path define by the cubic equations. Due to the proximity in each equation, Fig. 19 evaluates the error between them, showing an oscillatory trend of error variation, the maximum difference is the order of $10^{-2}$.

Fig. 17a. Set of forces obtained through equation (8) in each segment

Fig. 17b. Response of the system according of the forces

The obtained response Fig. 17. is desirable, but for another positions tests following the same cubic behavior, was founded stability problems represents in a leak of the desired path
system, this is evidenced in Fig. 19, where is tried to move both segments from 0° to 90° position.

![Fig. 18. Tracking error in each generalized coordinate](image1)

![Fig. 19. Trajectory deviation for each generalized coordinate in higher movements](image2)

This doesn’t mean that the flat systems theories doesn’t can be applied to certain positions, only that, for these solutions, flatness not guarantee the tracking and the necessary stability if applied in open loop, case of the examples seen before. It is necessary then, to change the focus, in order to take advantage of the benefits that flatness offers. This scheme implementing a controller that feedback information from real estate and compare it with the desired output, generated from this difference a control action that finally allows the system track the trajectory desired with a minimum error, this is made with the implementation of a controller into the system.

Called two degrees of freedom control (Fig. 20), any closed loop system contains a controller and a trajectories generator. The controller modifies the output in function of existing error between the actual output and nominated desired output (Van Nieuwstadt, 1997b).

This system had actually three degrees of freedom, such system contains a higher level that generate the output depending of the desired paths (output trajectory). An intermediate
level generates inputs based on these outputs, and finally the lower level stabilizes the system around the nominal trajectory. The uncertainties were variations caused by the gravitational, friction forces, and nonlinear in the models.

![Diagram of Two degree of Freedom Control System](image-url)

Fig. 20. Two degree of Freedom Control System (Van Nieuswadt, 1997b)

The solution gives an input trajectory that could be used into the real model. This assumption works in theory, however some interferences, could separate the real behaviour of the desired, thus, is necessary to apply a PD feedback control into each generalized coordinate, with this, is possibly to enforce the manipulator to describe a desired performance.

For the controller in Fig. 20 is established a feedback law (Franch & Agrawal, 2003):

$$\tau = \tau + k_p(z_n - x_n) + k_d(\dot{x}_n - \dot{x}_n)$$

(15)

Equation (15) ensures that the nominal state $X_n$ follow the desired path $Z_n$ over a correct choice of $k_p$ and $k_d$ constants. $k_p$ generates an action that modifies the inputs from the flatness system control in a proportional form. The derivative constant $k_d$ removes variations (oscillations) caused by the corrections in the proportional part. In Fig. 19 are visible the jumps in the position ranging from 100° to 800° in a small period of time, so hopefully abrupt change in the action of proportional control and consequently oscillations in the model. If each segment has its own control loop, with the constant $k_p=25$ and $k_d=0.5$ for the segment of the arm and $k_p=2.5$ and $k_d=0.5$ to the forearm. The results of the implementation of these values can be seen in Fig. 21 Obtained paths follow very near to the desired paths. Fig. 22 shows the performance of the arm according the inputs.

With this control, is easier to move the system using others trajectories, any position that is within the manipulator workspace, can be parameterized by a polynomial for then of differentially flatness systems theory, appropriate requirements. For example, is required that the arm makes a movement oscillating from a cosine function while the forearm elevate to 90° position following the same methodology of the previous path behavior, is obtained the behavior described in Fig 23.
Fig. 21. Trajectory tracking using the controller

Fig. 22. System behavior using the inputs defined by flatness

Fig. 23. Force behaviour obtained according with the cosine function implemented in the arm
The concept is also applicable to systems with more degrees of freedom, although the fact that the equations of motion are more complex it is possible to obtain path tracking and behaviors. But this implementation presents difficulties, inputs equations in function of four outputs are extensive and prevent the possibility of achieving an acceptable result for extensive times. Precisely this error, forced to resort to the use of a controller. Open loop system presented difficulties during the follow-up. The controller makes modifications imperceptibles on the inputs, but enough so that there is a stable trajectory. Fig. 26 visualize the behavior of one of these desired trajectories for a system that controls the four generalized coordinates. The ability to follow-up on trajectories using differential flatness systems on robotics manipulators depends on the quality of input values. Was founded that small changes in these settings offer better performance, proving that the resolution of inputs is an important part of the results.

The tests carried out previously, the models were analyzed over a time period in increments of 0.01 seconds. But testing with greater increases do not provide appropriate behavior, as it may be identified in Fig. 27, this makes it difficult to determine the values of the constants of the controller or even worse, that do not exist. This restriction to use short times make it difficult for the calculation of inputs due to the high consumption of computing resources, therefore, in to reconfigure a strategy for solution of this situation.
Fig. 26. Behavior of system for each trajectory, the dash line describes the actual movement while continuous line describes the desired trajectory.

Fig. 27. Behavior model with higher time period, are evidence a difficulty in the follow-up to the desired output.

5. Conclusions

This study gives to us the initial phases into the consecution of a trajectory control for an upper arm exoskeleton. Whit the forward kinematics study is possibly to generate a fist view on the system, this mathematical model achieved by means of a morphological study of the human arm gives a first view of the system behaviour, being this workspace a tool to predict the system capabilities in function of the mechanical restrictions and also the human movements. This one is an important implementation of the code made.
Therefore, the dynamical analysis provides the information about the system requirements in function of the stature of the wearer, the influences of the mechanical model was not considered into the dynamics, the movements was made considered the human arm as the objective to make an actuation, and the prototype as the element that actuated on the arm. Every consideration on the physical model of the system must be considered in the dynamical design. This dynamical analysis generates a mathematical model that is implementing into the problem of the trajectory tracking on the system. It was demonstrated that the arm is differential flat, and with this assumption, was possibly to generate any behaviour to be implemented into the system, it means a path in each generalized coordinate. In order to ensure a correct tracking, a PD controller was added into each generalized coordinate, the simulations shows that the model of the upper arm get a closely approach to the desired path, and with that is possibly the imposition of behaviours more specific, only defined by polynomial functions.

6. References


www.intechopen.com


This book brings together some of the latest research in robot applications, control, modeling, sensors and algorithms. Consisting of three main sections, the first section of the book has a focus on robotic surgery, rehabilitation, self-assembly, while the second section offers an insight into the area of control with discussions on exoskeleton control and robot learning among others. The third section is on vision and ultrasonic sensors which is followed by a series of chapters which include a focus on the programming of intelligent service robots and systems adaptations.

How to reference
In order to correctly reference this scholarly work, feel free to copy and paste the following:
