Hybrid ARQ Utilizing Lower Rate Retransmission over MIMO Wireless Systems

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1. Introduction

Hybrid automatic repeat request (Hybrid ARQ or HARQ), an extension of ARQ that incorporates forward error correction coding (FEC), is a retransmission scheme with error-control method employed in current communications systems. In standard ARQ, redundant bits are added to data to be transmitted using an error-detecting code, e.g., cyclic redundancy check (CRC). The contribution of HARQ is its efficient utilization of the available resources and the provision of reliable services in latest-generation systems.

This chapter focuses on wireless systems using HARQ with emphasis on the multiple-input multiple-output (MIMO) paradigm. In this chapter, the architecture of MIMO transceivers that are based on bit-interleaved coded modulation (BICM) and it employs HARQ is described. MIMO system is an attractive technique that can enhance the spectral efficiency through spatial multiplexing (SM) [Foschini 1995]. However, many wireless environments may suffer from ill-conditioned channels or multipath fadings, which degrade the system performance. The aim of this chapter is to find an efficient MIMO scheme for retransmission to combat the ill-ranked channel and enhancing the reliability.

In IEEE 802.16e [WiMAX 2007] standard, Space Time Coding (STC) subpacket combining, which retransmits with a different MIMO format, has been introduced. One possible way to combine the initial signal and the retransmitted signal is the utilization of Maximum Ratio Combining (MRC) in symbol level based on Virtual Space Time Block Coding (VSTBC) [Gao et. al , 2007]. However, such combining technique works properly only if the channel is quasi-static. In circumstances with high mobility, this technique does not provide satisfactory performance. Another approach is to combine the retransmitted and initial signals using symbol level combining (SLC) before the detector. However, the required buffer in SLC occupies large memory in receivers. Moreover, both SLC receiver architectures are only applicable when the initial and retransmitted symbols are aligned. Thus, they may be impractical for Incremental Redundancy (IR) HARQ [Lin et al. ,1984] and Constellation Rearrangement (CoRe). Although bit level combining (BLC) can solve the problem of huge buffer requirement, using ML-like MIMO detectors, e.g., List Sphere Decoder (LSD) [Damen et al. ,2003], can still increase the complexity significantly.
To reduce buffer size and power consumption, a lower rate MIMO mode in retransmission request, termed as Lower Rate Retransmission (LRR) Scheme [Chen et. al, 2009], is proposed. In this chapter, we define the rate as how much information can be transmitted in single time-frequency resource unit. Two examples of LRR schemes are introduced. For the first scheme, it applies rate-2 SM for initial transmission, and rate-1 STBC [Alamouti, 1998] or Space Frequency Block Coding (SFBC) [Kaiser, 2003] is used for retransmission. For the second scheme, rate-3 or rate-4 SM are leveraged for initial transmission, and a lower rate SM scheme, i.e., rate-2 or rate-3, is employed for retransmission. In order not to decrease the spectral efficiency, only partial coded bits are retransmitted in the proposed schemes.

In LRR, with fewer transmit antennae in retransmission, the acquired transmit power gain could be used to retransmit higher modulation symbols and keep the total retransmitted bits as close to initial transmission as possible. We tabulated this scheme as Lower Rate Retransmission combine with modulation step up (LRRMSU).

The notations of this paper are explained as following: Superscripts $T$ and $H$ indicate matrix transpose and hermitian, respectively. Superscripts $*$ indicates complex conjugate operation. Uppercase boldface denotes a matrix while lowercase boldface denotes a vector. $I_N$ denotes the $N \times N$ Identity matrix. $[A]_{i,j}$ and $[A]_{j,i}$ represent the element of $i$th row and $j$th column of matrix $A$, and $j$th column of matrix $A$, respectively. A circularly symmetric complex Gaussian vector $a$ with mean $m$ and covariance matrix $R$ is denoted as $a \sim NC [m, R]$. Finally, $nTX, M_t$ and $M_r$ refer to the n-th transmission, number of transmit antenna and receive antenna, respectively.

This article is organized as follows. In the next section the architecture of a single-input single-output (SISO) transceiver using BICM and HARQ is presented. The following section narrates conventional MIMO HARQ schemes. The LRR schemes are then elaborated with simpler receiver implementations. The following section contains some discussion of MIMO system design based on the employed HARQ scheme, receiver complexity, and storage requirements. To keep the number of retransmission bits, near to that of the initial transmission a LRRMSU scheme is illustrated to enhance the system throughput. Finally, some concluding remarks are provided.

2. MIMO HARQ scheme in 802.16

Figure 1 shows the block diagram of MIMO HARQ in 802.16e transmitter. It is illustrated that $k$ information bits $b, \{b \in b^{k \times 1} | b \in \{0,1\}\}$ are fit into one forward error control coding (FEC) block. These information bits are regarded as systematic bits $AB$. The mother code rate $R_{MC}$ of the Convolutional Turbo Code (CTC) in 802.16e is $\frac{1}{3}$. After CTC encoding, the encoded bit length is multiplied by 3, and the output $c, \{c \in b^{3k \times 1} | b \in \{0,1\}\}$, is consists of systematic bits $AB$, parity bits $Y_1Y_2$ and $W_1W_2$. In this example, the code rate $R$ is $\frac{1}{2}$, hence in the initial transmission, $W_1W_2$ are punctured in the bit selection block, and the remained encoded bits are $c_s, \{c_s \in b^{2k \times 1} | b \in \{0,1\}\}$. The bit selection procedure depends on
IR or Chase Combining (CC) HARQ [Chase, 1985] as well as the times of reception of NACK message.

Figure 2 gives an example of transmission with CC and IR modes. In CC mode, the same encoded bits are retransmitted after the request. For IR mode, on the other hand, additional parity check bits are retransmitted. Then, the selected bits, $s$, are modulated into $M$-ary QAM symbols,

$$s \in \mathbb{C}^ {2k \times 1}, \mathbb{C} \in \text{complex},$$

where $M = \log_2 M$.

It is then followed by a MIMO encoder. The encoder can be either in SM or spatial diversity (S-Div) mode. In SM mode, the MIMO encoder is simply a parser. In S-Div mode, it is Space Time Code (STC) or Space Frequency Code (SFC) encoder. The number of streams of SM and S-Div mode are denoted by $S_M$ and $S_D$, respectively. Note that a stream is defined as each output of the MIMO encoder. The value of $S_M$ is equivalent to the rate in SM mode. Moreover, in Figure 1, S-Div mode represents rate-1 Alamouti code. Finally, the output of the MIMO encoder is

$$x \in \mathbb{C}^{2k \times S_M S_D}.$$ 

The signal of ith MIMO encoder output $x_i = [x]_i$ is STC subpacket encoded based on Table 1, and STC subpacket encoding refers to the VSTBC of previous and current transmitted subpacket, as in 802.16e.
3. Lower Rate Retransmission (LRR) scheme

In practice, due to the propagation mechanisms, MIMO system may be suffered from high spatial correlation, which degrades the system capacity. If there is no feedback information regarding the channel rank, it is always a good approach to retransmit with a lower rate MIMO mode to provide a robust transmission. For instance, a rate-1 SFBC or STBC is recommended for retransmission for initial transmission in rate-2 SM mode. For rate-3 and rate-4 initial transmission in SM mode, on the other hand, rate-2 and rate-3 SM mode are recommended for retransmission. A list of possible MIMO mode selection for LRR is shown

Table 1. STC subpacket encoding
in Table 2. Since an open loop system is being considered, thus the transmitter does not possess channel state information (CSI). Note that LRR can be leveraged in association with a stream-to-antenna mapping technique such as precoding or antenna selection.

<table>
<thead>
<tr>
<th>Antenna</th>
<th>reTX</th>
<th>Initial TX=1TX</th>
<th>Request TX</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_t = 2$</td>
<td>rate-2 SM</td>
<td>rate-1 SFBC/STBC</td>
<td></td>
</tr>
<tr>
<td>$M_t = 3$</td>
<td>rate-3 SM</td>
<td>rate-2 SM</td>
<td></td>
</tr>
<tr>
<td>$M_t = 4$</td>
<td>rate-4 SM</td>
<td>rate-3 SM</td>
<td></td>
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</table>

Table 2. MIMO mode selection in LRR

In order to maintain the same spectral efficiency as the conventional HARQ schemes, fewer bits are encoded by LRR MIMO encoder. Although the number of retransmitted bit is reduced, the reliability is improved. Since only a portion of bits is retransmitted, the bit selection should be modified. An example of retransmissions with coding rate $R = \frac{1}{2}$ in CC and IR modes are shown as Figure 3.

![CC mode](image1.png) ![IR mode](image2.png)

Fig. 3. Different Bit selection methods for CC (left) and IR (right) with LRR scheme.

4. Receiver architectures

In this section, four types of MIMO receivers are elaborated. Firstly, we illustrate the combining methods for both VSTBC and STBC/SFBC in symbol and bit levels. Then, two SM detection algorithms in BLC are described: Soft Linear Minimum Mean Square Error (LMMSE)[Lee & Sundberg, 2007] algorithm and LSD algorithm. All algorithms use soft decision information generated from CSI. Finally, a complexity analysis is carried out in a rate-2 MIMO HARQ scheme.
4.1 VSTBC-MRC with SLC and STBC/SFBC-MRC with BLC

A flat-fading MIMO system can be expressed as:

\[ y = HPx + n = Hx + n, \]  

where \( H \) is the \( M_r \times M_t \) MIMO channel matrix of an OFDM subcarrier with unitary power complex Gaussian elements, \( x \) is the \( S_M \times 1 \) transmitted signal vector with unit total transmission power per subcarrier, and \( y \) is the \( M_r \times 1 \) received signal vector. Moreover, \( P \) is the \( M_t \times S_M \) precoding matrix, and \( n \sim N_0 \left[ 0, N_0 I_{M_r} \right] \) is the \( M_r \times 1 \) noise vector with complex Gaussian elements. Without loss of generality, a MIMO scenario with \( M_t = M_r = 2 \) and identity matrix \( P \) is described.

The signal in the odd transmission (as defined in Table 1) is:

\[ x_{\text{odd}} = \begin{bmatrix} x_0 \\ x_1 \end{bmatrix} \] (2)

And the signal in the even transmission is:

\[ x_{\text{even}} = \begin{bmatrix} -x_1^* \\ x_0^* \end{bmatrix} \] (3)

Here, subscripts \( \text{odd} \) and \( \text{even} \) indicate different subpacket information at odd and even retransmission. Moreover, the initial transmission is started with the first odd transmission. Hence, a pair of received signal in subpackets can be formed as:

\[ y_{\text{odd}} = H_{\text{odd}} x_{\text{odd}} + n_{\text{odd}} \] (4)

\[ y_{\text{even}} = (pH_{\text{odd}} + \Delta H) x_{\text{even}} + n_{\text{even}} \] (5)

Here, we assume \( H_{\text{even}} = pH_{\text{odd}} + \Delta H \) and \( \Delta H \sim N_0 \left[ 0, (1 - p^2) I_{M_r} \right] \) with \( 0 \leq p \leq 1 \) is induced from Doppler effect. Equation 4 and 5 can be further stacked as:

\[ \begin{bmatrix} y_{\text{odd}} \\ y_{\text{even}}^* \end{bmatrix} = \overline{H} x_{\text{odd}} + \begin{bmatrix} n_{\text{odd}} \\ n_{\text{even}}^* \end{bmatrix} \] (6)

Here,

\[ \overline{H} = \begin{bmatrix} [H_{\text{odd}}]_{1,1} & [H_{\text{odd}}]_{1,2} \\ [H_{\text{even}}]_{1,2} & -[H_{\text{even}}]_{1,1} \end{bmatrix} \]

and the combined signal vector by MRC is shown in eq. 7.

The MRC for signal operates as following:
Where $\hat{h}_0 s_0$ and $\hat{h}_1 s_1$ are the soft detection symbols of $s_0$ and $s_1$, respectively. The equivalent channel gain $\hat{h}_0$ and $\hat{h}_1$ can also be obtained by MRC in eq. 8.

The MRC for channel operates as following:

$$\begin{bmatrix} \hat{h}_0 \\ \hat{h}_1 \end{bmatrix} = \text{diag}(H^H H)$$

(8)

In equation 7, if the channel is not static for each retransmission, the orthogonality of Alamouti Code is destroyed and the interference is thereby induced. The mathematical description is as follows:

$$\hat{h}_0 s_0 + \left\{(1-\rho^2)(h_{00}^{\text{odd}} h_{11}^{\text{odd}} + h_{10}^{\text{odd}} h_{01}^{\text{odd}}) - \rho(h_{00}^{\text{odd}} \Delta h_{01}^{\text{odd}} + h_{10}^{\text{odd}} \Delta h_{10}^{\text{odd}} + h_{01}^{\text{odd}} \Delta h_{11}^{\text{odd}} + h_{11}^{\text{odd}} \Delta h_{11}^{\text{odd}}) - \Delta h_{00}^{\text{odd}} \Delta h_{01}^{\text{odd}} + \Delta h_{10}^{\text{odd}} \Delta h_{11}^{\text{odd}} \right\} s_1 + \hat{h}_1 h_{0}^{\text{even}} + h_{0}^{\text{even}} h_{0}^{\text{even}} + h_{1}^{\text{even}} h_{1}^{\text{even}} + h_{1}^{\text{even}} h_{1}^{\text{even}}$$

(9)

where the second term is the interference induced from signal $s_1$. The equivalent SNR of signal $s_0$ is:

$$\text{SNR} = \frac{8}{\left(1-\rho^2\right)^2 \gamma + 2\rho^2 \beta + \alpha + 4N_o}$$

(10)

with the assumptions of cross correlation

$$E\left[|h_{ij}^{\text{odd}} h_{mn}^{\text{odd}}|^2\right] = \gamma, \quad E\left[|h_{ij}^{\text{odd}} \Delta h_{mn}|^2\right] = \beta \quad \text{and} \quad E\left[|\Delta h_{ij}^{\text{odd}} \Delta h_{mn}|^2\right] = \alpha, \quad ij \neq mn.$$
For the proposed LRR scheme with rate-2 SM initial transmission, the scheme is simply tantamount to a SFBC after rate reduction, and it can be detected by MRC method and then combined with previous LLRs in BLC. The SFBC formats of the retransmitted subpacket are expressed as:

\[
\mathbf{x}_{\text{sub}_0} = \begin{bmatrix} x_0 \\ x_1 \end{bmatrix}
\]

and

\[
\mathbf{x}_{\text{sub}_1} = \begin{bmatrix} -x_1^* \\ x_0^* \end{bmatrix}
\]

where subscripts \( \text{sub}_0 \) and \( \text{sub}_1 \) indicate the subcarrier indices 0 and 1, respectively. The detection is performed for every two consecutive subcarriers and the former detected LLRs is stored in BLB, hence SLB is not required. The overall receiver architecture is shown in Figure 5.

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**Fig. 4. Subpacket Combining Process**

**Fig. 5. SFBC MRC with BLC**
4.2 Soft LMMSE with BLC

In BLC, there is no restrictions on symbol-alignment for the signals of odd and even retransmissions. Based on the system model in equation 1, and signal vector $x$ here is the same as $x_{\text{odd}}$, the detection signal can be expressed as in 13. The LMMSE for signal operates as:

$$\hat{x} = (H^H H + S,M,N_o I)^{-1} H^H y$$

Eq. (13)

The equivalent channel gain is the inverse of diagonal terms of MSE matrix. The LMMSE for channel gain operates as:

$$\begin{bmatrix} \frac{1}{\hat{h}_0} \\ \frac{1}{\hat{h}_1} \end{bmatrix} = \text{diag}\left(\left(H^H H + S,M,N,I\right)^{-1}\right),$$

Eq. (14)

where $\hat{h}_0$ and $\hat{h}_1$ are the equivalent soft CSI gains and the overall scheme is shown in Figure 6.

Fig. 6. Soft LMMSE processing with BLC

4.3 LSD with BLC

There are many simplified LLR algorithms for soft values computation for a SM system in the existing literature. Here, we focus on an exhaustive search which results in no penalty in performance. The LLR of the $k$th bit on the transmitted symbol vector $x$ (which contains $S_M \times M$ bits) is:

$$LLR_k \approx \frac{1}{M_i N_o} \left( - \min_{k \neq k+1} \|y - H x_{k,1}\|^2 + \min_{k \neq k-1} \|y - H x_{k,0}\|^2 \right).$$

Eq. (15)
where $x_{k,1}$ and $x_{k,0}$ are the symbol vectors with $k$th bit equals +1 and -1, respectively. The block diagram of LSD with BLC is shown in Figure 7:

![Figure 7. LSD processing with BLC](image)

### 4.4 Complexity analysis

An example based on 802.16e Partial Usage of Subchannels (PUSC) format is given here to analyze the complexity of different schemes under our consideration. The resource unit is constructed with one slot with 48 data subcarriers. For sake of simplicity, only the number of complex multiplication operations (CMLs) is considered. A real division operation is assumed to be equivalent to one CML. Besides, the complexity of LSD with BLC is not examined, because its complexity depends on the modulation order and it is undoubtable to be more complicated than the other detectors. The complexity of a MIMO receiver with $M_t = M_r = 2$ is shown as follows.

1. **VSTBC-MRC with SLC:**

   In this case, there are 48 subcarriers in total to be implemented with the MRC operation, and each of them takes eight CMLs in MRC for signal and eight CMLs (only diagonal elements are needed) in MRC for channel. Hence, it takes 768 CMLs in total.

2. **STBC/SFBC-MRC with BLC:**

   Every 2-subcarrier pair is to be implemented by one MRC operation in this case, hence the number of required operations is half of VSTBC-MRC with SLC. Thus, the total number of required operations is 384 CMLs.

3. **Soft LMMSE with BLC:**

   In this scenario, we should consider 48 subcarriers with $2 \times 2$ matrix operations in MMSE filtering. Each subcarrier requires eight CMLs to compute $\mathbf{H}^H \mathbf{H}$, eight CMLs for matrix inversion (two CMLs for the determinant and six CMLs for the real divisions), the
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Multiplication of \( \left( H^H H + S_M N e I \right)^{-1} \) with \( H^H \) takes eight CMLs, and finally, \( \left( H^H H + S_M N e I \right)^{-1} H^H y \), needs further four CMLs. Hence, there are 1344 CMLs in total.

To recapitulate, our proposed scheme reduces the complexity by about 50% and 70% as compared to VSTBC-MRC with SLC and soft LMMSE with BLC, respectively.

5. Lower Rate Retransmission (LRR) combined with Modulation Step Up (LRRMSU) scheme

There are several pros of the LRR scheme such as 1. a robust retransmission, because the inter-stream interference is reduced, 2. the transmitter side acquires additional transmit power gain for each retransmitted stream, 3. frequency and spatial diversity is gained, because different resource allocation will be automatically guaranteed. However, one possible cons, the total retransmitted bits will be reduced as compare to number of bits in initial transmission, might degrade its performance in high coding rate scenarios. To overcome this deficiency, higher order modulation or called modulation step up is introduced and combined with the LRR scheme in Figure 8. In the initial transmission, the transmission mode operates with 4 transmission antennae with QPSK in each stream. In retransmission, the number of transmission antennae is reduced to 3 but the modulation order is step up to 16 QAM. Therefore, we keep the number of retransmission bits very close to that of traditional scheme.

Fig. 8. LRRMSU retransmission scheme

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6. Numerical results

In order to verify the superiority of the proposed scheme, the simulation based on a low correlation MIMO model [WiMAX, 2007] with \( M_t = M_r = 2 \) is undertaken here. In particular, we show two examples of comparison in this paper: \textit{VSTBC in SLC v.s. SFBC in BLC} and \textit{SFBC in BLC v.s. SM in BLC}. The delay profile of each path is evaluated under ITU-R [ITU-R, 2000] Pedestrian Type-B 3km/hr (PB3) or Vehicular Type-A 60km/hr (VA60). Furthermore, PUSC with 10Mhz bandwidth is assumed and the coding scheme is based on 802.16e CTC. In addition, we postulate that the receiver has perfect CSI. The HARQ round trip interval is 10ms, and the subpacket will be shifted by 3 subchannel length to gain higher diversity in frequency domain. The subpacket size for each coding rate is summarized in Table 3, where \( N_{EP} \) is the number of information bits before feeding into FEC encoder. Note that we concentrate on CC mode and the comparison with IR mode is beyond our scope due to page restrictions.

<table>
<thead>
<tr>
<th>modulation coding scheme</th>
<th>( N_{EP} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>QPSK 1/2</td>
<td>480</td>
</tr>
<tr>
<td>QPSK 3/4</td>
<td>432</td>
</tr>
<tr>
<td>16QAM 1/2</td>
<td>480</td>
</tr>
<tr>
<td>16QAM 3/4</td>
<td>432</td>
</tr>
<tr>
<td>64QAM 1/2</td>
<td>432</td>
</tr>
<tr>
<td>64QAM 2/3</td>
<td>384</td>
</tr>
<tr>
<td>64QAM 3/4</td>
<td>432</td>
</tr>
<tr>
<td>64QAM 5/6</td>
<td>480</td>
</tr>
</tbody>
</table>

Table 3. \( N_{EP} \) size of different code rate with CC

**Example 1: VSTBC in SLC v.s. SFBC in BLC.**

We focus on simulation results of PB3 and VA60. In 1TX, the receiver is a soft MMSE detector for both cases, and in 2TX the VSTBC-MRC or SFBC-MRC is used. Simulation results of 2TX packet error rate (PER) in Figure 9 and 10 show that our proposed scheme has poorer error performance in low mobility. However, the interference terms in equation 9 will impact the performance of VSTBC in SLC scheme as the mobility increases. Hence, the proposed scheme becomes superior in moderate and higher speed scenarios. Nevertheless, a Doppler estimator is generally not available at the receiver; hence conventional 802.16e cannot be guaranteed to be superior at all mobility levels. The PER in 3TX is not evaluated, because the retransmitted subpacket cannot be combined with previous ones in BLC, in 3TX of a 802.16e MIMO-HARQ scheme.
Fig. 9. VSTBC SLC v.s. SFBC BLC in 2TX PB3 CC

Fig. 10. VSTBC SLC v.s. SFBC BLC in 2TX VA60 CC
Example 2: SFBC in BLC v.s. SM in BLC.

1TX, the SM detection is the same for soft MMSE/LSD for 802.16e and the proposed scheme. For later retransmissions than 1TX, the detection method is soft MMSE/LSD in 802.16e and SFBC-MRC in the proposed scheme. The maximum number of retransmission assumed in our simulation is 3. We first show that the PER results in 11 and 12, and it then follows by

Fig. 11. Soft MMSE BLC v.s. SFBC MRC BLC in 3TX PB3 CC

Fig. 12. LSD BLC v.s. SFBC MRC BLC in 3TX PB3 CC
throughput comparison in 13 and 14. Thus, when the channel model is PB3 with low spatial correlation, it is shown that the proposed scheme always outperforms the ones in 802.16e, especially in higher coding rate scenarios. In terms of throughput, the proposed scheme can achieve higher throughput in low signal to noise ratio (SNR) region with the same coding rate. Nonetheless, the throughput curves are similar in high SNR region. The results also show that the proposed scheme is less sensitive to improper link adaptation.

Fig. 13. Soft MMSE BLC v.s. SFBC MRC BLC in PB3 CC

Fig. 14. LSD BLC v.s. SFBC MRC BLC in PB3 CC
7. Conclusions

In this chapter, we proposed a novel scheme to gain better performance, which reduces the receiver complexity (by 50% ~ 70% in 2 x 2 MIMO scenario) as well as the buffer requirement. From the simulation, it is observable that the proposed scheme can achieve better performance than conventional 802.16e schemes in scenarios with moderate or high mobility. In another set of simulations, we have compared the proposed scheme with the BLC in 802.16e architecture. The results have verified that our scheme is less sensitive to inappropriate link adaptation.

8. References


This book has been prepared to present the state of the art on WiMAX Technology. The focus of the book is the physical layer, and it collects the contributions of many important researchers around the world. So many different works on WiMAX show the great worldwide importance of WiMAX as a wireless broadband access technology. This book is intended for readers interested in the transmission process under WiMAX. All chapters include both theoretical and technical information, which provides an in-depth review of the most recent advances in the field, for engineers and researchers, and other readers interested in WiMAX.

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