1. Introduction

The efficiency of planetary gearboxes mainly depends on the tooth- and bearing friction losses. This work shows the new mathematical model and the results of the calculations to compare the tooth and the bearing friction losses in order to determine the efficiency of different types of planetary gears and evaluate the influence of the construction on the bearing friction losses and through it on the efficiency of planetary gears. In order to economy of energy transportation it is very important to find the best gearbox construction for a given application and to reach the highest efficiency.

In transmission system of gas turbine powered ships, power stations, wind turbines or other large machines in industry heavy-duty gearboxes are used with high gear ratio, efficiency of which is one of the most important issues. During the design of such equipment the main goal is to find the best constructions fitting to the requirements of the given application and to reduce the friction losses generated in the gearboxes. These heavy-duty tooth gearboxes are often planetary gears being able to meet the following requirements declared against the drive systems:

- High specific load carrying capacity
- High gear ratio
- Small size
- Small mass/power ratio in some application
- High efficiency.

There are some types of planetary gears which ensure high gear ratio, while their power flow is unbeneficial, because a large part of the rolling power (the idle power) circulate inside the planetary gearbox decreasing the efficiency. In the simple planetary gears there is no idle power circulation. Therefore heavy-duty planetary drives are set together of simple planetary gears in order to transmit megawatts or even more power, while they must be compact and efficient.

2. Planetary gearbox types

The two- and three-stage planetary gears consisting of simple planetary gears are able to meet the requirements mentioned above [Fig. 1(a)–1(d)].
Varying the inner gear ratio (the ratio of tooth number of the ring gear and of the sun gear) of each simple planetary gear stage KB the performance of the whole combined planetary gear can be changed and tailored to the requirements.

There are special types of combined planetary gears containing simple KB units (differential planetary gears), which can divide the applied power between the planetary stages thereby increasing the specific load carrying capacity and efficiency of the whole planetary drives [Fig. 1(b)-1.(d)]. Proper connections between the elements of the stages in these differential planetary gears do not result idle power circulation.

![Fig. 1. (a) Gearbox KB+KB; (b). Planetary gear PKG; (c). Planetary gear PV; (d). Planetary gear GPV](image)

The efficiency of planetary gears depends on the various sources of friction losses developed in the gearboxes. The main source of energy loss is the tooth friction of meshing gears, which mainly depends on the arrangements of the gears and the power flow inside the planetary gear drives. The tooth friction loss is influenced by the applied load, the entraining speed and the geometry of gears, the roughness of mating tooth surfaces and the viscosity of lubricant. Designers of planetary gear drives can modify the geometry of tooth profile in order to lower the tooth friction loss and to reach a higher efficiency [Csobán, 2009].
3. Friction loss model of roller bearings

It is important to find the parameters (such as inner gear ratio, optimal power flow) of a compound planetary gear drive which result its highest performance for a given application. The power flow and the power distribution between the stages of a compound gearbox is also a function of the power losses generated mainly by the friction of mashing teeth and the bearing friction.

This is why it is beneficial, when, during the design of a planetary gear beside the tooth friction loss also the friction of rolling bearings is taken into consideration even in the early stage of design. In this work a new method is suggested for calculate the rolling bearing friction losses without knowing the exact sizes and types of the bearings.

In this model first the torque and applied loads (loading forces and, if possible, bending moments) originated from the tooth forces between the mating teeth have to be determined. Thereafter the average diameter of the bearing \(d_m\) can be calculated as a function of the applied load and the prescribed bearing lifetime. Knowing the average diameter \(d_m\), the friction loss of bearings can be counted using the methods suggested by the bearing manufacturers based on the Palmgren model [SKF 1989].

For determining the functions between the bearing average diameter and between the basic dynamic, static load, inner and outer diameter [Fig. 2-6.], the data were collected from SKF catalog [SKF 2005].

The functions between the bearing parameters (inner diameter \(d_b\), dynamic basic load \(C\)) and the average diameters \(d_m\) being necessary for calculation of the friction moment and the load can be searched in the following form:

\[
Y = \bar{c} \cdot \bar{d}_m^3
\]

(1)

The equations of the diagrams [Fig. 2-6.] give the values of \(c\) and \(d\) for the inner diameter of the bearings \(d_b\) and for the basic dynamic loads \(C\) of the bearings.

Knowing the torque \(M_{24}\) and the strength of the materials of the shafts (\(\tau_m\) \(\sigma_m\)) the mean diameter of the bearing for central gears (sun gear, ring gear) necessary to carry the load can be calculated using the following formula:

\[
d_{m,2,4}(d) = \bar{d} \sqrt[3]{\frac{16 \cdot M_{2,4}}{\tau_m \cdot \pi}}
\]

(2)

Calculating the tangential components of the tooth forces the applied radial loads of the planet gear shafts \(F_r\) can be determined (which are the resultant forces of the two tangential components \(F_{t2}\) and \(F_{t4}\)). The shafts of the planet gears are sheared and bended by the heavy radial forces, this is why, in this analysis, at the calculation of shaft diameter, once the shear stresses, then the bending stresses are considered.

Calculating the maximal bending moment \(M_{liba}\) of the planet gear shafts, and the allowable equivalent stress \(\sigma_m\) of planet gear pins, the bearing inner diameter \(d_b\) necessary to carry the applied load of the planet gear shaft and the average bearing diameter \(d_{m3}\) can be calculated:

\[
d_{m3}(d) = \bar{d} \sqrt[3]{\frac{32 \cdot M_{liba}}{\sigma_m \cdot \pi}}
\]

(3)
The functions between the bearing geometry and load carrying capacity for deep groove ball bearings [Fig. 2(a)-2(d)]. The points are the average data of the bearings taken from SKF Catalog [SKF 2005] and the continuous lines are the developed functions between the parameters.

\[
y = 0.3984x^{1.1179}, \quad R^2 = 0.9996
\]

\[
y = 1.7374x^{0.9364}, \quad R^2 = 0.9998
\]

\[
y = 109.09x^{1.3236}, \quad R^2 = 0.9882
\]

\[
y = 12.603x^{1.7564}, \quad R^2 = 0.9912
\]

Fig. 2. (a) The average inner diameter of the deep groove ball bearing as a function of its average diameter. (b). The average outer diameter of deep groove ball bearing as a function of the average diameter. (c). The average basic dynamic load of deep groove ball bearing as a function of the average diameter. (d). The average static load of deep groove ball bearing as a function of the average diameter.
The functions between the bearing geometry and load carrying capacity for cylindrical roller bearings [Fig. 3(a)-3(d)].

![Graphs showing relationships between bearing parameters and load carrying capacity.](image-url)

**Fig. 3.** (a) The average inner diameter of the cylindrical roller bearing as a function of its average diameter. (b) The average outer diameter of cylindrical roller bearing as a function of the average diameter. (c) The average basic dynamic load of the cylindrical roller bearing as a function of its average diameter. (d) The average static load of different types of cylindrical roller bearing as a function of the average diameter.
The functions between the bearing geometry and load carrying capacity for full complement cylindrical roller bearings [Fig. 4(a)-4(d)].

Fig. 4. (a) The average inner diameter of the full complement cylindrical roller bearing as a function of its average diameter. (b). The average outer diameter of the full complement cylindrical roller bearing as a function of the average diameter. (c). The average basic dynamic load of the full complement cylindrical roller bearing as a function of its average diameter. (d). The average static load of different types of full complement cylindrical roller bearing as a function of the average diameter.
The functions between the bearing geometry and load carrying capacity for spherical roller bearings [Fig. 5(a)-5(d)].

Fig. 5. (a) The average inner diameter of the spherical roller bearing as a function of its average diameter. (b) The average outer diameter of spherical roller bearing as a function of the average diameter. (c) The average basic dynamic load of spherical roller bearing as a function of its average diameter. (d) The average static load of spherical roller bearing as a function of its average diameter.
The functions between the bearing geometry and load carrying capacity for CARB toroidal roller bearings [Fig. 6(a)-6(d)].

Fig. 6. (a) The average inner diameter of CARB toroidal roller bearing as a function of its average diameter. (b). The average outer diameter of CARB toroidal roller bearing as a function of the average diameter. (c). The average basic dynamic load of CARB toroidal roller bearing as a function of its average diameter. (d). The static load of CARB toroidal roller bearing as a function of the average diameter.
The $V$ shear load of the planet gear shaft is equal with the applied load $F_r$ divided by the number of sheared areas $A$ of the shaft. Knowing the $V$ shear load and the allowable equivalent stress $\tau_m$ of planet gear pins, the bearing inner diameter $d_b$ necessary to carry the applied load of the planet gear shaft and the average bearing diameter $d_{m3}$ can be calculated:

$$d_{m3}(d) = \sqrt[3]{\frac{16 \cdot V}{3 \cdot \tau_m \cdot \pi}}$$  \hspace{1cm} (4)

The average diameters of bearings necessary to reach the prescribed lifetime $L_{2h}$ was determined using the SKF modified lifetime equation [SKF 2005] ($C$ is the basic dynamic load, $F_r$ is the radial bearing load and $a_1$ is the bearing life correction factor) as follows:

$$d_m(L_{2h}) = \sqrt[6]{\frac{L_{2h} \cdot 60 \cdot n}{10^6 \cdot a_1 \cdot F_r}}$$  \hspace{1cm} (5)

From the two calculated average diameters of bearings ($d_m(d)$ and $d_m(L_{2h})$) the larger ones have to be chosen. This biggest average diameter can be called resultant average (ball or roller) bearing diameter ($d_{m\text{res}}$).

$$d_{m\text{res}} = \left[ d_m(d) + \frac{1}{2} \left( \left( d_m(L_{2h}) - d_m(d) \right) + \left( d_m(L_{2h}) - d_m(d) \right) \right) \right]$$  \hspace{1cm} (6)

### 3.1 Calculating the friction losses and efficiency of roller bearings

The sun gears and the ring gears are well balanced by radial components of tooth forces; the friction losses of their bearings are not depending on the applied load. The energy losses of these bearings are determined by the entraining speed of the bearings, the viscosity of lubricant and the bearing sizes.

The calculation of the component of friction torque $M_0$ being independent of the bearing load can be performed using the following equations [SKF 1989].

When

$$\nu \cdot n \geq 2000$$

$$M_0 = 10^{-7} \cdot f_0 \cdot (\nu \cdot n)^{2/3} \cdot d_m^3$$  \hspace{1cm} (7)

and when

$$\nu \cdot n < 2000$$

$$M_0 = 160 \cdot 10^{-7} \cdot f_0 \cdot d_m^3$$  \hspace{1cm} (8)

At bearings of planet gears the component of friction torques $M_1$ depending on the bearing loads was calculated using the following simple equation [SKF 1989]:

$$M_1 = f_1 \cdot P_1^n \cdot d_m^n$$  \hspace{1cm} (9)

Using the average bearing diameters the friction torques of the bearings can be determined:

$$M_0 \left( d_{m\text{res}} \right) = M_0 \left( d_{m\text{res}} \right) + M_1 \left( d_{m\text{res}} \right) + ...$$  \hspace{1cm} (10)
Knowing the friction torques of the sun gear its bearing efficiency can be calculated using the following equation:

\[ \eta_{2}\text{Bearing} = \frac{M_2 - M_2(d_{m_2})}{M_2 \cdot \omega_2} \cdot \omega_2 = 1 - \frac{M_2(d_{m_2})}{M_2} \] (11)

The bearing efficiency of planet gears can be determined with the following equation:

\[ \eta_{3}\text{Bearing} = \frac{M_3 - M_3(d_{m_3})}{M_3 \cdot \omega_3} \cdot \omega_3 = 1 - \frac{M_3(d_{m_3})}{M_3} \] (12)

The power loss generated only by the bearings in the gearbox can be calculated as (the rolling efficiency of a simple stage and the gearbox efficiency is a function of only the bearing efficiencies):

\[ \eta_{g} = \eta_{2}\text{Bearing} \cdot \eta_{3}\text{Bearing} \rightarrow \eta_{\text{gearboxBearing}} \]

\[ v_{\text{Bearing}} = P_{in} \cdot (1 - \eta_{\text{gearboxBearing}}) \] (13)

The power loss generated by the tooth friction can be calculated with the following equations (the rolling efficiency of a simple stage and the gearbox efficiency is a function of only the tooth efficiencies):

\[ \eta_{g} = \eta_{23} \cdot \eta_{34} \rightarrow \eta_{\text{gearboxTooth}} \]

\[ v_{\text{Tooth}} = P_{in} \cdot (1 - \eta_{\text{gearboxTooth}}) \] . (14)

The rolling efficiency of a simple planetary gear stage can be calculated as:

\[ \eta_{g} = \eta_{23} \cdot \eta_{34} \cdot \eta_{2}\text{Bearing} \cdot \eta_{3}\text{Bearing} \rightarrow \eta_{\text{gearbox}} \] (15)

The total power loss generated in the planetary gear drive as a function of the gearbox efficiency:

\[ \Sigma v = P_{in} \cdot (1 - \eta_{\text{gearbox}}) \] (16)

The power loss ratios show the dominant power loss component. The tooth power loss ratio is the tooth power loss component divided by the total power loss:

\[ \frac{v_{\text{Tooth}}}{\Sigma v} \] (17)

The bearing loss ratio is the power loss generated by the bearing friction divided by the total power loss:

\[ \frac{v_{\text{Bearing}}}{\Sigma v} \] (18)

The bearing selecting and efficiency calculation algorithm can be seen in figure 10.
The Bearing Friction of Compound Planetary Gears in the Early Stage Design for Cost Saving and Efficiency

<table>
<thead>
<tr>
<th>Bearing Types/function</th>
<th>( \dot{c} )</th>
<th>( \ddot{d} )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Deep groove ball bearing</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( d - d_m )</td>
<td>0,3984</td>
<td>1,1179</td>
</tr>
<tr>
<td>( D - d_m )</td>
<td>1,7374</td>
<td>0,9364</td>
</tr>
<tr>
<td>( C - d_m )</td>
<td>109,09</td>
<td>1,3236</td>
</tr>
<tr>
<td>( C_0 - d_m )</td>
<td>12,603</td>
<td>1,7564</td>
</tr>
<tr>
<td><strong>Cylindrical roller bearings</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( d - d_m )</td>
<td>0,4167</td>
<td>1,0966</td>
</tr>
<tr>
<td>( D - d_m )</td>
<td>1,7114</td>
<td>0,9481</td>
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<tr>
<td>( C - d_m )</td>
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<td>1,6675</td>
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<tr>
<td>( C_0 - d_m )</td>
<td>22,552</td>
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<td><strong>Full complement cylindrical roller bearings</strong></td>
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</tr>
<tr>
<td>( d - d_m )</td>
<td>0,4595</td>
<td>1,0951</td>
</tr>
<tr>
<td>( D - d_m )</td>
<td>1,6961</td>
<td>0,9399</td>
</tr>
<tr>
<td>( C - d_m )</td>
<td>444,04</td>
<td>1,3465</td>
</tr>
<tr>
<td>( C_0 - d_m )</td>
<td>164,13</td>
<td>1,6125</td>
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<tr>
<td><strong>Spherical roller bearings</strong></td>
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<td>( d - d_m )</td>
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<td>( C_0 - d_m )</td>
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<tr>
<td><strong>CARB® toroidal roller bearings</strong></td>
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</tr>
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<td>( d - d_m )</td>
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<td>( C - d_m )</td>
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<td>1,6435</td>
</tr>
<tr>
<td>( C_0 - d_m )</td>
<td>59,869</td>
<td>1,8637</td>
</tr>
</tbody>
</table>

Table 1. Parameters for the bearing selection

Fig. 7. The bearing selecting and efficiency calculation algorithm
4. Comparing the properties of planetary gears

The performance of a planetary gear drive depends on its kinematics, its inner gear ratios and the connections between the planetary stages. Only detailed calculations can reveal the behavior of planetary gears and show their best solutions for given applications. To calculate the gear ratios and the gearbox efficiencies of various planetary gears (Fig. 1(a)-1(d).) the following equations were developed:

The gear ratio of planetary gear KB+KB (Fig. 1(a).) (sun gears drive and carriers are driven):

\[ i_{KB+KB} = (1 - i_{b^+}) \cdot (1 - i_{b^-}) \]  

The efficiency of planetary gear KB+KB:

\[ \eta_{KB+KB} = \frac{(1 - i_{b^+} \cdot \eta_{g^+}) \cdot (1 - i_{b^-} \cdot \eta_{g^-})}{(1 - i_{b^+}) \cdot (1 - i_{b^-})} \]  

The gear ratio of planetary gear PKG (Fig. 1(b).):

\[ i_{PKG} = (i_{b^+} + i_{b^-} - i_{b^+} \cdot i_{b^-}) \]  

The efficiency of planetary gear PKG:

\[ \eta_{PKG} = \frac{i_{b^+} \cdot \eta_{g^+} + i_{b^-} \cdot \eta_{g^-} - i_{b^+} \cdot i_{b^-} \cdot \eta_{g^+} \cdot \eta_{g^-}}{i_{b^+} + i_{b^-} - i_{b^+} \cdot i_{b^-}} \]  

Power distribution between the stages (the power of the driven element of the first stage \( P_{k^+} \) divided by the output power \( P_{out} \)):

\[ \frac{P_{k^+}}{P_{out}} = \frac{1}{1 + \frac{i_{b^+} \cdot \eta_{g^+} - i_{b^+} \cdot \eta_{g^-}}{i_{b^+} \cdot \eta_{g^+}} - i_{b^-} \cdot \eta_{g^-}} \]  

The gear ratio of planetary gear PV (Fig. 1(c).):

\[ i_{PV} = 1 + i_{b^+} \cdot i_{b^-} - i_{b^+} \]  

The efficiency of planetary gear PV:

\[ \eta_{PV} = \frac{1 - i_{b^+} \cdot \eta_{g^+} + i_{b^-} \cdot i_{b^+} \cdot \eta_{g^+} \cdot \eta_{g^-}}{1 - i_{b^+} + i_{b^-} \cdot i_{b^+}} \]  

Power distribution between the stages (the power of the driven element of the first stage \( P_{k^-} \) divided by the output power \( P_{out} \)):

\[ \frac{P_{k^-}}{P_{out}} = \frac{1}{1 + \frac{i_{b^+} \cdot i_{b^-} \cdot \eta_{g^+} \cdot \eta_{g^-}}{1 - i_{b^-} \cdot \eta_{g^-}}} \]  

The gear ratio of planetary gear GPV (Fig. 1(d).):
\[ i_{GPV} = 1 - i_b - i_{b_p} + i_{b_p} \cdot i_b + i_b \cdot i_{b_p} \]  

The efficiency of planetary gear GPV:

\[ \eta_{GPV} = \left[ \frac{(1 - i_b \cdot \eta_g) \cdot (1 - i_{b_p} \cdot \eta_{b_p}) + i_b \cdot i_{b_p} \cdot \eta_g \cdot \eta_{b_p}}{1 - i_b - i_{b_p} + i_b \cdot i_{b_p} + i_b \cdot i_{b_p}} \right] \]  

Power distribution between the stages (the power of the driver element of the first stage \( P_{2'} \) divided by the power of the driver element of the second stage \( P_2 \)):

\[ \frac{P_{2'}}{P_2} = \left( \frac{1}{i_{b_p}} - 1 \right) \left( 1 - i_{b_p} \right) \]  

5. Results of calculations

Calculations were to compare the tooth and the bearing friction losses in order to determine the efficiency of different types of planetary gears and evaluate the influence of the construction on the bearing friction losses and the efficiency of planetary gears. Comparing the calculated power losses caused by only the friction of tooth wheels or only by the bearing friction with the total power losses of the gearboxes, it is obvious that the bearing friction loss is a significant part of the whole friction losses. Behavior of various types of two- and three-stage and differential planetary gears were investigated and compared using the derived equations, following a row of systematical procedures. If the input power, the input speed and lubricant viscosity are known, the calculation can be performed. The first step is to choose various inner gear ratios for every stage and to combine them creating as many planetary gear ratios as possible. Using the equations presented above (1-29) the efficiency and the bearing power loss of every gear can be calculated. Some results are presented in diagrams (Fig. 8-17). Comparing the calculated values of efficiency and power loss ratios the optimal gearbox construction can be selected. The beneficial inner gear ratio of each stage and the power ratios were determined for all the four types of planetary gears. When the optimal inner gear ratios are known, the tooth profile ensuring the lowest tooth friction can be calculated for every planetary gear stage by varying the addendum modification of tooth wheels [Csobán 2009]. The calculations were performed for all planetary gears presented above for transmitting a power of 2000 kW at a driving speed of 1500 rpm. In the calculations the parameters of Table 2 and 3 were used.

<table>
<thead>
<tr>
<th>(\sigma_F) [MPa]</th>
<th>(\eta_M) [mPas]</th>
<th>(R_{a23}) [(\mu m)]</th>
<th>(R_{a34}) [(\mu m)]</th>
<th>(P_{in}) [kW]</th>
<th>(n_{in}) [1/min]</th>
<th>(\beta) [°]</th>
<th>(x_2) [-]</th>
<th>(N) [-]</th>
<th>(b/d_w) [-]</th>
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<tr>
<td>500</td>
<td>63</td>
<td>0,63</td>
<td>1,25</td>
<td>2000</td>
<td>1500</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>0,8</td>
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Table 2. Other important parameters for the analyses

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>(f_0)</th>
<th>(f_1)</th>
<th>c(d_{s1})</th>
<th>d(d_{s1})</th>
<th>c(L_{1h})</th>
<th>d(L_{1h})</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>7,5</td>
<td>0,00055</td>
<td>0,4595</td>
<td>1,0951</td>
<td>444,04</td>
<td>1,3465</td>
</tr>
</tbody>
</table>

Table 3. Parameters for calculate the bearing friction losses
The results of calculation are presented in (Fig. 8-17). On the diagrams only those results can be seen, where the gears have no undercut or too thin top land.

**Fig. 8.** The power loss ratio of planetary gear KB+KB as a function of gear ratio. Prescribed gearbox lifetime=5000[h]

**Fig. 9.** The power loss ratio of planetary gear KB+KB as a function of gear ratio. Prescribed gearbox lifetime=50000[h]
The Bearing Friction of Compound Planetary Gears in the Early Stage Design for Cost Saving and Efficiency

Fig. 10. The power loss ratio of planetary gear PV as a function of gear ratio. Prescribed gearbox lifetime=5000[h]

Fig. 11. The power loss ratio of planetary gear PV as a function of gear ratio. Prescribed gearbox lifetime=50000[h]
Fig. 12. The power loss ratio of planetary gear PKG as a function of gear ratio. Prescribed gearbox lifetime=5000[h]

Fig. 13. The power loss ratio of planetary gear PKG as a function of gear ratio. Prescribed gearbox lifetime=50000[h]
The power loss ratios of the three-stage GPV planetary gearbox were investigated at the same gear ratio range as the two-stage differential gears have (Figure 14-15).

Fig. 14. The power loss ratio of planetary gear GPV as a function of gear ratio. Prescribed gearbox lifetime=5000[h], \( \text{ib}''=2 \)

Fig. 15. The power loss ratio of planetary gear GPV as a function of gear ratio. Prescribed gearbox lifetime=50000[h], \( \text{ib}''=2 \)
The GPV gearbox can operate with higher gear ratios than the two-stage gears. The gear ratio range was changed with increasing the inner gear ratio of the first stage while the inner gear ratios of the second and third stage were changed and combined (Figure 16-17).

Fig. 16. The power loss ratio of planetary gear GPV as a function of gear ratio. Prescribed gearbox lifetime=5000[h], $ib''=8$

Fig. 17. The power loss ratio of planetary gear GPV as a function of gear ratio. Prescribed gearbox lifetime=50000[h], $ib''=8$
6. Conclusions

Comparing the results of the analysis the following can be stated:

- It is obvious that the bearing friction loss is a significant part of the friction losses.
- Higher gear ratios can be realized with the planetary gear PKG than with planetary gear PV.
- It can be stated that thanks to relatively low predicted lifetime, smaller bearings have to be build in the gearbox. Having smaller bearings, the tooth power loss ratio will be higher (at the same types of bearings).
- If longer bearing life is needed larger bearings have to build in the gearbox. Larger bearings lead to higher power losses (at the same types of bearings).
- Varying the inner gear ratios of the investigated planetary gear drives the values of the power loss rates change significantly only in the range of the lower gear ratios.
- Depending on the gear ratios and prescribed lifetime the values of the tooth power loss ratio change between 80% to 40% while the values of the bearing power loss ratio change between 20% to 60%.

Using the bearing power loss model presented above all types of bearings can be considered for a given planetary gearbox optimization and application and all the important parameters like efficiency, size and even cost can be compared easily.

7. Acknowledgment

This work is connected to the scientific program of the "Development of quality-oriented and harmonized R+D+I strategy and functional model at BME" project. This project is supported by the New Hungary Development Plan (Project ID: TÁMOP-4.2.1/B-09/1/KMR-2010-0002).

8. Nomenclature

\[\begin{array}{ll}
2 & \text{sun gear}, \\
3 & \text{planet gear}, \\
4 & \text{ring gear}, \\
a, b & \text{exponents [-]}, \\
a_1 & \text{factor for bearing life correction \((a_1 = 0.21 \ldots 1)\) [-]}, \\
C & \text{the basic dynamic load} \,[\text{N}], \\
c(d_b;L_1h) & \text{developed constant for bearing calculation} \\
d(d_b;L_1h) & \text{developed constant for bearing calculation} \\
\bar{c} ; d & \text{constant and exponent (table 1.),} \\
d_m & \text{average diameter of bearing} \,[\text{mm}], \\
d_m;res & \text{resultant average bearing diameter} \,[\text{mm}], \\
f_0 & \text{coefficient (which is a function of bearing type and size) [-]}, \\
f_1 & \text{coefficient (which is a function of bearing type and load) [-]}, \\
F_r & \text{is the radial bearing load} \,[\text{N}], \\
\eta_s & \text{is the rolling efficiency of a simple planetary gear stage} \, KB, \\
\eta_M & \text{viscosity at operating temperature} \,[\text{Pas}], \\
I & \text{is the gear ratio}, \\
i_b & \text{is the ratio of the number of teeth of sun gear and ring gear at the third stage}, \\
i_{b_1} & \text{is the ratio of the number of teeth of sun gear and ring gear at the second stage}, \\
\end{array}\]
\( i_{b} \) is the ratio of the number of teeth of sun gear and ring gear at the first stage,
\( k \) planetary carrier,
\( L_{th} \) prescribed lifetime [h],
\( M_{0} \) load independent friction torque [Nmm],
\( M_{1} \) load dependent friction torque [Nmm],
\( M_{2,4} \) sun or ring gear torque [Nm],
\( M_{3} \) planet gear torque [Nm],
\( n \) bearing velocity [rpm],
\( n_{in} \) driving speed [rpm],
\( \nu \) kinematical viscosity at operating temperature [mm\(^2\)/s],
\( P_{1} \) load of the bearing [N],
\( P_{in} \) driving power [W],
\( R_{a} \) average surface roughness (CLA),
\( \sigma_{F} \) bending strength of teeth [MPa],
\( \sigma_{m}, \tau_{m} \) allowable equivalent and shear stress components [MPa],
\( \Sigma \nu \) total power loss [W],
\( \nu \) entraining speed [m/s],
\( V \) shear load of planet gear pin [N],
\( \nu_{bearing} \) bearing friction loss component [W],
\( \nu_{tooth} \) tooth power loss component [W],
\( \omega_{3g} \) angle velocity of planet gear [rad/s],
\( Y \) calculated parameter \((d_{p}, C)\).

9. References


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In the past decades, significant advances in tribology have been made as engineers strive to develop more reliable and high performance products. The advancements are mainly driven by the evolution of computational techniques and experimental characterization that leads to a thorough understanding of tribological process on both macro- and microscales. The purpose of this book is to present recent progress of researchers on the hydrodynamic lubrication analysis and the lubrication tests for biodegradable lubricants.