1. Introduction
The study of non-Newtonian fluids is of fundamental importance in practically all branches of science and engineering that deal with incompressible fluid flow. Blood rheology, food processing, petroleum engineering, polymer blending and pharmaceutical product development are only but a few areas in which non-Newtonian fluids play a major role. Inelastic fluids with shear rate dependent viscosities are an example of non-Newtonian fluids, such fluids are called Generalized Newtonian fluids. Fluids that exhibit elastic effects are another example of non-Newtonian fluids, these fluids are called Viscoelastic fluids. We focus attention on viscoelastic fluids whose viscosities are either independent of applied shear-rates (Boger fluids) or whose viscosities are shear-rate dependent (e.g. the Generalized Oldryd-B fluids). In either case the fluid viscosity will be considered temperature dependent and our investigations will focus on the fluids’ heat transfer characteristics in simple flows. As in Chinyoka (2008; 2009a;b; 2010; 2011) the viscoelastic fluid behavior is compared to that for corresponding inelastic (Newtonian and/or Generalized Newtonian) fluids and it is demonstrated that depending on the physical application, viscoelasticity may or may not be favorable. For a comprehensive overview of non-Newtonian flows in general and viscoelastic fluid phenomena in particular, we refer to the excellent treatises of Bird et al. (1987); Ferry (1981).

Investigations of heat transfer in fluid flow have mostly been conducted for inelastic fluids. Temperature dependent flows of viscoelastic fluids have been largely limited by the slow development of the relevant universally accepted non-isothermal constitutive models. The mathematical discussion of the constitutive modeling of non-isothermal effects in the flow of viscoelastic fluids is still underway and the references Dressler et al. (1999); Hütter et al. (2009); Peters & Baajens (1997); Sugend et al. (1987); Wapperom & Hulsen (1998) provide a clear picture as to the current developments. What is now beyond doubt, among these representative cited works, is that temperature changes in such flowing polymeric systems should at the very minimum capture the effects of the three processes; conductive heat transfer effects, entropic effects due to stress work and energetic effects due to the changes in the polymer orientations. Secondary effects, say due radiation and chemical reactions can be included or neglected depending on the exact nature of the physical situation. The major difference between the most recent work Hütter et al. (2009) and previous works is the realization in Hütter et al. (2009) that the usual modeling of energetic effects using the conformational tensor may fail to capture those energetic effects that may arise from fast deformation/relaxation processes due to microscopic changes, say, resulting from continual
changes of orientation of adjacent atoms in a polymer chain. A local fast variable is thus
used in Hütter et al. (2009) as opposed to the slow tensor used in Dressler et al. (1999); Peters & Baaijens (1997); Wapperom & Hulsen (1998) and all the similar related works. Microscopically-based models might thus currently not be our best choice since their energy equations are still an area of very active discussion. In the current work, we will thus focus on conformational tensor based models for the energetic effects. We however leave open the possibility of revisiting this in the future, once the results of Hütter et al. (2009) have been fully tested experimentally and hence once the numerical values of the local variables needed are readily available for actual polymeric systems.

Simple flows of liquids undergoing exothermic reactions is a model problem in industrial processes involving chemically reactive lubricants and most petroleum products. In lubrication applications it is of paramount importance to use fluids with delayed susceptibility to thermal runaway phenomena else the lubricants are easily degraded and expensive material exposed to wear. Similarly it would be important to understand and be able to anticipate and hopefully delay the onset of such thermal runaway phenomena in the pipeline transport of reactive petroleum products in order that the end-products remain usable. Investigations into reactive lubricants subjected to shear flow were carried out, say, in Chinyoka (2008) where it was demonstrated that viscoelastic (Oldroyd-B) liquids can withstand higher values of the reaction parameter without undergoing thermal runaway as opposed to corresponding Newtonian lubricants. A similar investigation in Chinyoka (2010) also showed that the linear Phan-Thien-Tanner fluids also display better thermal loading properties compared to corresponding Newtonian fluids. A question thus naturally arises; is it always the case that viscoelastic fluids show better thermal loading properties (and hence delayed thermal runaway) under exothermic reactions than corresponding Newtonian fluids? Our current investigations with Johnson-Segalman fluids show that the answer is negative. The viscoelastic models used in Chinyoka (2008; 2009a;b; 2010; 2011) exhibit a monotonic shear stress/shear rate relationship in simple shear flows. The Johnson-Segalman fluid, allows for a non-monotonic relationship between the shear stress and rate of shear. For certain values of the viscoelastic material parameters, the Johnson-Segalman model exhibits a non-monotonic stress-strain relationship. Under certain values of such parameters, only weak solutions are admissible in pressure driven channel flow of Johnson-Segalman liquids. Physically, the weak solutions manifest as shear banding, where fluid near to the walls moves rapidly with high shear rates whereas the bulk fluid within the channel largely exhibits plug flow with very low shear rates. This leads to jumps in the shear rates leading to zones of markedly different shear-rates within the fluid and hence the name shear-banding. The shear banding phenomenon has been observed experimentally in the flow of certain classes of viscoelastic fluids. The phrase “Johnson-Segalman fluid” refers to a class of fluids that exhibit shear banding in experiment. The presence of high shear rate regions in the pressure driven flow of Johnson-Segalman fluids can lead to larger increases in the fluid temperature and hence easy susceptibility to thermal runways phenomena (in case of reacting flows) compared to even the inelastic fluids.

The main objective of this work is thus to demonstrate conditions under which fluid viscoelasticity may or may not enhance the thermal loading properties in real fluid flow situations. The question of whether solutions of partial differential equations exist globally in time or develop singularities in finite time remains a focus of attention in the scientific community. The fact that problems of industrial and engineering significance are governed by these equations makes the discussion all the more important. For a comprehensive overview of the typical examples where finite-time blow-up, or at least very rapid growth, occurs in mechanical systems and in particular those of thermal-fluid mechanics refer to
Effects of Fluid Viscoelasticity in Non-Isothermal Flows

Straughan (1998). In chemical kinetics, such finite time blow up of physical temperatures is commonly referred to as thermal runaway and is closely connected with the so-called Kamenetskii parameter, named after the pioneering work of Frank-Kamenetski (1969). The need to understand and control such phenomena as thermal runaway provides the impetus for investigations like those chronicled in this work.

The work in this chapter is organized as follows. Section (2) summarizes the constitutive and mathematical models that are relevant in non-isothermal flow of polymeric (viscoelastic) fluids. In section (3), we look at the relative significance of the energy elastic effects in flow of polymeric fluids, basically summarizing the works in Chinyoka (2010); Hütter et al. (2009). In sections (4,5,6), we give some applications to reacting flows in lubrication, heat exchangers and convection driven flows respectively. The issues related to the pressure driven flow of Johnson-Segalman fluids are summarized in section (7) in which we also highlight some current investigations involving non-isothermal flow of viscoelastic fluids with shear-rate dependent viscosity. Concluding remarks follow in section (8).

2. Mathematical modeling

The dimensionless governing equations for the velocity \( u \), temperature \( T \) and extra stress components \( \tau \) are,

\[
\nabla \cdot u = 0, \tag{1}
\]

\[
Re \ \frac{Du}{Dt} = -Re \ \nabla p + \nabla \cdot g + F, \tag{2}
\]

\[
Re Pr \ \frac{DT}{Dt} = \nabla^2 T + Br Q_D + \delta, \tag{3}
\]

\[
\tau + We \lambda \left( \frac{\nabla u}{\tau} - \frac{D}{Dt} \ln(1 + aT) \right) + f(\tau) = 2 \beta \mu_p S. \tag{4}
\]

Here \( g = \tau + 2\mu_s (1 - \beta) S \) is the extra stress tensor, \( S = (\nabla u + (\nabla u)^T)/2 \) is the deformation rate tensor and \( \tau \) is the polymeric tensor, \( F \) represents body forces for the momentum equation, \( Q_D \) is the dissipation function that takes into account both entropic and energetic effects, \( \delta \) represents source terms for the energy equation, \( f(\tau) \) is a nonlinear function of the polymer stress tensor, \( Re \) is the Reynolds number, \( Pr \) is the Prandtl number, \( Br \) is the Brinkman number, \( We \) is the Weissenberg number, \( \beta \) is the ratio of polymer to solvent viscosity. The time derivatives are defined by:

\[
\frac{DA}{Dt} = \frac{\partial A}{\partial t} + (u \cdot \nabla)A, \tag{5}
\]

and

\[
\nabla \tau = \left( 1 - \frac{\xi}{2} \right) \nabla \tau + \frac{\xi}{2} \triangle \tau, \quad 0 \leq \xi \leq 2, \tag{6}
\]

where

\[
\nabla \tau = \frac{D}{Dt} \tau - \nabla u \cdot \tau - \tau \cdot (\nabla u)^T, \quad \triangle \tau = \frac{D}{Dt} \tau + \tau \cdot \nabla u + (\nabla u)^T \cdot \tau. \tag{7}
\]

The Johnson Segalman model has \( 0 < \xi < 2 \), the lower convected Maxwell model has \( \xi = 2 \) and all the other viscoelastic fluid models considered in this work have \( \xi = 0 \). Polymeric fluids have \( 0 < \beta \leq 1 \) so that \( \beta = 0 \) corresponds to inelastic fluids. The expressions for the dimensionless dissipation function \( Q_D \) depends on the particular fluid model and will be specified independently for each case.
The temperature dependence of the solvent viscosity, polymer viscosity and relaxation time is given by $\mu_s$, $\mu_p$ and $\bar{\lambda}$ respectively. These will be specified in terms of William-Landel-Ferry (WLF), Arrhenius or Nahme shift factors.

### 3. Energy elastic effects

It is concluded in Hütter et al. (2009) that the usual modeling of energetic effects using the conformational tensor may fail to capture those energetic effects that may arise from fast deformation/relaxation processes due to microscopic changes, say, resulting from continual changes of orientation of adjacent atoms in a polymer chain. We illustrate these issues using the work in Chinyoka (2010). In this case, the dissipation function takes the form,

$$Q_D = \gamma (\tau : \dot{\Sigma}) + (1 - \gamma) \frac{\text{Tr}(\tau)}{2\text{We}_\lambda},$$

(8)

where the parameter $\gamma$ signifies the ability of viscoelastic fluids to store energy due to their elastic behavior. In particular, $0 \leq \gamma \leq 1$, where $\gamma = 0$ corresponds to the case of pure energy elasticity and $\gamma = 1$ corresponds to pure entropy. Allowance for exothermic reactions is modeled via Arrhenius kinetics,

$$\delta = \delta_1 \exp \left( \frac{T}{1 + \alpha T} \right),$$

(9)

where the reaction parameter, $\delta_1$ is also called the Frank-Kamenetskii parameter. The nonlinear polymer stress function for the Phan-Thien-Tanner (PTT) model is given by,

$$f(\tau) = \epsilon \text{We} \frac{\beta}{\text{Tr}(\tau)} \tau,$$

(10)

where $\epsilon$ is a dimensionless quantity depending on the fluid. For the temperature dependence of the viscosities and relaxation time respectively, we use the Nahme and WLF models. In particular, the solvent viscosity is modeled via a Nahme-type law:

$$\mu_s(T) = \exp(-\epsilon \alpha T),$$

(11)

and the polymer viscosity and relaxation time are modeled via the WLF equation,

$$\mu_p(T) = \exp \left( - \frac{\alpha c_1 T}{c_2 + \alpha T} \right),$$

(12)

$$\bar{\lambda}(T) = \exp \left( - \frac{\alpha c_1 T}{c_2 + \alpha T} \right).$$

(13)

The parameters $c_1$ and $c_2$ are respectively assigned the values 15 and 50 corresponding to the case where the initial fluid temperature is close to the glass transition temperature and hence the polymer viscosity and relaxation times are strongly temperature dependent. The parameter $\alpha$ is an activation energy parameter and isothermal flows have $\alpha = 0$. We consider both shear and pressure driven channel flow and consider no-slip conditions for velocity on the walls. The channel wall temperatures are also kept constant (and uniform) and continuity of the stress tensors at the walls is enforced. We employ semi-implicit finite difference schemes for the solution process of the highly nonlinear and transient problems. Such schemes were
given in, say, Chinyoka et al. (2005) for isothermal viscoelastic flow and then modified and extended to the energy equation in Chinyoka (2008; 2009a;b; 2010; 2011).

3.1 Poiseuille flow results

Fig. 1. shows the temperature distribution in a pressure driven channel flow of a PTT fluid for a range of the parameter $\gamma$ ranging from the case of pure entropy to pure energy elasticity.

![Temperature distribution in a pressure driven channel flow of a PTT fluid](image)

**Fig. 1.** Non-isothermal Poiseuille flow of PTT fluid in a channel

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>0.0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.8</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{\text{max}}$</td>
<td>0.0137</td>
<td>0.0139</td>
<td>0.0141</td>
<td>0.0146</td>
<td>0.0148</td>
</tr>
</tbody>
</table>

**Table 1.** Maximum temperatures for various choices of $\gamma$.

The results show that fluid elasticity can be used to reduce the growth of fluid temperature. We next show a comparison of the results of Fig. 1. and Table 1. with results for inelastic (Newtonian) fluids. Fig. 2. shows the temperature distribution in a pressure driven channel flow of a PTT and a Newtonian fluid for the values of the parameter $\gamma$ corresponding to pure entropy to pure energy elasticity. We notice that the highest temperatures are recorded for the Newtonian fluid and that for the viscoelastic fluids, attainable temperatures increase with increasing $\gamma$.

As with the conclusions of Peters & Baaijens (1997), for the non-isothermal flow of a PTT fluid around a cylinder, the difference in maximum attainable temperatures for the cases of pure entropy and pure energy are relatively small at the given parameter values. If we increase the shear-rates, by increasing the driving pressure gradient see Fig. 3. and Table 3., we notice that the aforementioned differences in the maximum temperatures increase significantly in line with the conclusions of Hütter et al. (2009). We similarly notice that the highest temperatures, besides being much higher than before, are also recorded for the Newtonian fluid. We thus conclude that energy elastic effects may be neglected at low shear rate flows. In practice, pressure driven channel flows usually obtain at relatively low shear rates.
Fig. 2. Non-isothermal Poiseuille flow of PTT & Newtonian fluids

<table>
<thead>
<tr>
<th></th>
<th>Newtonian</th>
<th>PTT, γ = 0</th>
<th>PTT, γ = 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>T&lt;sub&gt;max&lt;/sub&gt;</td>
<td>0.0180</td>
<td>0.0137</td>
<td>0.0148</td>
</tr>
</tbody>
</table>

Table 2. Maximum temperatures for various choices of γ.

Fig. 3. Non-isothermal Poiseuille flow of PTT & Newtonian fluids (Higher Pressure Gradient)
Table 3. Maximum temperatures for various choices of $\gamma$.

3.2 Shear flow results

In Figs. 4., 5. & 6. and Tables 4., 5. & 6., we repeat the computations of the previous subsection but in this case for shear driven flow instead. As before, the results show that energy elastic effects become important for flows involving large shear rates (and hence large deformations and fast relaxation processes). In practice shear driven flows (say in fibre spinning operations) can lead to large attainable shear rates in the flow field.

![Graph of shear flow results](image)

Fig. 4. Non-isothermal Couette flow of PTT fluid

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>0.0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.8</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{\text{max}}$</td>
<td>0.0358</td>
<td>0.0411</td>
<td>0.0464</td>
<td>0.0571</td>
<td>0.0626</td>
</tr>
</tbody>
</table>

Table 4. Maximum temperatures for various choices of $\gamma$.

<table>
<thead>
<tr>
<th>Model</th>
<th>Newt</th>
<th>PTT, $\gamma = 0$</th>
<th>PTT, $\gamma = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{\text{max}}$</td>
<td>0.1456</td>
<td>0.0358</td>
<td>0.0626</td>
</tr>
</tbody>
</table>

Table 5. Maximum temperatures for various choices of $\gamma$.

<table>
<thead>
<tr>
<th>Model</th>
<th>Newt</th>
<th>PTT, $\gamma = 0$</th>
<th>PTT, $\gamma = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{\text{max}}$</td>
<td>0.3320</td>
<td>0.0601</td>
<td>0.1237</td>
</tr>
</tbody>
</table>

Table 6. Maximum temperatures for various choices of $\gamma$. 
3.3 Thermal runaway

The long term behavior of the fluid maximum temperature with respect to higher values of either $\delta_1$ or time is not directly obvious. There could be blow-up of the solutions (thermal runaway) if $\delta_1$ exceeds certain threshold values as is demonstrated say in Chinyoka (2008).
and in related works cited therein. In Fig. 7, the maximum temperatures are recorded at convergence for each value of the reaction parameter until a threshold value of the reaction parameter is reached at which blow-up of the temperature is observed. We notice that the threshold value of $\delta_1$ is increased when we use increasingly polymeric liquids. The explanations relate to the ability of viscoelastic fluid to store energy due to their elastic character. Thus while Newtonian fluids would dissipate all the mechanical energy as heat in an entropic process, viscoelastic fluids on the other hand will partially dissipate some of the energy and store some.

![Fig. 7. Thermal Runaway](image)

4. Thermally decomposable lubricants

In this section, we summarize the work in Chinyoka (2008) for the flow of a thermally decomposable lubricant described by the Oldroyd-B model. In this case, the dissipation function takes the form,

$$Q_D = 2\mu_s (1 - \beta) : \nabla u + \gamma \varepsilon : \varepsilon + (1 - \gamma) \frac{\hat{G}}{2\text{We}\lambda(T)} (I_1 + \text{Tr} (b^{-1}) - 6), \quad (14)$$

where the conformation tensor $b$ is related to the extra stress tensor $\varepsilon$ by:

$$\varepsilon = \frac{\hat{G}}{1 - \xi} (b - I_1). \quad (15)$$

$I_1$ denotes the first invariant of $b$ and $\hat{G}$ is the shear modulus. As before, the allowance for exothermic reactions is modeled via Arrhenius kinetics. The nonlinear polymer stress function for the Oldroyd-B model is identically zero,

$$f(\varepsilon) \equiv 0. \quad (16)$$
The temperature dependence of the viscosities and relaxation time respectively follow a Nahme-type law:

\[ \mu_s(T) = \mu_p(T) = \exp(-\alpha T), \quad \lambda(T) = \frac{1}{1 + \alpha T} \exp(-\alpha T). \quad (17) \]

The boundary and initial conditions for the current problems are similar to those considered in the previous section. The results for current work are qualitatively similar to those displayed in Fig. 7., see Chinyoka (2008), and will not be repeated here. It thus follows that polymeric lubricants (of the Oldroyd-B type) are able to withstand much higher temperature build ups than those designed from corresponding inelastic fluids.

### 5. Flow in heat exchangers

The lubricant fluid dynamics of the previous section is an important problem as far as physical (industrial and engineering) applications are concerned. An equally important problem is that of coolant fluid dynamics, which is necessarily related to heat exchanger design. Three major types of heat exchangers are in existence, parallel flow, counter flow Chinyoka (2009a) and cross flow Chinyoka (2009b) heat exchangers. The parallel flow heat exchangers are quite inefficient for industrial scale cooling processes and will not be discussed any further. Car radiators employ the cross flow heat exchanger design in which liquid coolant is cooled by a stream of air flowing tangential to the direction of flow of the liquid coolant. Counterflow heat exchanger arrangements are normally employed in industrial settings (say distillation processes and food processing) in the form of pipe-in-a-pipe heat exchangers, in which the main fluid to be cooled flows in the inner pipe in the opposite direction to the “colder” fluid flowing in the outer annulus.

A choice of the coolant fluid which optimizes performance is undoubtedly of major importance as far as physical applications are concerned. In particular, the coolant fluid should be capable of resisting large temperature increases as well as also being able to rapidly lose heat. This thus provides the impetus for a comparative study of the thermal loading properties of inelastic versus viscoelastic coolants. In most industrial settings, the focus may instead be on the cooling characteristics and properties of fluids whose elastic properties are predetermined and not subject to choice, say the fluids extracted from distillation processes. The works referenced in this section can still be used to determine the cooling properties of such fluids whether they are inelastic or viscoelastic. Such conclusions can be obtained from investigations such as those in Chinyoka (2009a;b). In these two cited works, the Giesekus model is employed for the viscoelastic fluids. In this case, the dissipation function takes the form,

\[
Q_D = 2\mu_s (1 - \beta) \mathcal{S} : \nabla \mathbf{u} + \gamma \mathcal{T} : \mathcal{S}
\]

\[
+ \frac{(1 - \gamma) \mathcal{G}}{2 \text{We} \lambda(T)} [(1 - \varepsilon)(I_1 + \text{Tr}(b^{-1}) - 6) + \varepsilon (b : b - 2I_1 + 3)]
\]

(18)

where \(\varepsilon\) is the Giesekus nonlinear parameter such that,

\[ f(\mathcal{T}) = \varepsilon \mathcal{T}^2. \quad (19) \]

As before, the allowance for exothermic reactions is modeled via Arrhenius kinetics and the temperature dependence of the viscosities and relaxation time respectively follow a Nahme-type law. The velocity and stress boundary and initial conditions for the current
Effects of Fluid Viscoelasticity in Non-Isothermal Flows

problems are similar to those considered previously. Convective temperature boundary conditions are employed at the interfaces and initial conditions are specified appropriately. Typical results for the fluid temperature are displayed in Fig. 8. The figure shows the results for a double pipe (pipe in a pipe) counterflow heat exchanger. The inner pipe is referred to as the core and we use $T_c$ to represent the core temperature. The outer shell temperature is represented by $T_s$. The flow is from left to right in the core and from right to left in the shell and the figure shows, as expected, that the core fluid temperature decreases downstream (since it is being cooled by the shell fluid) whereas the shell fluid temperature increases downstream. As in the previous sections, a viscoelastic core fluid leads to lower temperatures than an inelastic fluid Chinyoka (2009a;b).

6. Convection reaction flows

The one dimensional natural convection flow of Newtonian fluids between heated plates has received considerable attention, see for example the detailed work in Christov & Homsy (2001) and the references therein. In fact the steady state case easily yields to analytical treatment, White (2005). In physical applications lubricants, coolants and other important industrial fluids are usually exposed to shear flow between parallel plates. Differential heating of the plates thus indeed lead to natural or forced convection flow as illustrated in Christov & Homsy (2001). The previous sections have highlighted the need to employ viscoelastic fluids in such industrial applications involving lubricant and coolant fluid dynamics especially if thermal blow up due, say, to exothermic reactions is a possibility. In this section we revisit the shear flow of reactive viscoelastic fluids between parallel heated plates and in light of the observations just noted, we investigate the added effects of natural or forced convection, in essence summarizing the results of Chinyoka (2011).

As before, we use the Giesekus model for the viscoelastic fluid. The model problem consists of a viscoelastic fluid enclosed between two parallel and vertical plates. For simplicity, we consider the case in which the left hand side plate moves downwards at constant speed and the right hand side plate moves upwards at a similar speed. This creates a shear flow within the enclosed fluid. Additionally, the differential heating of the plates leads to convection currents developing in the flow field. Relevant body forces that account for the convection flow are added to the momentum equation. These body forces are of the form:

$$
F = i \frac{Gr}{Re^2} T,
$$

where $i$ is the unit vector directed vertically downwards, $Gr$ is the Grashoff number and $T$ is the fluid temperature. Typical results are displayed in Figs. 9. - 12.

As is expected from the results of the preceding sections and as also shown in Chinyoka (2011) the maximum temperatures attained are lower for the viscoelastic Giesekus fluids than for corresponding inelastic fluids.

7. Current and future work

In this section we summarize at a couple of current investigations that may in the future have an impact on the conclusions drawn thus far.

7.1 Shear rate dependent viscosity

The viscoelastic fluids chronicled in the preceding sections were all of the Boger type and hence all had non shear-rate dependent viscosities. The reduction of these fluids to inelastic
Fig. 8. Surface & Contour plots of Temperature
thus lead directly to Newtonian fluids! All the comparisons made were thus for viscoelastic fluids against Newtonian fluids. We note that the viscoelastic fluids are part of the broader class of non-Newtonian fluids. It may be important to compare the performance of viscoelastic fluids against other (albeit inelastic) non-Newtonian fluids, i.e. the Generalized Newtonian fluids, which are characterized by shear-rate dependent viscosities. The current work in Chinyoka et al. (Submitted 2011b) for example uses Generalized Oldroyd-B fluids, which contain both shear-rate dependent viscosity (described by the Carreau model) as well as elastic properties.

7.2 Non-monotonic stress-strain relationships
The viscoelastic fluids used in the preceding sections are also all described by a monotonic stress versus strain relationship. No jump discontinuities are thus expected in the shear rates for any of these viscoelastic models and hence they all lead to smooth (velocity, temperature and stress) profiles in simple flows. The viscoelastic Johnson-Segalman model however allows for non-monotonic stress-strain relationships in simple flow under certain conditions Chinyoka Submitted (2011a). Under such conditions, jump discontinuities may appear in the shear-rates and hence no smooth solutions would exist, say, for the velocity Chinyoka Submitted (2011a). In particular only shear-banded velocity profiles would be obtainable. If the flow is non-isothermal, as in Chinyoka Submitted (2011a), the large shear rates obtaining in the flow would lead to drastic increases in the fluid temperature even beyond the values attained for corresponding inelastic fluids. This would thus be an example of a viscoelastic fluid which does not conform to the conclusions of the preceding sections in which viscoelastic fluids always resisted large temperature increases as compared to corresponding inelastic fluids.

\[ T(x,y,t) \]

![Temperature distribution in absence of convection flow.](image-url)
Fig. 10. Velocity vectors in absence of convection flow.

Fig. 11. Pressure contours in absence of convection flow.
Fig. 12. Temperature distribution in presence of convection flow.

8. Conclusion

We conclude that non-Newtonian fluids play a significant role in non-isothermal flows of industrial importance. In particular, viscoelastic fluids are important in industrial applications which require the design of fluids with increased resistance to temperature build up. For improved thermal loading properties, energetic and entropic effects of the (viscoelastic) fluids however need to be carefully balanced, say by varying the elastic character of the fluids. Viscoelastic fluids, say of the Johnson-Segalman type, that exhibit shear banding in experiment however may not be suitable for the aforementioned applications as they can lead to rapid blow up phenomena, faster than even the corresponding inelastic fluids. All the quantitative (numerical) and qualitative (graphical) results displayed were computed using semi-implicit finite difference schemes.

9. References

T. Chinyoka, Computational dynamics of a thermally decomposable viscoelastic lubricant under shear, Transactions of ASME, J. Fluids Engineering, December 2008, Volume 130, Issue 12, 121201 (7 pages)


T. Chinyoka, Suction-injection control of shear banding in non-isothermal and exothermic channel flow of Johnson-Segalman liquids, submitted.

T. Chinyoka, S. Goqo, B.I. Olajuwon, Computational analysis of gravity driven flow of a variable viscosity viscoelastic fluid down an inclined plane, submitted.


The theoretical analysis and modeling of heat and mass transfer rates produced in evaporation and condensation processes are significant issues in the design of a wide range of industrial processes and devices. This book includes 25 advanced and revised contributions, and it covers mainly (1) evaporation and boiling, (2) condensation and cooling, (3) heat transfer and exchanger, and (4) fluid and flow. The readers of this book will appreciate the current issues of modeling on evaporation, water vapor condensation, heat transfer and exchanger, and on fluid flow in different aspects. The approaches would be applicable in various industrial purposes as well. The advanced idea and information described here will be fruitful for the readers to find a sustainable solution in an industrialized society.

How to reference
In order to correctly reference this scholarly work, feel free to copy and paste the following:
