Small-Bang versus Big-Bang Cosmology

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1. Introduction

During the 1920’s the U.S. astronomer Hubble observed that the red shifts, from distant galaxies, were increasing with distance. The similarity with the well known Doppler Effect gave the way to a rapidly spreading idea: that the universe was expanding. Galaxies were thought to be receding from us at a speed proportional to their distance. Considering the universe as a “gas” of galaxies, each galaxy similar to one molecule in a gas, the expansion clearly implied that the universe was getting cooler and thinner with age. We know today that this cooling and thinning is correct: the universe is very old and its known temperature and density for today are very low: 2.7ºK (the cosmic microwave background radiation) of temperature and about \( \sim 10^{-29} \text{ grams/cc} \) average density.

Now, if we imagine a thought experiment and reverse the time, going backwards, we get the idea of a very hot and very dense universe at its initial stages. Going to the limit, getting closer and closer to a theoretically zero time, we have a mathematical singularity: infinite temperature and infinite density. As a result of this initial picture, we can imagine that these infinites were the result of a very big and sudden explosion: and that it expanded rapidly to a lower and lower temperature and density. Today we observe a cool temperature and a thin “gas”. The British astronomer and cosmologist Fred Hoyle ironically called this a “Big-Bang”. But if we take the imagination of a Big-Bang explosion as a fact, as many people have done, we are entitled to take as a fact too that a gas after an explosion decelerates to a lower and lower speed of expansion. Today we should observe a decelerating universe. And this is not the case.

Initial expansion (according to a hypothetical Big-Bang), and present acceleration of the universe, as observed thanks to the astrophysics of the supernovae Type Ia, are two very different things. While the expansion is very well based on observation, following the Hubble’s red shift findings, an initial explosion at a space-time point, the Big-Bang, is the result of a mathematical extrapolation, and therefore so far it is only speculation. On the other hand, the accelerated expansion of the universe is based on observation [1]. It is the result of the successful application of the scientific method, like the case of the expansion of the universe. Accelerated expansion is a very well based observation on scientific grounds, and in a direct way. This is not so for the assumed Big-Bang initial explosion.

It is very interesting to note that the cosmological model of a Big-Bang, as a frame of work, has been and still is the underpinning of the majority of the research work done in cosmology. It had, and still has, many drawbacks. One of them, a very serious one, was related to the fact that it could not explain the present size of the universe. Following the initial developments of this model the present size of the universe would be very small: may
be of the order of meters. Obviously this is not the case, and one had to look for an
explanation. Instead of looking for an alternative model, something that the many
drawbacks of the model has demanded many times, the main stream of the scientific
community in cosmology has always decided to add more and more “ad hoc” explanations
to keep this frame of work alive. And it appears that this is going to go on for a long time.
There is so much work, interests, beliefs, efforts, etc, behind the Big-Bang idea that the
overall worldwide inertia created by this cosmological model is very big indeed.
At any rate, some of the “ad hoc” explanations to sustain the main stream ideas could be
good ones. I mean good ones when one considers them isolated, independent of the reason
that made them come into existence. For example: INFLATION. A very fast exponential
expansion at the very early stages of the universe would bring it close to a reasonable size to
avoid discrepancies. It has some predictions, flat universe, critical density, cosmic
microwave background radiation properties (CMBR) etc. that have been observed. Then, it
seems to be a good idea, a good scientific approach supported by the confirmation of some
of its predictions.
Again, if one accepts INFLATION as a beginning of the universe, a fast exponential
expansion during a very short time, one immediately imagines that after inflation a period
of deceleration should follow. This is the case, but there is more to it. As mentioned above,
during the last half of the age of the universe there is an observed accelerated expansion.
And of course this must have a reason, a physical push to expand the universe. This
physical mechanism must be of universal significance, because it has been accelerating the
whole universe during the last half of its age. And during the first half it counteracted the
inertial deceleration after inflation due to the gravitational universal attraction. And it
reversed the deceleration giving the accelerated expansion we observe today. About half
way in time the deceleration-acceleration transition implied zero acceleration. We see no
need of an initial point like explosion. Inflation does the job.
History has already gone through this state of affairs. Almost one hundred years ago, when
Einstein developed his cosmological equations, the general belief was to imagine the
universe in a static state. Since gravitation was well known, as an attractive force, soon it
was realized that a collapse was inevitable due to the pull of gravity. But no collapsing
universe was observed. Then a pushing mechanism should be balancing gravitation to get a
static universe. And Einstein introduced his well known cosmological constant, the lambda
constant. Today we observe the universe in an accelerated expansion during the last half of
its age. Then a kind of pushing mechanism is again required to explain this observation.
And it could be a question of strength: the pushing force due to the $\Lambda$ “constant” seems to
be growing as the universe expands. On the contrary, the overall gravitational force is
constant. This is enough to explain by itself the present acceleration of the expansion of the
universe. And it may arrive at a disaggregation of everything in a finite time: expansion to
infinity. Since the lambda constant is a very well known physical construct, the attention of
most cosmologists is again in favor of such solution. The point is that such a $\Lambda$ constant
implies energy, and the immediate and easy way out is to imagine the existence of a kind of
dark energy to explain $\Lambda$, so dark that no one has seen it yet. We do not know of any
interaction between this postulated dark energy and any other well known energy we are
used to observe and identify. So far, the dark energy is just a theoretical construct. But we
have more choices to explain this pushing force. Aside from believing in dark energy one
can believe in an equivalent mechanism to explain the push: creation of matter, as we will see
[2]. Then the sequence of events to explain the dynamics of the universe would be: fast
exponential inflation, and then a slow deceleration followed by a slow acceleration as of today. And our prediction is that this late acceleration is increasing and that it will disperse the whole universe to infinity in a finite time. Like a kind of second inflation at the end of the time of the universe as we know it. We may be now at about one half of the total age of our universe. The creation pressure [2] is always present, growing, and its effects are permanently present till the final stage.

Following the arguments given above, we can make now a straightforward proposal: there was no big bang at all. Instead we can say that we are the result of an initial small bang, just after inflation of an initial fluctuation, an initial quantum black hole whose inflation a little later decelerated. But this deceleration was overcome by the push of the creation pressure, the continuous creation of matter [2], [3], [4] and [5]. As we will see, most physical properties of the universe are subject to this increase with time.

The above considerations are in agreement with the idea that the universe is a kind of black hole [6]. Black holes have a characteristic mass-size relation. Taking the gravitational constant $G$ and the speed of light $c$ as units, $G = c = 1$, the black hole mass $M$ is equal to his size $L$, within a factor of 2. Then, dividing the size $L$ by the speed of light $c$ one gets a characteristic time $t$ for the black hole. In these units 1 second equals $3 \times 10^{10}$ cm, and this equals $10^{40}$ grams. We then have:

$$(-2) M = L = t \quad (1)$$

For the universe $M = L = t = 10^{56}$ grams $- 10^{28}$ cm $- 10^{10}$ years. For the Planck scale, a quantum black hole, one has to divide (1) by $\sim 10^{61}$ to get the Planck’s units $m = l = t = 10^{-5}$ grams $- 10^{-33}$ cms $- 10^{-44}$ seconds. Possibly this may be the first quantum of everything in our universe. All the basic physical properties at the Planck scale (the so-called natural units) differ by the factor $10^{61}$ from the scale of the universe.

### 2. Scale cosmology

It looks like the universe can be considered to be structured in different scales. Each scale is a quantum black hole, as we will see, and is in itself a universe too. A black hole has its mass $M$ and its size $L$ connected by the simple relation

$$(-2) GM/c^2 = L \quad (2)$$

On the other hand, a quantum black hole is characterized by its size $L$ being equal to its the de Broglie wavelength (with a generalized Planck’s constant $H$)

$$L = H/Mc \quad (3)$$

Now, combining (2) and (3) we get (for a general quantum black hole) the mass $M$, length $L$ and time $t$ as follows

$$M = (Hc/G)^{1/2}$$

$$L = (GH/c^3)^{1/2} \quad (4)$$

$$t = (GH/c^5)^{1/2}$$

If we use the natural Planck’s constant $\hbar$ in (4) we get the Planck’s units
\[
m_p = (hc/G)^{1/2} \approx 2.177 \times 10^{-5} \text{ grams}
\]
\[
l_p = (Gh/c^3)^{1/2} \approx 1.616 \times 10^{-33} \text{ cms}
\]
\[
t_p = (Gh/c^3)^{1/2} \approx 5.39 \times 10^{-44} \text{ sec}
\]

The scale of our universe is found to be the Planck's scale (5) multiplied by the factor \(10^{61}\) or, equivalently, by using a universal Planck's constant \(H \approx 10^{122} \hbar\) giving

\[
M_u = (Hc/G)^{1/2} \approx 10^{56} \text{ grams}
\]
\[
L_u = (GH/c^3)^{1/2} \approx 10^{28} \text{ cms}
\]
\[
t_u = (GH/c^5)^{1/2} \approx 5 \times 10^{17} \text{ sec}
\]

There is a new scale that can be defined below the Planck's scale. The point is that the quantum of gravity [7] has a mass \(m_g\) given by

\[
m_g = \hbar/c^2 t_u \approx 2 \times 10^{-66} \text{ grams}
\]

and it defines a scale like Planck's scale multiplied by, once again, the factor \(10^{61}\). This is equivalent to obtain this new scale by using an equivalent generalized Planck's constant \(H \approx 10^{-122} \hbar\) giving the sub-Planck scale

\[
m_{sp} = 10^{61} (hc/G)^{1/2} \approx 2 \times 10^{-66} \text{ grams}
\]
\[
l_{sp} = 10^{61} (Gh/c^3)^{1/2} \approx 10^{94} \text{ cms}
\]
\[
t_{sp} = 10^{61} (Gh/c^5)^{1/2} \approx 10^{-104} \text{ sec}
\]

The physical meaning of the sub-Planckian scale (8) is not yet very well known, except for the concept of the quantum of gravity \(m_g\) that we have introduced [7] in the past. It may also have a meaning related to information [8]: in a parallel way it can be given a sense as the unit of information, the bit, with the physical properties in (8). We can also give some sense for a quantum of time, defined as the minimum interval of time obtained using the mass of the universe

\[
t_{sp} = h/M_u c^2 \approx 10^{-104} \text{ sec}
\]

This is a very suggestive relation: it means that the sub-Planckian scale (8) gives us the minimum quantum of mass, length and time. The three quantum black hole scales, (5), (6) and (8) are then the minimum scale (8), sub-Planckian, the “natural” scale (5), Planckian, and the scale of the universe (6) where we live.

There is a new physical parallel that gives a meaning to the sub-Planckian “quantum”. It may be regarded as the unit of information, the bit [8]

### 3. Gravity as an emerging entropic force

Verlinde [9] has introduced the concept of the force of gravity as due to a gradient of entropy \(S\), i.e. gravity as an emergent entropic force. Though the change in entropy \(S\) may be due to internal redistribution of masses in the system, it may also be due to a cosmological increase of mass with time, as we will see here. The basic idea can be expressed as the
relation between temperature T, entropy S and energy $Mc^2$, according to the thermodynamic relation

$$T \Delta S = \Delta Mc^2 = \Delta t/2$$  \hspace{1cm} (10)

We have used the Machian black hole relation $2GM/c^2 = ct$ to obtain the last term in (10). Dealing with a “quantum” black hole universe with $H \approx 10^{122} \hbar$, we have equations (1), and (2), and using the Hawking [10] and Bekenstein [11] black hole relation for the entropy $S$

$$S = 4\pi k/\hbar c GM^2$$ \hspace{1cm} (11)

we get from (11), with $G = c = \hbar = k = 1$

$$\Delta S = 4\pi 2M \Delta M$$ \hspace{1cm} (12)

And using (10) and (12) we have

$$T \Delta S = 2\pi Tt \Delta t = \Delta t/2$$ \hspace{1cm} (13)

i.e.

$$4\pi Tt = k/\hbar = 1$$ \hspace{1cm} (14)

Then temperature varies inversely proportional to cosmological time. This is a well known relation in our universe. But here we have a surprising possibility: since the temperature $T$ is a statistical parameter, then the time $t$ may have this character too. The mass of the universe must be time varying [14], so that the gradient of $M$ in (12) is responsible for the increase in entropy $\Delta S$, and therefore for the force of gravity. Verlinde’s ideas [9] may be extended to a distribution of mass in the whole universe varying with cosmological time.

4. The cosmological constant versus the pressure of creation

The cosmological constant $\Lambda$ has been related to the vacuum energy, and therefore to a negative pressure, to explain the accelerated expansion of the universe. Recently we have an interesting suggestion [1]: it implies that there is no cosmological constant. Its theoretical need can also be fulfilled by a creation pressure $p_c$. At any rate, either $\Lambda$ or a creation pressure implies (with $c = 1$), from Einstein cosmological equations:

$$\Lambda \approx 1/t^2$$ \hspace{1cm} (15)

And from (1) and (11) we get

$$S \approx t^2 \quad \text{i.e.} \quad \Delta S \approx 1$$ \hspace{1cm} (16)

The creation pressure $p_c$ [1] has been presented as equivalent to the effect of a cosmological constant $\Lambda$. A creation pressure expressed as $\Omega_{cp}$, a dimensionless parameter i.e.

$$\Omega_{cp} = (8\pi/3) Gp_c / (c^2H^2)$$ \hspace{1cm} (17)

as usually done in cosmology, is equivalent to the effect of a cosmological constant $\Lambda$, with omega parameter $\Omega_\Lambda$, if and only if the following relation holds:

$$- \Omega_{cp} \equiv 3\Omega_\Lambda$$ \hspace{1cm} (18)
This follows from the first of the cosmological equations of Einstein, i.e.
\[ 1 - 2q + \Omega_p + \Omega_k = 3 \Omega_\Lambda \] (19)
Here \( q \) is the deceleration parameter and \( \Omega_k \) the curvature. If we consider a creation pressure instead of a cosmological constant, usually taken as the dark energy constituent of the universe, we get from (18) and (19)
\[ 1 - 2q + \Omega_k = - \Omega_{cp} \] (20)
And using the present observations that give \( \Omega_k << 1 \) we finally get for the creation pressure, instead of a cosmological constant
\[ \Omega_{cp} = 2q - 1 \] (21)
The present estimates [15] of the numerical values of the deceleration parameter \( q \) are: for very high red shift, close to the initial stages of the universe, \( q \approx 0.5 \) which implies \( \Omega_{cp} \approx 0 \). The initial creation pressure is very small, corresponding to a small dark energy component, if any. At this stage we should expect a small acceleration of the initial expansion that balances the gravitational attraction (may be after inflation has finished in a very short time). At the present time [15] we have the approximate value \( q \approx -0.5 \), which implies \( \Omega_{cp} \approx -2 \). The present creation pressure is then pretty high. From (18) it would correspond to a value of \( \Omega_\Lambda \approx 2/3 \), in complete agreement with the very well known value of this parameter for today. There is no known reason for this negative increase in the creation pressure (positive increase in \( \Lambda \) and therefore in accelerating the expansion of the universe) to stop in the near future. We can extrapolate and consider the rather strong possibility that the universe will spread to infinity, in a finite time, due to an ever increasing accelerated expansion [8].
The creation pressure is related to the creation rate \( \Gamma \) of the mass \( M \) [1] by the following expression
\[ \Gamma = \dot{\rho} / \rho + 3 \dot{R}/R = d (\ln \rho \ R^3)/dt = d (\ln M)/dt = \dot{M}/M \] (22)
The creation pressure \( p_{cp} \) is defined in terms of the creation rate \( \Gamma \) and other physical quantities [1] and is
\[ p_{cp} = - \rho \ c^2 \ (\Gamma/3H) \] (23)
If we consider the universe as a black hole [6] then we have
\[ 2 GM/c^2 = R \quad \text{i.e.} \quad \dot{M}/M = \dot{R}/R = H = \Gamma \] (24)
where \( H \) is the Hubble parameter. The creation pressure in (23) becomes
\[ p_{cp} = - (1/3) \rho \ c^2 \] (25)

5. The cosmological constant versus the energy of the information

We can think of our universe as a kind of “quantum” black hole [6] and apply the Hawking-Bekenstein [10] and [11] formulation for its entropy \( S \). Using the black hole relation (2) between its mass \( M \) and its size \( a(t) \)
\[ 2GM/c^2 = a(t) \] (26)
and combining (11) and (26) we get (with the linear relation \( a(t) \approx ct \))

\[
S = 4\pi \frac{\hbar}{bc} G\left(\frac{c^4}{2G}\right)^2 = \pi \frac{k}{\hbar} \left(\frac{c^3}{G}\right) t^2
\]  

(27)

And in natural units \( G = c = \hbar = k = 1 \) we finally get

\[
S \approx t^2
\]  

(28)

Going on using natural units, in Planck’s units of time we have then from (28)

\[
S \approx 10^{122}
\]  

(29)

The entropy of the universe increases with time and will arrive at a maximum at \( x = 2 \), its lifetime, and has a value of the order of \( -10^{122} \).

The quantum of gravity with mass \( m_g \) has been presented [7] as

\[
m_g = \frac{\hbar}{c^2 t} \approx 10^{-65} \text{ grams} 
\]  

(7)

Since the mass of the universe \( M \) is about \( 10^{56} \) grams, one has the number of gravity quanta \( N_g \) in the universe as

\[
N_g \approx \frac{M}{m_g} \approx 10^{122}
\]  

(30)

The two very large numbers in (29) and (30), being of the same order of magnitude, give us a very strong reason to believe that the entropy \( S \) of the universe is the number of gravity quanta, as proposed 10 years ago [7], and this is the number of bits \( I \) that it contains:

\[
I \approx S \approx N_g \approx \frac{M}{m_g} \approx 10^{122}
\]  

(31)

Then, the unit of information, the bit, can be interpreted as having a mass \( m_g \) and an energy \( m_g c^2 = \hbar/t \), i.e. the quantum of gravitational energy \( \hbar \omega \approx 10^{-45} \) ergs.

Now we can check the holographic principle, [12] and [13], for the universe: the amount of information (31) inside the whole universe is equal to the area of the event horizon in Planck’s units (28) and (29).

6. The accelerated expansion of the universe

The deceleration parameter \( q \) was defined in terms of the scale factor \( a(t) \) and its derivatives as follows:

\[
q = -\frac{\ddot{a}}{a} \frac{a'}{(a')^2} 
\]  

(32)

We see that \( \ddot{a} \) being a deceleration one has \( \ddot{a} < 0 \) and then the parameter \( q \) should be \( q > 0 \) for deceleration and \( q < 0 \) for acceleration. We can take into account the definition of the Hubble parameter \( H \)

\[
H = \frac{\dot{a}}{a}
\]  

(33)

So that equation (32) transforms to

\[
H' + (1 + q) H^2 = 0
\]  

(34)
The importance of this equation cannot be overestimated. It means that given the measured values of $q$ [15] one can approach its time variation by the linear relationship:

$$q(x) = -x + \frac{1}{2}$$  \hspace{1cm} (35)

where we have defined $x = t/t_0$ the ratio of any age of the universe $t$ to the present age of the universe $t_0 \approx 1.37 \times 10^{10}$ years. Then close to the beginning of the universe we have $q \approx \frac{1}{2}$ (i.e. $x = \varepsilon \ll 1$) and today $q \approx -\frac{1}{2}$ ($x_0 \approx 1$). Rearranging equation (34) with the change $dt = t_0 \, dx$ we get

$$\frac{\dot{H}}{H^2} = -\frac{d}{(t_0 \, dx)} = -[1 + q(x)]$$  \hspace{1cm} (36)

And integrating we have

$$\frac{1}{H} \frac{dt}{t_0} = \int [1 + q(x)] \, dx + \text{constant}$$  \hspace{1cm} (37)

Using (35) we get

$$\frac{1}{H} \frac{dt}{t_0} = 1.5 \, x - 0.5 \, x^2 + \text{constant}$$  \hspace{1cm} (38)

Choosing the limits of integration from 0 to $x$ and taking into account that the present value of $H$ is $H_0 \approx 1/t_0$, for $x = 1$, the constant in (38) has the value zero. With (33) equation (38) is then equivalent to

$$t_0 \, \frac{\dot{a}}{a} = [1.5 \, x - 0.5 \, x^2]^{-1} = \frac{d}{dx} \ln a$$  \hspace{1cm} (39)

Integrating once more we get

$$\ln a = \int [1.5 \, x - 0.5 \, x^2]^{-1} \, dx + \ln a_0$$  \hspace{1cm} (40)

where $a_0$ is the present value of the cosmological scale parameter $a(t_0)$

$$a/a_0 = \exp \{ \int [1.5 \, x - 0.5 \, x^2]^{-1} \, dx \}$$  \hspace{1cm} (41)

integrating (41) we have

$$a/a_0 = \exp\{\frac{2}{3} \ln \frac{2x}{(3-x)}\} = \left[\frac{2x}{(3-x)}\right]^{2/3}$$  \hspace{1cm} (42)

The plot of this expression is shown below in Fig. 1.

Fig. 1. In this figure 1 we have the plot of the scale factor of the universe (vertical axis), relative to its present value $a_0$, in terms of time $t$ (horizontal axis), relative to the present age of the universe $t_0$. An infinite expansion appears at $t = 3t_0$. 

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7. The final inflation

Having used only the relations (32), (33), (34) and (35), without using the field equations of general relativity (only the observed values of the deceleration parameter $q$), the predicted final “inflation” at $t_f = 3t_0$ is a result of an extrapolation towards the future. The present day observations of $q$ cover 1/3 of this time interval and strongly support the final expansion, the finite lifetime of the universe in a surprisingly rather short time from now (only 2 aeons).

We speculate that the initial inflation may have started from the Planck’s quantum black hole, bringing the universe close to its present size. After that, an almost linear expansion goes on due to the creation pressure, thus bringing the universe to its present size. The final inflation follows at about $4 \times 10^{10}$ years of age, giving a finite lifetime for our universe. This is clearly an unexpected result that comes from the present observations of the values of the deceleration parameter $q$.

8. Conclusions

The generalization of the concept of a quantum black hole (giving the sub-Planckian scale, the Planckian scale and the scale of our universe) shows that there is a numerical factor, $\sim 10^{61}$, that is equivalent to the total age of the universe in Planck’s units. It looks like this is the characteristic lifetime of a universe, in terms of the successive factors for the different scales, $10^{-61}, 1, 10^{61}$, (or in terms of the generalized Planck’s constant, $10^{-122}, 1, 10^{122}$). The age of a universe is intimately related to the choice of the unit of time interval. For the sub-Planckian scale we have $10^{-104}$ seconds, for the Planck scale $5 \times 10^{-44}$ seconds and for our universe about $5 \times 10^{17}$ seconds.

The picture that arises for the evolution of the universe is: no big-bang, an initial inflation (an exponential expansion) of a quantum black hole, Planck’s type, a slow deceleration followed by a slow acceleration. Then we have an almost linear expansion at the present time. And a final disaggregation to infinity at about $4 \times 10^{10}$ years of age, the lifetime of our universe.

The cosmological constant $\Lambda$ can be substituted by a creation pressure. This is in line with the idea of gravitation being an emerging entropic force. For the existence of this force an increase in mass with time (a Mass-Boom, [14]) is necessary, giving a positive gradient of entropy for the universe and therefore the emergent gravitation.

9. Appendix

We are going to calculate now the following important cosmological parameters, in terms of the dimensionless age, $x = t/t_0$, and relative to the present size of the universe $a_0 = 1$:

1. The speed of expansion of the universe $a'(t)$
2. The Hubble parameter $a'(t)/a(t)$
3. The acceleration of the expansion $a''(t)$
4. The deceleration parameter $q = -a''(t) a(t)/a'(t)^2$
   1. The speed of expansion of the universe $a'(t)$. We find the derivative of the scale factor $a(t)$ in (42) as

$$a'(t)/a(1) = 4 (2x)^{-1/3} (3-x)^{3/3}$$  \hspace{1cm} (43)
Aspects of Today’s Cosmology

Fig. 2. The speed of expansion of the universe as in (43). There are two vertical asymptotes at \( x = 0 \) and at \( x = 3 \). They imply the initial inflation \( (x = 0) \) and the final disaggregation to infinity \( (x = 3) \) at about \( 4 \times 10^{10} \) years.

2. The Hubble parameter \( H = \frac{a'(t)}{a(t)} \). If we divide the expression (43) by the expression (42) we get for \( H \)

\[
H = \frac{2}{x(3-x)} \tag{44}
\]

The following figure 3 gives the graph of this expression:

Fig. 3. The Hubble parameter \( H \) in terms of age \( x \). We see again the initial inflation \( (x=0) \) and the final \( (x=3) \) disaggregation given by the two vertical asymptotes.

3. The acceleration of the expansion \( a''(t) \). Differentiating once more the expression (43) we get for the acceleration of the universe

\[
a'' = -\frac{8}{3} \left( \frac{1}{2x} \right)^{4/3} \left( \frac{1}{3-x} \right)^{5/3} + \frac{20}{3} \left( \frac{1}{2x} \right)^{1/3} \left( \frac{1}{3-x} \right)^{8/3} \tag{45}
\]
Fig. 4. The acceleration of the expansion of the universe is seen here again with two vertical asymptotes. Close to the origin the negative acceleration suggests the action of gravitation balancing the inflation phase. After half of the present age of the universe we see a positive acceleration, growing, and due to the pushing force that grows with the increasing size of the universe.

4. The deceleration parameter

\[ q = \frac{-a''(t) a(t)}{(a'(t))^2} = 0.5 - x \]  (46)

Fig. 5. The deceleration parameter. Using the expressions (42, 43) and (45) that define \( q \) gives back the original function assumed for \( q \) in (35).

9. Acknowledgement

I am thanking the owners of the Wolfram Mathematica Online Integrator that I have used to obtain the Fig. 1, 2, 3, 4 and 5 of this work.
10. References

This book presents some aspects of the cosmological scientific odyssey that started last century. The chapters vary with different particular works, giving a versatile picture. It is the result of the work of many scientists in the field of cosmology, in accordance with their expertise and particular interests. Is a collection of different research papers produced by important scientists in the field of cosmology. A sample of the great deal of efforts made by the scientific community, trying to understand our universe. And it has many challenging subjects, like the possible doomsday to be confirmed by the next decade of experimentation. May be we are now half way in the life of the universe. Many more challenging subjects are not present here: they will be the result of further future work. Among them, we have the possibility of cyclic universes, and the evidence for the existence of a previous universe.

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