Semi-Active Control of Civil Structures Based on the Prediction of the Structural Response: Integrated Design Approach

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1. Introduction

Various methodologies for vibration control of civil structures have been proposed so far. The traditional scheme is passive vibration control, i.e., dissipation of the vibration energy to the outside of the structural systems with dampers or mass dampers etc.. Passive control is quite simple and popular still, however it has some limitations, e.g., insufficient performance and/or difficulty in tuning such devices for a case of multi-mode vibration control etc.. Active vibration control is a candidate for a breakthrough to overcome the above drawbacks of passive control and has been studied extensively these decades ((Spencer et al., 1998) and the references therein). Although many studies show that the active control methodology achieves the quite high control performance on vibration suppression, it requires a large energy source to produce the control force and this fact has been an obstacle in applying active methods to general vibration control problems.

Semi-active control, which is not necessarily new (Karnop et al., 1974) either, can be recognized as an intermediate between passive and active schemes in the sense of not only the performance on vibration suppression but also the complexity of the control system. In most semi-active control vibration suppression is achieved by changing the damping coefficient of the semi-active damper. In civil structures semi-active control technique is getting more realistic recently (Casciati et al., 2006) along with the development of a large scale damper whose damping property is able to be changed (Sodeyama et al., 1997). In semi-active control, the damping coefficient of the semi-active damper is changed mainly based on the following two strategies:

• An active controller is synthesized with a standard control theory (e.g., sky-hook, LQ or $\mathcal{H}_\infty$ control, etc.) firstly as a target for the semi-active control. Then the semi-active damper is operated so as to emulate the targeted active control force as much as possible(Dyke et al., 1996; Karnop et al., 1974; Kurata et al., 1999).

• The damping coefficient of the semi-active damper is determined so as to maximally accelerate energy dissipation of the structural systems(Gavin, 2001).

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The latter method is based on the Lyapunov theory. The Lyapunov function has been defined as the total energy, i.e., the sum of the kinetic and potential energies of the structural system in most cases. The control law for maximizing the rate of energy dissipation is generally given as a bang-bang type change of the damping coefficient.

In this chapter we propose a new semi-active control method that is based on a one-step-ahead prediction of the structural response for the seismic disturbance. In the present semi-active control the vibration control device (VCD), which has been developed by the authors to reduce the structural vibration, is employed. The VCD is a mechanism consists of a ball screw, a flywheel and an electric motor connected to the ball screw. The VCD is installed between neighboring two floors of buildings as general dampers. Unlike general dampers, the VCD generates two types of resistance forces, i.e., a damping force proportional to the relative velocity and an inertial force proportional to the relative acceleration between two floors. The damping force is generated by the electric motor and the inertial force is produced by the mechanism of the ball screw and the flywheel. The damping coefficient of the VCD can be adjusted in a real-time manner by changing the electric resistance connected to the motor and we use this capability of the VCD for the semi-active control.

We assume the command signal for changing the damping coefficient of each VCD takes two values, i.e., the command to take the maximum or minimum damping coefficient in the present chapter. Under the assumption we obtain the optimal command signal among all possible combinations of command signals \(2^n_{\text{VCD}}\), \(n_{\text{VCD}}\): The number of VCDs) in a real-time manner. The optimal command signal is selected from all candidates of the command signals so that the weighted Euclidean norm of the predicted one-step-ahead structural response, calculated by a numerical integration method, e.g., Runge-Kutta method, is minimized.

To accomplish the further performance improvement of the semi-active control system an integrated design approach of structural and semi-active control systems using the above predictive control law is proposed. The stiffness distribution between neighboring two floors of the structure and some parameters of the VCDs are taken as adjustable structural design parameters. The weighting matrix that is used in the predictive semi-active control law is defined as the control design parameters. Those design parameters are simultaneously optimized so that the seismic responses of the semi-active control system subject to various recorded and artificial earthquake waves become good. The control performance is evaluated by an objective function that is calculated by the simulated structural response of the semi-active control system. The Genetic Algorithm (GA) is adopted for the optimization.

As a design example the a simulation study for a fifteen-story building with three VCDs is presented. The control performance of the semi-active control system is significantly improved by the proposed integrated design methodology.

The rest of the chapter is organized as follows: In §2 the principle of the VCD and the mathematical model of the civil structure with the VCD are introduced. The semi-active control law based on the prediction of the structural response is given in §3. The integrated design problem of the structural and the semi-active control is formulated in §4. The simulation results for a fifteen-story building are shown in §5 and the conclusion of the chapter is presented in §6.

Notations are as follows: \(t\): time, \(\mathbb{R}^{m \times n}\): the set of \(m \times n\) real matrices, \(\mathbb{R}^m\): the set of \(m\)-dimensional real vectors, \(\mathbb{S}^n\): the set of \(n\)-dimensional real symmetric matrices, \(0_{m \times n}\): an \(m \times n\) zero matrix, \(I\): an identity matrix with an appropriate dimension, \(1_{m \times n}\): an \(m \times n\) matrix whose all elements equal to 1, \(T\): the transposition of a vector or a matrix.
2. Mathematical model

2.1 Vibration Control Device (VCD)

The schematic diagram of the VCD is shown in Fig. 1. The VCD is installed between neighboring two floors of a structure, e.g., the $i$-th and $(i-1)$-th floors. In Fig. 1, $q_i(t)$, $m_i^{\text{VCD}}$ and $d_i^{\text{VCD}}(t)$ are the displacement of the $i$-th floor, an equivalent mass and a time-varying damping coefficient of the VCD, $j$-th VCD, $j = 1, \ldots, n_{\text{VCD}}$ respectively. The resistance force produced by the relative motion of the two floors is denoted by $f_j^{\text{VCD}}(t)$. The VCD is a dynamic system which has two input signals, i.e., the relative acceleration and velocity between two floors. The output signal of the VCD is the resistance force $f_j^{\text{VCD}}(t)$. The mathematical model of the VCD is described as the following:

$$f_j^{\text{VCD}}(t) = m_j^{\text{VCD}}(\ddot{q}_i(t) - \ddot{q}_{i-1}(t)) + d_j^{\text{VCD}}(t)(\dot{q}_i(t) - \dot{q}_{i-1}(t))$$  \hspace{1cm} (1)

The VCD whose resistance force property is given as Eq. (1) has been developed by the authors in recent years (Ohtake et al., 2006). The assembly and the design parameters of the VCD in (Ohtake et al., 2006) are shown in Fig. 2 and Table 1 respectively. The VCD is a mechanism consists of a ball screw, a flywheel, an electric motor and an electric circuit to control the damping property of the VCD. The ball screw and the flywheel produce a large inertia force (the term $m_j^{\text{VCD}}(\ddot{q}_i(t) - \ddot{q}_{i-1}(t))$ in Eq. (1)) which is proportional to the relative acceleration between two floors. The damping force denoted by $d_j^{\text{VCD}}(t)(\dot{q}_i(t) - \dot{q}_{i-1}(t))$ in Eq. (1) is generated by the electric motor (generator) and the electric circuit for the energy dissipation. The coefficients $m_j^{\text{VCD}}$ and $d_j^{\text{VCD}}(t)$ in Eq. (1) are given as (Ohtake et al., 2006):

$$m_j^{\text{VCD}} = K_s(I_1 + r^2 I_2), \quad d_j^{\text{VCD}}(t) = K_s \frac{r^2 K_f K_e}{R_a + R},$$  \hspace{1cm} (2)

where $K_s$, $r$, $I_1$ and $I_2$ are the constant related to the ball screw, the gear ratio, the moment of inertia of the flywheel and that of the electric motor respectively. In the damping coefficient $d_j^{\text{VCD}}(t)$, $K_s$, $K_e$, $R_a$ and $R$ are the torque constant, the back-emf (back electromotive force) constant, the electric resistance of the motor and the resistance connected to the motor terminal respectively. From Eq. (2) the damping coefficient of the VCD can be controlled by changing the resistance $R$. We have developed an electronic circuit to change the electronic resistance according to a command voltage that is produced by an implemented semi-active control law. More detailed dynamical properties of the VCD are presented in (Ohtake et al., 2006) and we are currently developing the VCD that can produce a larger resistance force.

2.2 Model of civil structures with VCD

Consider a civil structure with the $n_{\text{VCD}}$ VCDs, whose inertial and variable damping coefficients are denoted by $m_j^{\text{VCD}}$ and $d_j^{\text{VCD}}(t)$ ($j = 1, \ldots, n_{\text{VCD}}$) respectively. The equation of motion of the structure is given as

$$M\ddot{q}(t) + D(t)\dot{q}(t) + Kq(t) = b_2\dot{w}(t) + b_1(t)w(t) + b_0w(t),$$  \hspace{1cm} (3)

$$M = M^0 + M^{\text{VCD}}, \quad D(t) = D^0 + D^{\text{VCD}}(t), \quad b_1(t) = b_1^0 + b_1^{\text{VCD}}(t), \quad b_2 = b_2^0 + b_2^{\text{VCD}},$$  \hspace{1cm} (4)

where $q(t) \in \mathbb{R}^n$ and $w(t) \in \mathbb{R}^{n_w}$ are the displacement and the disturbance vectors of the structure respectively. Matrices $M, D, K \in S^n$ and $b_j \in \mathbb{R}^{n \times n_w}$ ($j = 0, 1, 2$) are the
Table 1. Design parameters of the VCD (Ohtake et al., 2006)

<table>
<thead>
<tr>
<th>Design parameters</th>
<th>Value [Unit]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>0.699 [m]</td>
</tr>
<tr>
<td>Weight</td>
<td>16.8 [kg]</td>
</tr>
<tr>
<td>Maximum load</td>
<td>30 [kN]</td>
</tr>
<tr>
<td>Stroke</td>
<td>±6.0 × 10⁻² [m]</td>
</tr>
</tbody>
</table>

mass, damping, stiffness and influence coefficient matrices of the structure with the VCDs respectively. In Eq. (4) the matrices $M^0$, $D^0$, $b^0_1$ and $b^0_2$ are the mass, damping and influence coefficient matrices of the original structure without VCDs and the matrices $M^{\text{VCD}}$, $D^{\text{VCD}}(t)$, $b^1_{\text{VCD}}(t)$ and $b^2_{\text{VCD}}$ are those of the installed VCDs respectively. Note that the matrices $D$ and $b_1$ accordingly become time varying because they contain the variable damping coefficient of the VCD.

With the equation of motion in Eq. (3) the state-space form of the structure with $n_{\text{VCD}}$ VCDs is given by

$$
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu_x(t), \\
z(t) &= C_zx(t) + D_zu_z(t),
\end{align*}
$$

where

$$
A := \begin{bmatrix}
0_{n \times n} & I_n \\
-M^{-1}K & -M^{-1}D(t)
\end{bmatrix},
B := \begin{bmatrix}
0_{n \times 3n_w} \\
I_n \\
-M^{-1}\left[b_0 b_1(t) b_2\right]
\end{bmatrix}.
$$

The vector $z(t) \in \mathbf{R}^{n_{\text{D}}}$ is the output vector to evaluate the performance of the structural system with VCDs. The output vector $z(t)$ will be used to construct the semi-active control law that will be proposed in the next section. Note that all coefficient matrices in Eq. (5) can become time varying because possibly they contain the variable damping coefficient of the VCDs, i.e., matrices $A$, $B$, $C_z$, $D_z$ are functions on the variable damping coefficient ($d^1_{\text{VCD}}(t)$, $j = 1, \ldots, n_{\text{VCD}}$) of the VCDs. In this chapter we assume each variable damping coefficient of the VCD can be controlled in a following range:

$$
d^1_{\text{VCD}}(t) \leq d^2_{\text{VCD}}(t) \leq d^3_{\text{VCD}}, \quad j = 1, \ldots, n_{\text{VCD}}
$$

where $d^1_{\text{VCD}} \geq 0$ and $d^2_{\text{VCD}} \geq 0$ are the maximum and minimum damping coefficients of the $j$-th VCD respectively.
3. Semi-active control methodology

We propose a semi-active control using the variable damping capability of the VCD. The control law is based on a prediction of the seismic response of the structure. The prediction of the structural response can be obtained through a numerical integration of the state-space model defined in Eq. (5) with the current sensor data of the state variable and the earthquake disturbance. The prediction of the structural response is obtained for all candidates of command signals each VCD. Then the optimal (combination of) command signal is selected among all candidates of the command signals in a real time manner. Essentially the command signal for the VCD can take a continuous value in the range given as Eq. (6). In this chapter, however, to make the prediction-based semi-active control realistic, the command signal for each VCD denoted by $d^c_j(t)$, $j = 1, \ldots, n_{VCD}$ is quantized to the following one bit signal:

$$d^c_j(t) = \begin{cases} d^VCD_j & \text{for } j = 1, \ldots, n_{VCD} \end{cases}$$

With the above quantization the possible number of command combinations for $n_{VCD}$ VCDs becomes $2^{n_{VCD}}$. In the present chapter the optimal combination is selected among all possible combinations so that the following performance index $J_p$ is minimized:

$$J_p := \sqrt{z^T(t_0 + \Delta t)Q_w z(t_0 + \Delta t)}$$

where $t_0$ and $\Delta t > 0$ are the current time instant and the length of the prediction respectively. The matrix $Q_w = Q^T_w \geq 0$ is an weighting matrix, defined by a control system designer, for evaluating the predicted control performance.

From Eq. (5) the prediction of the output vector $z(t_0)$ is given as

$$z(t_0 + \Delta t) = C_z x(t_0 + \Delta t) + D_z u_z(t_0 + \Delta t),$$

where the prediction of the state vector $x(t_0 + \Delta t)$ and the output vector $z(t_0 + \Delta t)$ are approximated by

$$x(t_0 + \Delta t) \approx x(t_0) + \Delta t \dot{x}(t_0), \quad u_z(t_0 + \Delta t) \approx u_z(t_0) + \Delta t \dot{u}_z(t_0).$$

To have the (approximated) predicted output vector $z(t_0 + \Delta t)$ we need to obtain $\dot{x}(t_0)$ and $\dot{u}_z(t_0)$. The time derivative of the state vector can be obtained with the current state vector $x(t_0)$ and the disturbance $u_x(t_0)$ at $t = t_0$ by using the state-space form Eq. (5), that is,

$$\dot{x}(t_0) = Ax(t_0) + Bu_x(t_0).$$

Fig. 2. Assembly of the VCD (Ohtake et al., 2006)
We can apply various numerical integration methods, e.g., Euler method or Runge-Kutta method etc., to obtain (the approximated) \( \dot{x}_0(t_0) \). Furthermore the time derivative of the disturbance \( u_z(t_0) \) can be obtained from the following simple relationship:

\[
\dot{u}_z(t_0) = \frac{d}{dt} \begin{bmatrix} w(t_0) \\ \dot{w}(t_0) \end{bmatrix} = \begin{bmatrix} \dot{w}(t_0) \\ \ddot{w}(t_0) \end{bmatrix}
\]  
(12)

In the present chapter all \( 2^{n_{\text{VCD}}} \) values of \( J_p 's \), the weighted Euclidean norm of the predicted output vector \( z(t_0 + \Delta t) \), are computed for all possible combinations of the command signals for the VCDs at every time instant. Then the optimal combination that minimizes \( J_p \) in Eq. (8) is selected among all \( 2^{n_{\text{VCD}}} \) candidates of command signals.

Similar to currently available devices that realize a variable damping property, such as MR damper etc., the VCD in the present chapter also has a dynamic delay for a given command signal. In other words the \( i \)-th damping coefficient of the VCD, denoted by \( d_{VCD}^i(t) \), cannot perfectly track the command signal \( d_c^i(t) \) defined as Eq. (7). We assume the characteristic of the delay is modeled as the following first order dynamics:

\[
\frac{d(d_{VCD}^j(t))}{dt} = -\frac{1}{T_d} d_{VCD}^j(t) + \frac{1}{T_d} d_c^j(t), \quad j = 1, \ldots, n_{\text{VCD}}
\]  
(13)

where \( T_d > 0 \) is the time constant representing the delay of the VCD.

Several features of the proposed prediction-based semi-active control are summarized as follows:

- In the semi-active control the predicted performance index \( J_p \) in Eq. (8) is obtained for all possible combinations of the command signals and the optimal command that achieves the minimum \( J_p \) is selected. As shown in §1 conventional semi-active control laws consider the norm of the closed-loop system (in the phase of the targeted active control design) or the energy dissipation rate of the structural systems. Such performance indices are used in general control system design (not only in semi-active control design) as a measure for evaluating the closed-loop performance. However in the semi-active control of civil structures we can not know if a satisfactory result is obtained or not with the adopted semi-active control in the sense of the vibration control of civil structures when we employ one of the conventional semi-active control methods because a mismatch exists between the above performance index and key performance measures used in vibration control of civil structures, e.g., the relative displacement between neighboring two floors and the absolute acceleration of each floor. In other words there is a gap between the performance index for the control system design and the quantities for the performance evaluation in a semi-active control of civil structural systems. On the other hand the performance index \( J_p \) is the Euclidean norm of the predicted output vector \( z(t) \) in Eq. (5). The elements of the vector \( z(t) \) can be defined as the relative displacement and velocity between neighboring two floors and the absolute displacement and velocity of each floor, etc. (Note: Clearly, there is a strong relationship between the absolute acceleration and the displacement or velocity of each floor.). Therefore the gap mentioned above is small in the proposed control method. By comparing the proposed prediction-based approach with the conventional semi-active control laws in §1 the prediction-based method is a more direct method because the performance measure on vibration suppression of the civil structural systems can be directly dealt with to obtain the optimal command for the VCDs in a real-time manner.
• The proposed semi-active control is quite flexible because it is a fully algorithm-based control law. As the most noteworthy example of the flexibility we show that a robust performance property within the present semi-active control framework can be recovered easily with a simple method that is shown as follows. In general model-based semi-active control methods including the proposed prediction-based approach the performance of the control system depends on the accuracy of the model of the control object. Because the perfect mathematical model of the control object is generally unavailable we need to make the control law to be robust in some sense. In the present semi-active control the predicted performance index \( J_p \) in Eq. (8) is obtained for all \( 2^{n_{\text{VCD}}} \) candidates of the command signals and the optimal command that achieves the minimum \( J_p \) is selected. Let \( J_{p}^{\text{max}} \) denote the predicted performance index obtained by setting the command signals for all VCDs to their maximum values, i.e., \( d_i^c(t) = d_i^{\text{VCD}}, \ i = 1, \ldots, n_{\text{VCD}} \). As a simple method to recover the robustness we redefine the performance index \( J_{p}^{\text{max}} \) as \( \alpha J_{p}^{\text{max}} \), \( 0 \leq \alpha < 1 \) and then the optimal command signal is selected in the same way. After the redefinition the command signal corresponding to (the redefined) \( J_{p}^{\text{max}} \) tends to be selected more often because the command \( d_i^c(t) = d_i^{\text{VCD}}, \ \forall \ i = 1, \ldots, n_{\text{VCD}} \) is overvalued among all candidates of the optimal command signals. By making \( 0 \leq \alpha < 1 \) smaller, the performance of the semi-active control approaches asymptotically to the case where all damping coefficients are kept their maximum. Actually in \( \alpha = 0 \) the semi-active control is coincident with the passive control where all damping coefficients are fixed their maximum because the command signal \( d_i^c(t) = d_i^{\text{VCD}}, \ \forall \ i = 1, \ldots, n_{\text{VCD}} \) is always selected during the semi-active control as a result. The above methodology is reasonable because the maximum damping case guarantees a certain level of vibration suppression independent of the mathematical model of the structural system although the resulted performance of the semi-active control becomes conservative. Note that in general control design there is a trade-off between the performance and the robustness like this situation. As the authors’ knowledge, among so many studies about semi-active control of civil structures classified in §1 there are no theoretical considerations about the robustness of the semi-active control system although all model-based semi-active control laws highly rely on the mathematical model of the structural systems. The authors would like to emphasize the fact that we should not expect the semi-active control methodologies aiming to realize the desired active control as much as possible, e.g., clipped optimal control (Dyke et al., 1996), have a performance robustness comparable with the targeted active control even if the targeted active control law is robustly designed in some sense. It is because the semi-active devices can not produce the same control force as the targeted active control perfectly. With the above discussion we can say that the present approach is simple but useful because the performance robustness of the present semi-active control can be easily recovered by making the factor \( \alpha \) smaller and the degree of the robustness also can be adjusted by tuning the parameter \( \alpha \).

• As other examples of the flexibility of the control law we present following two features:
  – A constraint on the hardware can be included quite easily, e.g., a constraint on the command signal such that all commands for VCDs must take the same value at some instant.
  – In the present study we use the Euclidean norm (2-norm) of the predicted output. Other norms such as 1-norm and \( \infty \)-norm also can be used.
• Additionally, with the bang-bang control nature of the method the implementation of the control law is relatively easier than general feedback control methods that require the continuous controller output.

4. Integrated design approach

Under the predictive semi-active control law in §3, an integrated design problem of structural and control systems is formulated. The integrated design problem is the control system design problem that the controller and the structural parameters, e.g., the stiffness and the damping characteristics of structural systems, are dealt with as adjustable design parameters.

The integrated design problem was firstly considered in the control system design of large space structures (LSS) in 1980’s and has been studied around this three decades in the control system and the structural design communities (Grogoriadis et al., 1996; Hiramoto et al., 2000; Hiramoto & Grigoriadis, 2006; Onoda & Haftka, 1987). In the integrated design paradigm we can expect better control performance than that of the standard structural or control design because we can adjust the design parameters simultaneously, i.e., we can use the full design freedom of the formulated design problem. However, we currently have the result that the general integrated design problem becomes a BMI (Bilinear Matrix Inequality) optimization problem that cannot be obtained the globally optimal solution with the practically acceptable amount of computation even in the simplest case. Several methods have been proposed so far to obtain a locally optimal solution (Grogoriadis et al., 1996; Hiramoto et al., 2000; Hiramoto & Grigoriadis, 2006; Onoda & Haftka, 1987).

In this chapter the integrated design problem of the civil structural system with the predictive semi-active control presented in the previous section is formulated. As the structural design parameters we take the structural stiffness between neighboring two floors denoted by $k_i$, $i = 1, \ldots, n$, the equivalent mass and the maximum value of the variable damping coefficient of each VCD that are denoted by $m_{jVCD}$ and $d_{jVCD}$, $j = 1, \ldots, n_{VCD}$ respectively. Each structural design parameter is able to be adjusted in the ranges as follows:

$$ (k_i)_l \leq k_i \leq (k_i)_u, \quad i = 1, \ldots, n \quad (14) $$

$$ (m_{jVCD})_l \leq m_{jVCD} \leq (m_{jVCD})_u, \quad j = 1, \ldots, n_{VCD} \quad (15) $$

$$ (d_{jVCD})_l \leq d_{jVCD} \leq (d_{jVCD})_u, \quad j = 1, \ldots, n_{VCD} \quad (16) $$

Note that subscripts $u$ and $l$ mean that the maximum and minimum values of the structural design parameters respectively.

The control design parameters are defined as $[Q_w]_{kk}$, $k = 1, \ldots, n_z$ that are the diagonal elements of the weighting matrix $Q_w$ in Eq. (8). The range of $[Q_w]_{kk}$, $k = 1, \ldots, n_z$ is given by

$$ 0 \leq ([Q_w]_{kk})_l \leq [Q_w]_{kk} \leq ([Q_w]_{kk})_u, \quad k = 1, \ldots, n_z. \quad (17) $$

The objective function in the present integrated design problem is defined as the following:

$$ J = \lambda J^s + (1 - \lambda) J^h, \quad 0 < \lambda < 1, \quad (18) $$

$$ J^s = \sum_{i=1}^{n_e} f_i^s, \quad f_i^s = \sum_{j=1}^{4} g^i (f_j^s)_j, \quad g^i \geq 0, \quad i = 1, \ldots, n_e, \quad j = 1, \ldots, 4, \quad (19) $$
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\[
(J_i^s)_1 = \sum_{k=1}^{n} \left( \frac{\text{RMS} \left( \frac{^s r_k^i (t)}{0 r_k^i (t)} \right)}{\text{RMS} \left( ^0 r_k^i (t) \right)} \right),
\]

(20)\[
(J_i^s)_2 = \sum_{k=1}^{n} \left( \frac{\text{RMS} \left( ^s a_k^i (t) \right)}{\text{RMS} \left( ^0 a_k^i (t) \right)} \right),
\]

(21)\[
(J_i^s)_3 = \sum_{k=1}^{n} \left( \frac{\max_{0 \leq t \leq t_f} \left| \frac{^s r_k^i (t)}{0 r_k^i (t)} \right|}{\max_{0 \leq t \leq t_f} \left| ^0 r_k^i (t) \right|} \right),
\]

(22)\[
(J_i^s)_4 = \sum_{k=1}^{n} \left( \frac{\max_{0 \leq t \leq t_f} \left| ^s a_k^i (t) \right|}{\max_{0 \leq t \leq t_f} \left| ^0 a_k^i (t) \right|} \right),
\]

(23)

\[
J^h = \sum_{i=1}^{n_e} J_i^h, \quad J_i^h = \sum_{j=1}^{h_i} \left( J_i^h \right)_j, \quad h_i \geq 0,
\]

(24)\[
(J_i^h)_1 = \sum_{k=1}^{n} \left( \frac{\text{RMS} \left( h r_k^i (t) \right)}{\text{RMS} \left( ^0 r_k^i (t) \right)} \right),
\]

(25)\[
(J_i^h)_2 = \sum_{k=1}^{n} \left( \frac{\text{RMS} \left( h a_k^i (t) \right)}{\text{RMS} \left( ^0 a_k^i (t) \right)} \right),
\]

(26)\[
(J_i^h)_3 = \sum_{k=1}^{n} \left( \frac{\max_{0 \leq t \leq t_f} | h r_k^i (t) |}{\max_{0 \leq t \leq t_f} | ^0 r_k^i (t) |} \right),
\]

(27)\[
(J_i^h)_4 = \sum_{k=1}^{n} \left( \frac{\max_{0 \leq t \leq t_f} | h a_k^i (t) |}{\max_{0 \leq t \leq t_f} | ^0 a_k^i (t) |} \right),
\]

(28)

where \( n_e \) and \( T_f^i, i = 1, \ldots, n_e \) are the number and the duration time of the \( i \)-th earthquake waves that are used to obtain the simulated structural response in the present integrated design respectively. The relative displacement between neighboring \( k \)-th and \((k - 1)\)-th floors and the absolute acceleration of \( k \)-th floor are denoted by \( r_k^i \) and \( a_k^i \), \( i = 1, \ldots, n_e, k = 1, \ldots, n \) respectively. Superscripts \( ^s, h \) and \( ^0 \) show the case of semi-active control, the case where the damping coefficient of all VCDs are kept their maximum throughout the earthquake event (Passive on) and the case without VCDs. The scalar \( 0 < \lambda < 1 \) and \( ^g_j^i, i = 1, \ldots, n_e \) are weighting factors that are determined by the designer.

The components \( J_i^s, i = 1, \ldots, n_e \) are the sum of ratios between the RMS or the peak values of structural responses \( (r_k^i, a_k^i, i = 1, \ldots, n_e, k = 1, \ldots, n) \) that are obtained respectively in the semi-active control case and the passive on case for the \( i \)-th earthquake wave. Hence, if we get a solution to the integrated design problem that the component \( J_i^s, i = 1, \ldots, n_e \) is small, the amount of the performance improvement is large when the predictive semi-active control law is applied.
On the other hand the components \( J_h^i, i = 1, \ldots, n_e \) are the sum of ratios between the RMS or the peak values of \( r_k^i \) and \( a_k^i, i = 1, \ldots, n_e, k = 1, \ldots, n \) in the passive on case and the case without VCDs. When the component \( J_h^i, i = 1, \ldots, n_e \) is small, the performance enhancement by installing VCDs (and keeping the damping coefficients maximum) is large.

The performance index \( J \) in Eq. (18) is the weighted sum of \( J^s \) representing the performance improvement with the semi-active control and \( J^h \) related to the effect of the VCD itself. By minimizing the performance index \( J \) we can obtain the structural and control design parameters that achieve the good control performance on vibration suppression.

The formulated integrated design problem that minimizes the objective function \( J \) in Eq. (18) is a highly nonlinear and non-convex optimization problem on structural and the control design parameters because of the form of the objective function itself and the nonlinear nature of the semi-active control law. Furthermore the number of the design parameters is relatively high in the formulated optimization problem compared to the standard structural or the control system design problem because we need to optimize not only the structural or control design parameters only but parameters both in the structural and control system simultaneously. Therefore it is difficult to apply gradient-based optimization methods that achieve a local convergence of the objective function \( J \) because the amount of computation to get the gradient of \( J \) on the structural and control design parameters become enormous. In the present chapter the Genetic Algorithm (GA) is applied to obtain an optimal solution. In GA the global optimality of the obtained solution is not guaranteed. However no gradient information is required to get the solution and many successful applications of GA have been reported so far for various types of optimization problems.

5. Design example

Let us consider a fifteen-story building with three VCDs shown in Fig. 3. The one-dimensional earthquake disturbance whose displacement is defined as \( w(t) \) is assumed in the present example. The mass of each floor \( m_i \) and the nominal inter-story stiffness \( k_{i,nom} \), \( i = 1, \ldots, 15 \) are shown in Table 2. In the present design example the structural damping matrix \( D_0 \) is defined as

\[
D_0 = \beta K, \quad \beta > 0, \quad (29)
\]

where the scaler \( \beta > 0 \) is determined so that the damping ratio of the 1st mode of vibration becomes 2\%. The nominal parameter values of the VCDs are determined as

\[
\begin{align*}
(m_{VCD})^{nom} &= 9.0 \times 10^6 \ [kg], \\
(d_{VCD})^{nom} &= 3.6 \times 10^7 \ [Ns/m], \\
(d_{VCD})^{nom} &= 0, \quad j = 1, 2, 3. \quad (30)
\end{align*}
\]

The maximum and minimum values of the structural design parameters in Eqs. (14)-(16) are defined as follows:

\[
\begin{align*}
(k_i)_l &= 0.5k_{i,nom}^{nom}, \quad (k_i)_u = 2k_{i,nom}^{nom}, \quad i = 1, \ldots, 15, \\
(m_{VCD})_l &= 0.5(m_{VCD})^{nom}, \quad (m_{VCD})_u = 2(m_{VCD})^{nom}, \\
(d_{VCD})_l &= 0.5(d_{VCD})^{nom}, \quad (d_{VCD})_u = 2(d_{VCD})^{nom}, \quad j = 1, 2, 3.
\end{align*}
\]
Fig. 3. Fifteen story building with 3-VCDs

<table>
<thead>
<tr>
<th>Story</th>
<th>$m_i \times 10^3$ [kg]</th>
<th>$k_{nom} \times 10^6$ [N/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>882</td>
<td>291</td>
</tr>
<tr>
<td>14</td>
<td>470</td>
<td>294</td>
</tr>
<tr>
<td>13</td>
<td>470</td>
<td>305</td>
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<td>470</td>
<td>313</td>
</tr>
<tr>
<td>11</td>
<td>470</td>
<td>352</td>
</tr>
<tr>
<td>10</td>
<td>480</td>
<td>412</td>
</tr>
<tr>
<td>9</td>
<td>485</td>
<td>427</td>
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<td>8</td>
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<tr>
<td>2</td>
<td>505</td>
<td>644</td>
</tr>
<tr>
<td>1</td>
<td>900</td>
<td>1098</td>
</tr>
</tbody>
</table>

Table 2. Nominal structural parameters of the fifteen story building ($i = 1, \ldots, 15$)

The control output vector $\mathbf{z}(t)$ in Eq. (5) is defined as

$$
\mathbf{z}(t) = 
\begin{bmatrix}
\mathbf{r}(t) \\
\dot{\mathbf{r}}(t) \\
\mathbf{q}(t) \\
\dot{\mathbf{q}}(t)
\end{bmatrix}
= \mathbf{C}_z \mathbf{x}(t) + \mathbf{D}_z \mathbf{u}_z(t),
$$

(35)
The time constant $T_d$ in Eq. (13) to model the delay of the VCD for the command signal $d_l(t)$, $j = 1, 2, 3$ is defined as $T_d = 0.02$ [s].

From the definition of $z(t)$ in Eq. (35) the weighting matrix $Q_w$ in Eq. (8) has 60 diagonal elements. Because the number of elements is too many to conduct the GA-based optimization the matrix $Q_w$ is parameterized as

$$Q_w = \text{diag}(w_r^1 1_{1 \times 5}, w_r^m 1_{1 \times 5}, w_{dr}^1 1_{1 \times 5}, w_{dr}^m 1_{1 \times 5}, w_{d_q}^1 1_{1 \times 5}, w_{d_q}^m 1_{1 \times 5}),$$

where $w_r^l, w_r^m, w_{dr}^l, w_{dr}^m, w_{d_q}^l, w_{d_q}^m > 0$ are the weighting factors for the relative displacement and velocity for lower (1st-5th), middle (6th-10th) and higher (11th-15th) floors, and those for the absolute displacement and velocity for the floors divided like the above way respectively. With the parameterization the number of the control design parameters becomes 12. The maximum and minimum values of those weighting factors are determined to be 1 and 0 respectively.

To obtain the simulated structural responses that are used in the proposed GA-based optimal design we take four recorded or artificial earthquake waves ($n_e = 4$ in Eqs. (19) and (24)), i.e.,

<table>
<thead>
<tr>
<th>Design parameter [Unit]</th>
<th>Nominal</th>
<th>Optimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{VCD}^1$ [kg]</td>
<td>$9.00 \times 10^6$</td>
<td>$5.04 \times 10^6$</td>
</tr>
<tr>
<td>$m_{VCD}^2$ [kg]</td>
<td>$9.00 \times 10^6$</td>
<td>$4.78 \times 10^6$</td>
</tr>
<tr>
<td>$m_{VCD}^3$ [kg]</td>
<td>$9.00 \times 10^6$</td>
<td>$4.78 \times 10^6$</td>
</tr>
<tr>
<td>$\frac{d_{VCD}^1}{m}$ [Ns/m]</td>
<td>$3.60 \times 10^7$</td>
<td>$6.79 \times 10^7$</td>
</tr>
<tr>
<td>$\frac{d_{VCD}^2}{m}$ [Ns/m]</td>
<td>$3.60 \times 10^7$</td>
<td>$6.78 \times 10^7$</td>
</tr>
<tr>
<td>$\frac{d_{VCD}^3}{m}$ [Ns/m]</td>
<td>$3.60 \times 10^7$</td>
<td>$7.04 \times 10^7$</td>
</tr>
</tbody>
</table>

Table 3. Nominal and optimal design parameters of VCDs

where $q(t)$ and $r(t)$ are the vector of the absolute displacement of each floor and the relative displacement between neighboring two floors given as

$$q(t) = \begin{bmatrix} q_1(t) \\ q_2(t) \\ \vdots \\ q_{15}(t) \end{bmatrix}, \quad r(t) = \begin{bmatrix} r_1(t) \\ r_2(t) \\ \vdots \\ r_{15}(t) \end{bmatrix}.$$

Then coefficient matrices $C_z$ and $D_z$ in Eq. (5) are given by

$$C_z = \begin{bmatrix} \mathbf{R} & 0_{15 \times 15} \\ 0_{15 \times 15} & \mathbf{R} \\ \mathbf{I}_{30} \end{bmatrix}, \quad D_z = \begin{bmatrix} \mathbf{s} & 0_{15 \times 1} \\ 0_{15 \times 1} & \mathbf{s} \\ 0_{30 \times 30} \end{bmatrix}.$$

where

$$\mathbf{R} = \begin{bmatrix} 1 & 0 & 0 & 0 & \ldots & 0 \\ 0 & 1 & -1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & -1 & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \ddots \\ 0 & \ldots & \ldots & \ldots & \ldots & 1 & -1 \end{bmatrix}, \quad \mathbf{s} = \begin{bmatrix} -1 \\ 0_{14 \times 1} \end{bmatrix}.$$
El Centro NS (1940) \((i = 1)\), BCJL1 (artificial) \((i = 2)\), Hachinohe NS (1968) \((i = 3)\) and JMA kobe NS (1995) \((i = 4)\) waves. All earthquake waves are scaled such that the peak ground acceleration (PGA) becomes 4.0 \([\text{m/s}^2]\).

The optimal structural and control design parameters are obtained by minimizing the objective function \(J\) in Eq. (18) with GA. The optimized design parameters are shown in Figs. 4, 5 and Table 3. To compare the performance on vibration suppression between the optimal semi-active control system obtained by the present integrated design approach and some other cases, structural responses are obtained for the scaled BCJL2 (artificial, PGA: 4.0 \([\text{m/s}^2]\)) earthquake wave\(^1\) shown in Fig. 6 for seven cases given as follows:

- **NC (Nominal structure):** The case of the nominal structural design parameters without VCD
- **NC (Optimized structure):** The case of the optimized structural design parameters without VCD
- **Pon (Nominal structure):** The passive on case with the nominal structural design parameters
- **Pon (Optimized structure):** The passive on case with the optimized structural design parameters
- **SA (Nominal structure):** The semi-active control case with the nominal structural and control design parameters
- **SA (Optimized structure):** The semi-active control case with the optimized structural and control design parameters
- **Energy-based SA (Optimized structure):** The case of the semi-active control for the maximum energy dissipation(Gavin, 2001) with the optimized structural design parameters

The time histories of the structural responses \(r_{15}(t)\) (the relative displacement between the 15th and 14th floors) and \(a_{15}(t)\) (the absolute acceleration of the 15th floor) for the optimized structure are presented in Fig. 7. By comparing between the case NC, Pon and SA the structural responses are suppressed by introducing the VCDs. It is also clear that the further performance improvement is achieved with the proposed predictive semi-active control by comparing the responses of the case Pon and SA. The time histories of the variable damping coefficients of the three VCDs are also shown in Fig. 8. The damping coefficients of the VCDs are frequently changed between their maximum and minimum values by the predictive semi-active control. The intermediate values between the maximum and minimum values of the variable damping coefficients come from the delay of the VCDs modeled as Eq. (13).

To make a more quantitative evaluation of the control performance the RMS and the peak values of the relative displacement between neighboring two floors and the absolute acceleration of each floor for all the above seven cases are shown in Figs. 9-12. The results of the integrated design for the fifteen story building are summarized as follows:

1. By introducing VCDs the structural response for the earthquake disturbance is highly suppressed both in the nominal and the optimized structures. This result shows that the effectiveness of the VCD as the vibration control device for civil structures.

\(^1\) Note that this earthquake wave is not employed in the GA-based optimization.
Fig. 4. Nominal and optimal stiffness parameters $k_k$, $k = 1, \ldots, 15$

2. In the case Pon and the case SA the relative displacement $r_k$, $k = 1, \ldots, 15$ and the absolute acceleration $a_k$, $k = 1, \ldots, 15$ of the optimized structure are significantly improved compared to those of nominal structure especially in middle and higher floors.

3. In lower floors (1-3F) the values of the optimized stiffness $k_k$ becomes much lower than those of the nominal values while the stiffness in upper floors are increased. The result implies that so called “soft-story” type structure (Iwata et al., 1999), that aims at concentrating the structural deformation in lower floors by adopting small stiffness $k_k$ and moreover suppressing the vibration and the deformation in higher floors by adopting large stiffness $k_k$, is obtained as the result of the integrated design. Hence the relative displacement in lower floors becomes larger in the case NC for the optimized structure than that of the nominal structure. However, we can see that the large amount of the relative displacement in the lower floors of the optimized structure are well suppressed by the VCDs with the proposed predictive semi-active control.

4. From Figs. 9-12 the optimized semi-active control system (SA (Opt. str.)) shows the better performance on vibration suppression compared to that of the semi-active control proposed by (Gavin, 2001) (Energy-based SA (Opt. str.))². Especially the fairly better control performance on the absolute acceleration of each floor is achieved with the predictive semi-active control compared to that achieved with the he semi-active control law proposed in (Gavin, 2001). This result indicates that the importance of the proposed integrated design approach, i.e., the design of the structural system (the control object) with taking the employed control law into consideration.

² In the semi-active control law in (Gavin, 2001) the command signal for the VCD also becomes the bang-bang type that is same as the present predictive semi-active control.
Fig. 5. Optimal control design parameters

Fig. 6. BCJL2 wave (PGA=4.0 [m/s²])
Fig. 7. Time histories of $r_{15}$ and $a_{15}$ for BCJL2 wave (Optimized structure, PGA=4.0 [m/s$^2$])

Fig. 8. Time histories of the variable damping coefficients of VCDs for BCJL2 wave (Optimized structure PGA=4.0 [m/s$^2$])
Fig. 9. RMS values of the relative displacement $\text{RMS}(r_k)$, $k = 1, \ldots, 15$ (BCJL2 wave)
Fig. 10. RMS values of the absolute acceleration $\text{RMS}(a_k)$, $k = 1, \ldots, 15$ (BCJL2 wave)
Fig. 11. Peak values of the relative displacement $\max |r_k|$, $k = 1, \ldots, 15$ (BCJL2 wave)
Fig. 12. Peak values of the absolute acceleration max $|a_k|$, $k = 1, \ldots, 15$ (BCJL2 wave)
6. Conclusion

In this chapter the integrated design of civil structural systems and the semi-active control law is considered. The vibration control device (VCD) that is under development by the authors is adopted for the semi-active control. The VCD is the mechanism consists of the ball-screw and the flywheel that is for the inertial resistance force and the electric motor and the electric circuit for the damping resistance force with the variable damping coefficient. The semi-active control based on the one-step ahead prediction of the structural response is proposed. With the predictive semi-active control the stiffness property of the building, the parameters of VCDs and the weighting matrix used in the semi-active control are simultaneously optimized so that the control performance on vibration suppression for various recorded and artificial earthquake disturbances is optimized. The Genetic Algorithm is adopted for the optimization. The simulation study is conducted for the fifteen story building. The performance on vibration suppression of the semi-active control system obtained by the integrated design method is verified with the earthquake wave that is not employed in the GA-based optimization. The result of the simulation study shows the effectiveness of the proposed design methodology and the importance of the integrated design approach for control system design including semi-active control.

The future research subjects are summarized as follows:

- Integrated design of the semi-active control system including the optimization of the location of the VCDs
- Semi-active control for a simplified model of the real structural system
- Experimental study using a full-scale building with VCDs

7. References


This book focuses on the important and diverse field of vibration analysis and control. It is written by experts from the international scientific community and covers a wide range of research topics related to design methodologies of passive, semi-active and active vibration control schemes, vehicle suspension systems, vibration control devices, fault detection, finite element analysis and other recent applications and studies of this fascinating field of vibration analysis and control. The book is addressed to researchers and practitioners of this field, as well as undergraduate and postgraduate students and other experts and newcomers seeking more information about the state of the art, challenging open problems, innovative solution proposals and new trends and developments in this area.

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