1. Introduction

For some socioeconomic systems, it is desirable to provide real-time information or even a short-term forecast about dynamics. For instance, in stock markets it is advantageous to give a reliable forecast in order to maximize profit. In traffic flow, advanced traveler information systems (ATIS) provide real-time information about the traffic conditions to road users by means of communication such as variable message signs, radio broadcasts, or on-board computers (Adler & Blue, 1998). The aim is to help individual road users to minimize their personal travel time. Therefore traffic congestion should be alleviated, and the capacity of the existing infrastructure could be used more efficiently. Fig. 1 shows a schematic diagram of an information feedback system, which demonstrates that feedback information plays a significant role in the loop.

An Information Feedback System

Data Feedback

Data Generation → Data Collection → Data Processing

Fig. 1. The schematic diagram of an information feedback system.
Physics, other sciences and technologies meet at the frontier area of interdisciplinary research (Helbing, 1996; Chowdhury et al., 2000; Helbing, 2001; Nagatani, 2002). The concepts and techniques of physics are being applied to such complex systems as traffic systems. A lot of theories have been proposed such as car-following theory (Rothery, 1992), kinetic theory (Prigogine & Andrews, 1960; Paveri-Fontana, 1975; Helbing & Treiber, 1998) and particle-hopping theory (Nagel & Schreckenberg, 1992; Biham et al., 1992; Blue & Adler, 2001). These theories provide insights that help traffic engineers and other professionals to better manage congestion. Therefore these theories indirectly make contributions to alleviating traffic congestion and enhancing the capacity of existing infrastructure. Although dynamics of traffic flow with real-time traffic information have been extensively investigated (Friesz et al., 1989; Arnott et al., 1991; Ben-Akiva et al., 1991; Mahmassani & Jayakrishnan, 1991; Kachroo & Özbay, 1996; Yokoya, 2004), finding out a more efficient feedback strategy is still an overall task. Recently, some information feedbacks have been proposed to investigate the two-route scenario with the same length. Wahle et al. (2000 & 2002) firstly investigated the two-route scenario with travel time feedback strategy (TTFS). Subsequently, Lee et al. (2001) studied the effect of a different type of information feedback (MVFS), i.e. instantaneous average velocity. Wang et al. (2005) proposed a third type of information feedback (CCFS), i.e. instantaneous congestion coefficient which is defined as

\[ C = \sum_{i=1}^{q} n_i^2. \]  

(1)

where, \( n_i \) stands for vehicle number of the \( i \)th congestion cluster in which cars are close to each other without a gap between any two of them; \( q \) is the number of congestion clusters on the route. Then Dong et al. (2010b) put forward another type of information feedback (WCCFS), i.e. instantaneous weighted congestion coefficient which is defined as

\[ C_w = \sum_{i=1}^{p} F(n_m) n_i^2. \]  

(2)

where the definition of \( n_i \) is the same as above, \( F(n_m) \) is the weight function, and \( n_m \) stands for the position of the \( i \)th congestion cluster. Here, we use the result of median rounding \( \lfloor n_m \rfloor \) of the \( i \)th congestion cluster to represent its position. Furthermore, in order to provide road users with better guidance, Dong et al. (2009a; 2009b; 2010a; 2010d) proposed another two types of information feedback strategies named corresponding angle feedback strategy (CAFS) and prediction feedback strategy (PFS), respectively. The corresponding angle coefficient is defined as

\[ C_{\theta} = \sum_{i=1}^{q} \theta_i^2 = \sum_{i=1}^{q} \left( \arctan \left( \frac{n_{i,\text{first}}}{H} \right) - \arctan \left( \frac{n_{i,\text{first}} - l_i}{H} \right) \right)^2. \]  

(3)

where \( n_{i,\text{first}} \) stands for the position of the first vehicle in the \( i \)th congestion cluster, in which vehicles are close to each other without a gap between any two of them. \( l_i \) and \( \theta_i \) denote the length and the weight (corresponding angle) of the \( i \)th congestion cluster, respectively. \( H \) denotes the vertical distance from point \( T \) to the route (see Fig.2). In this chapter, we set \( H = 100 \). PFS is based on CCFS, and the predicted congestion coefficient (\( C_p \)) is defined as:

\[ C_p(t) = C(t + \Delta t) = \sum_{i=1}^{q} n_i^2(t + \Delta t). \]  

(4)
which indicates that PFS uses future road condition (the value of congestion coefficient $C$ at time $t + \Delta t$) as feedback information.

It has been proved that TTFS is the worst one which brings a lag effect to make it impossible to provide the road users with the real situation of each route (Lee et al., 2001); CCFS is more efficient than MVFS because the random brake mechanism of the Nagel-Schreckenberg (NS) model (Nagel & Schreckenberg, 1992) brings fragile stability of velocity (Wang et al., 2005); WCCFS is more efficient than CCFS for the reason that CCFS does not take the weights of different parts of the route into consideration (Dong et al., 2010b). However, WCCFS is still not the best one due to the fact that the weight function $F(n_m)$ does not contain any information related to the length of the congestion cluster while the corresponding angle $\theta$ takes both the length and route location of each cluster into account (Dong & Ma, 2010a). Though the road capacity adopting PFS is the best one among these feedback strategies (Dong et al., 2009a; Dong et al., 2009b; Dong et al., 2010d), the validity of PFS depends on the length of the route when the traffic system is multi-route (Dong et al., 2010d). Also, PFS is not easy to realize since it needs predicted road data, which will be discussed in detail in the following paragraphs. We report the simulation results adopting six different feedback strategies TTFS, MVFS, CCFS, WCCFS, CAFS and PFS in a two-route scenario with a single route following the NS mechanism.

The chapter is arranged as follows: In Section 2, several cellular automaton models of traffic flow will be mentioned including some analytical studies and a two-route scenario is briefly introduced, together with six feedback strategies of TTFS, MVFS, CCFS, WCCFS, CAFS and PFS all depicted in detail. In Section 3, some simulation results will be presented and discussed based on the comparison of six different feedback strategies. In Section 4, we will make some conclusions.

2. The model and feedback strategies

A. NS mechanism

Recently, a lot of cellular automaton models are proposed such as NS model (Nagel & Schreckenberg, 1992), FI model (Fukui & Ishibashi, 1996), VDR model (Barlovic et al., 1998), VE model (Li et al., 2001), BL model (Knospe et al., 2000), VE model (Li et al., 2001), Kerner-Klenov-Wolf model (Kerner et al., 2002; Kerner & Klenov, 2004; Kerner, 2004), FMCD model (Jiang & Wu, 2003; Jiang & Wu, 2005), VDDR model (Hu et al., 2007), and VA model (Gao et al., 2007). Also, a lot of analytical works based on statistical physics, such as the
spacing-oriented mean field theory, have been studied to investigate fundamental diagrams and asymptotic behavior of CA models, i.e., NS model, FI model, and NS & FI combined CA model (Wang et al., 2000a; Wang et al., 2000b; Mao et al., 2003; Wang et al., 2003; Wang et al., 2001; Fu et al., 2007). Among these CA models, the NS model is so far the most popular and simplest cellular automaton model in analyzing the traffic flow (Nagel & Schreckenberg, 1992; Chowdhury et al., 2000; Helbing, 2001; Nagatani, 2002; Wang et al., 2002), where the one-dimension CA with periodic boundary conditions is used to investigate highway and urban traffic. This model can reproduce the basic features of real traffic like stop-and-go wave, phantom jams, and the phase transition on a fundamental diagram that plots vehicle flow versus density. Thus we still adopt NS model when comparing the effects of different feedback strategies in this chapter. In the following paragraphs, the NS mechanism will be briefly introduced as a basis of analysis.

The road is subdivided into cells (sites) with a length of $\Delta x=7.5$ m. The route length is set to be $L = 2000$ cells (corresponding to 15 km). $N$ denotes the total number of vehicles on a single route of length $L$. The vehicle density can be defined as $\rho = N/L$. A time step corresponds to $\Delta t = 1{s}$, the typical time a driver needs to react. $g_n(t)$ refers to the number of empty sites in front of the $n$th vehicle at time $t$, and $v_n(t)$ denotes the speed of the $n$th vehicle, i.e., the number of sites that the $n$th vehicle moves during the time step $t$. In the present paper, we set the maximum velocity $v_{\text{max}} = 3$ cells/time step (corresponding to 81 km/h and thus a reasonable value) for simplicity. The rules for updating the position $x$ of a car are as follows. (i) Acceleration: $v_i = \min(v_i + 1, v_{\text{max}})$. (ii) Deceleration: $v'_i = \min(v_i, g_i)$ so as to avoid collisions, where $g_i$ is the spacing in front of the $i$th vehicle. (iii) Random brake: with a certain probability $p$ that $v''_i = \max(v'_i - 1, 0)$. (iv) Movement: $x_i = x_i + v''_i$.

The fundamental diagram characterizes the basic properties of the NS model which has two regimes called "free-flow" phase and "jammed" phase. The critical density, basically depending on the random brake probability $p$, divides the fundamental diagram to these two phases. The transition such as from "free-flow" phase to "jammed" phase is called transition on a fundamental diagram (Nagel & Schreckenberg, 1992).

B. Two-route scenario

Recently, Wahle et al. (2000) investigated a two-route model. In their model, a percentage of drivers (referred to as dynamic drivers) choose one of the two routes according to the real-time information displayed on the roadside. In their model, the two routes $A$ and $B$ are of the same length $L$. A new vehicle will be generated at the entrance of the traffic system at each time step. If a driver is a so-called static one, he enters a route at random ignoring any advice. The density of dynamic and static travelers are $S_{\text{dyn}}$ and $1 - S_{\text{dyn}}$, respectively. Once a vehicle enters one of two routes, the motion of it will follow the dynamics of the NS model. In our simulation, a vehicle will be removed after it reaches the end point. It is important to note that if a vehicle cannot enter the preferred route, it will wait till the next time step rather than entering the un-preferred route.

The simulations are performed by the following steps: first, we set the routes and boards empty; second, let vehicles enter the routes randomly during the initial 100 time steps; third, after the vehicles enter the routes, according to four different feedback strategies, information will be generated, transmitted, and displayed on the board at each time step. Finally, the dynamic road users will choose the route with better conditions according to the dynamic information at the entrance of two routes.

C. Related definitions
The road conditions can be characterized by the fluxes of two routes. The flux of the $i$th route is defined as follows:

$$F_i = V_{\text{mean}}^i \rho_i = V_{\text{mean}}^i \frac{N_i}{L_i}$$

(5)

where $L_i$ represents the length of the $i$th route, $V_{\text{mean}}^i$ and $N_i$ denote the mean velocity of all the vehicles and the vehicle number on the $i$th route, respectively. In this chapter, the physical sense of flux $F$ is the number of vehicles passing the exit of the traffic system each time step. Therefore the larger the value of $F$, the better processing capacity the traffic system has.

We assume the two-route system has only one entrance and one exit as shown in Fig.3. In reality, there are different paths for drivers to choose from one place to another place. In this chapter, we focus on a two-route system. Different drivers departing from the same place could choose two different paths to get to the same destination which corresponds to the “one entrance and one exit” system. Thus the road condition in present work is closer to reality than some previous works (Wahle et al., 2000; Lee et al., 2001; Wang et al., 2005). The rules at the exit of the two-route system are as follows:

![Fig. 3. The one entrance and one exit two-route traffic system.](image)

a) The special velocity update mechanism for the vehicle nearest to the exit:
   i velocity(t)=Min(velocity(t)+1,3), (probability: 75%);
   ii velocity(t)=Max(velocity(t)-1,0), (probability: 25%);

b) Rules at the exit when vehicles competing for driving out:
   i At the end of two routes, the vehicle nearer to the exit goes first.
   ii If the vehicles at the end of two routes have the same distance to the exit, the faster a vehicle drives, the sooner it goes out.
   iii If the vehicles at the end of two routes have the same speed and distance to the exit, the vehicle in the route which has more vehicles drives out first.
   iv If the rules (i), (ii) and (iii) are satisfied at the same time, then the vehicles go out randomly.

c) velocity(t)=position(t)-position(t-1), where position(t)=L=2000; (valid only for the vehicles failed in competing for driving out at exit);

Here we want to stress that the vehicle nearest to the exit will not obey the NS mechanism but the special mechanism as shown in rule (a). However, vehicles following the vehicle closest to the exit still obey the NS mechanism. One should also be aware that if the vehicle nearest
to the exit does not compete with the vehicle on the other route for driving out or wins in the competition, the vehicle will ignore rule (c). The special velocity update mechanism (rule (a)) is equivalent to the situation that 75% drivers exhibit aggressive behavior and 25% drivers exhibit timid behavior near the exit, which is similar to the recent work studied by Laval & Leclercq (2010). Please note that drivers exhibit timid behavior may also exhibit aggressive behavior at next time step otherwise the timid drivers may stop at the exit all the time. Then we describe six different feedback strategies as follows:

TTFS: The information of travel time on the board is set to be zero until one car leave the traffic system. Each vehicle will be recorded the time when it enters and leaves one of the routes. We use the difference between this two values as the feedback information. A new dynamic driver will choose the road with shorter time shown on the information board.

MVFS: Every time step, the traffic control center will receive the velocity of each vehicle on the route from GPS. They will deal with the information and display the mean velocity of vehicles on each route on the information board. Road users at the entrance will choose one road with larger mean velocity.

CCFS: Every time step, the traffic control center will receive the position of each vehicle on the route from GPS. The work of the traffic control center is to compute the congestion coefficient of each route and display it on the information board. Road users at the entrance will choose one road with smaller congestion coefficient. The congestion coefficient is defined as

\[ C = \sum_{i=1}^{q} n_i w_i. \]  

where \( n_i \) stands for vehicle number of the \( i \)th congestion cluster in which cars are close to each other without a gap between any two of them, and \( q \) denotes the total number of congestion clusters on one route. Every cluster is evaluated by a weight \( w \), where \( w = 2 \) and one can check out that \( w > 2 \) leads to the similar results with \( w = 2 \) (Wang et al., 2005).

WCCFS: Every time step, the traffic control center will receive data from the navigation system (GPS) like CCFS, and the work of the center is to compute the congestion coefficient of each road with a reasonable weighted function and display it on the information board. Road users at the entrance will choose one road with smaller weighted congestion coefficient. The weighted congestion coefficient is defined as Eq.(2).

After we try some functions such as \( F(x) = \cos(ax) + b \) and Gaussian function, we find \( F(x) = kx + b \) is the optimal one in terms of improving the capacity of the road. Here, we set \( b \neq 0 \) for the reason that it will cause the absolute weight value of the first route site always to be the smallest when \( b = 0 \). In this chapter, we set \( b = 2.0 \). Then we get the function as follows:

\[ F(x) = k \times x + b = k \times \frac{n_m}{2000} + 2.0. \]  

Finally the expression of \( C_w \) becomes

\[ C_w = \sum_{i=1}^{q} F(n_m) n_i^2 = \sum_{i=1}^{q} (k \times \frac{n_m}{2000} + 2.0) \times n_i^2. \]  

We also find that how efficient the new strategy to improve the road capacity depends on the value of the weight factor (slope - \( k \)) which we will discuss in detail in Section 3.

CAFS: Every time step, the traffic control center will receive data from the navigation system (GPS) like CCFS. The work of the traffic control center is to compute the corresponding angle
of each congestion cluster (see Fig.2) on the lane, sum square of each corresponding angle up and display it on the information board. Road users at the entrance will choose one road with smaller corresponding angle coefficient. The corresponding angle coefficient is defined as Eq.(3).

PFS: Every time step, the traffic control center will receive data from the navigation system (GPS) like CCFs. The work of the center is to compute the congestion coefficient of each route, simulate the future road condition based on the current road condition by using CCFs, and display the results on the information board. Road users at the entrance will choose one route with smaller predicted congestion coefficient. For example, if the prediction time, \( T_p \), is 50 seconds and the current time is the 100th second, the traffic control center will simulate the road conditions in the next 50 seconds adopting CCFs, predict the road condition at the 150th second, and show the result on the information board at the entrance of the route. Finally, road users at the 100th second will choose one route with smaller predicted congestion coefficient at the 150th second. By the same token, road users at the entrance at the 101th second will choose one route with smaller predicted congestion coefficient at the 151th second like explained above. The predicted congestion coefficient is defined as Eq.(4).

In the following section, performance by using six different feedback strategies will be shown and discussed in detail.

3. Simulation results

Fig. 4 (a) shows the dependence of average flux on weight factor \((k)\) by using WCCFS. As to the routes’ processing capacity, we can see that in Fig.4 (a) there is a positive peak structure at the vicinity of \(k \sim -1.98\). Thus we will use \(k=-1.98\) in the following paragraphs. Then Eq.(8) will become \(C_w = \sum_{i=1}^{q} (-1.98 \times \frac{n_i}{2000} + 2.0) \times n_i^2\). In Fig.4 (b), we present the weight value of each site on one route. One can find the weight value of the entrance is much larger than that of the exit when using WCCFS and the reasons can be described as follows. First, in practice both acceleration and deceleration shock waves travel at final velocities and – at least on average – vehicles tend to be more affected by local traffic conditions than by conditions far ahead on the roadway. This suggests that greater weight should be given to traffic conditions on the upstream sections of each route. Second, the smaller weight value at the end of the route will alleviate the negative effect of congestion caused by the traffic jam.

Since Fig.4 (a) shows the weight value of the entrance is larger than that of the exit when adopting WCCFS, the point \(T\) located above the entrance of the route when adopting CAFS (see Fig.2) is reasonable. It makes the weight of the entrance the largest. Furthermore, the corresponding angle of each congestion cluster can reflect not only the weight of the route but also the length of the congestion cluster. Therefore the weight value is more reasonable than before (Dong et al., 2010a). Fig.4 (c) shows the dependence of average flux on position of the pillar (point \(T\)) by using CAFS. As to the routes’ processing capacity, we can see that the position of the pillar will directly affect the average flux. The average flux is much larger when point \(T\) locates at the entrance of the route while the value is pretty lower when point \(T\) locates at the end of the lane. Thus the result shown by Fig.4(c) is in accordance with that indicated by Fig.4 (a). Also, this can be understood as shown in Fig.5, where the congestion cluster on route \(A\) locates at the entrance of lane and the congestion cluster on route \(B\) locates at the end of the lane. From Fig.5, we can see clearly that \(C_\theta\) of route \(A\) is larger than that of route \(B\), so the road user should enter route \(B\) instead of route \(A\). If point \(T\) locate at the end of the route, \(C_\theta\) of route \(A\) will smaller than that of route \(B\), which will cause the vehicle to enter route \(A\). It will make the cluster larger or the vehicle even cannot enter the route.
Fig. 4. (a) Average flux vs weight factor ($k$). The parameters are $L=2000$, $p=0.25$, and $S_{dyn}=0.5$.
(b) Weight value of each site on one route. The parameters are $L=2000$, $p=0.25$, and $S_{dyn}=0.5$.
(c) Average flux vs position of pillar (point $T$). The parameters are $L=2000$, $p=0.25$, $S_{dyn}=1.0$ and vertical distance ($H$) is fixed to be 100.
(d) Average flux vs prediction time ($T_p$). The parameters are $L=2000$, $p=0.25$, and $S_{dyn}=0.5$.

Fig. 4 (d) shows the dependence of average flux on prediction time ($T_p$) by using PFS. As to the routes’ processing capacity, we can see that in Fig. 4 (d) there is a positive peak structure at the vicinity of $T_p \sim 60$. Thus we will use $T_p=60$ in the following paragraphs. PFS is more difficult to realize than other five feedback strategies since PFS is based on the future road condition. As demonstrated by our previous work (Dong et al., 2010), more routes the traffic system has, longer prediction time ($T_p$) PFS needs (see Fig. 6). The workload adopting CCFS is equivalent to the workload adopting PFS when $T_p = 0$ (which is equivalent to CCFS). Thus in a two-route scenario, the workload of PFS is sixty times greater than that of CCFS because the prediction time equal to 60 time steps ($T_p = 60$). Given the time interval of every time feedback is one time step, there is no doubt that PFS requires higher performance computers to operate than the other five feedback strategies. It indicates that PFS will cost more to realize than others.

Fig. 7 shows simulation results of applying TTFS in a two-route scenario with respect to flux, number of cars, and average speed all versus time step. The fluxes of two routes adopting TTFS show oscillation (see Fig. 7) obviously due to the information lag effect (Lee et al., 2001). This lag effect can be understood. For TTFS, the travel time reported by a driver at the end of two routes only represents the road condition in front of him, and perhaps the vehicles behind him have got into the jammed state. Unfortunately, this information will induce more vehicles
Fig. 5. The locations and corresponding angles of vehicle congestion clusters on route A and route B.

Fig. 6. (a) Average flux vs prediction time ($T_p$) in a three-route traffic system and $T_p(F_{\text{max}}) \sim 260$. (b) Average flux vs prediction time ($T_p$) in a four-route traffic system and $T_p(F_{\text{max}}) \sim 1020$. The parameters are $L=2000$, $p=0.25$, and $S_{\text{dyn}}=0.5$.

to choose his route until a vehicle from the jammed cluster leaves the system. This effect apparently does harm to the system. Another reason for the oscillation is that the two-route system only has one exit and the vehicle nearest to the exit obeys the special velocity update mechanism; therefore at most one vehicle can go out each time step. It will result in the traffic jam happening at the end of the route. Vehicle number versus time step shows almost the same tendency as flux versus time step (see Fig.7 (b)) and the average velocity is around 2.4 cells per time step (refer to Fig.7(c)).

Fig.8 shows simulation results of applying MVFS in a two-route scenario with respect to flux, vehicle number, and average speed all versus time step. The fluxes of two routes adopting
Fig. 7. (Color online) (a) Flux of each route with travel time feedback strategy. (b) Vehicle number of each route with travel time feedback strategy. (c) Average speed of each route with travel time feedback strategy. The parameters are $L=2000$, $p=0.25$ and $S_{dyn}=0.5$. 
MVFS showing oscillation (see Fig. 8) is primarily due to two reasons. First, for MVFS, we have mentioned that the NS model has a random brake scenario which causes the fragile stability of velocity, thus MVFS cannot completely reflect the real condition of routes. The other reason for the disadvantage of MVFS is that flux consists of two parts, mean velocity and vehicle density, but MVFS only grasps one part and lacks the other part of flux (Wang et al., 2005). Also, the one exit structure of the traffic system and the special velocity update mechanism for the vehicle closest to the exit can also cause oscillation as explained in the last paragraph.

Vehicle number versus time step shows almost the same tendency as flux versus time step (see Fig. 8(b)) and the average velocity is around 2.3 cells per time step (refer to Fig. 8(c)).

Fig. 9 show the dependence of flux, number of cars, average speed on time step by using CCFS. Compared with TTFS and MVFS, the performance of CCFS is good. The reason is primarily due to that CCFS takes the congestion cluster effects into account by adding a weight to each cluster. This can be explained by the fact that travel time of the last vehicle of the cluster from the entrance to the destination is obviously affected by the size of cluster. With the increasing of cluster size, travel time of the last vehicle will be longer, and the correlation between cluster size and travel time of the last vehicle is nonlinear. For simplicity, an exponent \( w \) is added to the size of each cluster to be consistent with the nonlinear relationship. To some extent, CCFS reduces the oscillation, and increases the vehicle number of each route (see Fig. 9(a) & (b)) while decreases the average velocity that is approximate to 2.2 cells per time step (refer to Fig. 9(c)).

The dependence of flux, number of vehicles, average speed on time step by adopting WCCFS is shown in Fig. 10. WCCFS further reduces the oscillation and increases the flux due to the fact that WCCFS takes the weights of different parts of the route into consideration. From Fig. 4(b) we can see that weight values at the end of the route are always smaller, which is equivalent to alleviating the negative effect of congestion caused by the traffic jam; therefore WCCFS may improve the road condition. Compared to CCFS, the performance adopting WCCFS is improved at some points, not only on the value but also the stability of the flux.

Fig. 11 shows the relationship between flux, number of vehicles, average speed and timestep by using CAFS. In contrast with CAFS, the fluxes of two routes adopting TTFS, MVFS, CCFS and WCCFS show larger oscillation (see Fig. 7-10). This oscillation effect can be understood for several reasons besides those discussed above. First, TTFS, CCFS and MVFS cannot reflect the weights of different parts of the route. Additionally, though WCCFS can reflect the route weights, the weighted function \( F(n_m) \) is independent of the cluster length. We use the median rounding \( n_m \) of the \( i \)th congestion cluster to represent its position when adopting WCCFS. However, CAFS takes both the length and location of the congestion cluster into account, which can give the road user with better guidance. For example, if there exit congestion clusters at the end of both routes, the road user will choose the route with shorter cluster length. The reason is that there is a positive correlation between the value of \( C_\theta \) and the length of the cluster when the locations of clusters are the same. If the clusters have the same length but locate at different positions of the routes as shown in Fig. 5, the road user will choose route \( B \) with smaller \( C_\theta \). Compared to WCCFS, the performance adopting CAFS is further improved, not only on the value but also the stability of the flux.

In Fig. 11(b), vehicle number versus time step shows that the routes’ accommodating capacity is greatly enhanced with an increase in average vehicle number. Thus perhaps the high fluxes of two routes with CAFS are mainly due to the increase of vehicle number. In Fig. 11(c), speed versus time step shows that although the speed is more stable by using CAFS, it becomes lower than the other four strategies discussed above. The reason is that the routes’
Fig. 8. (Color online) (a) Flux of each route with mean velocity feedback strategy. (b) Vehicle number of each route with mean velocity feedback strategy. (c) Average speed of each route with mean velocity feedback strategy. The parameters are $L=2000$, $p=0.25$ and $S_{dyn}=0.5$. 
Fig. 9. (Color online) (a) Flux of each route with congestion coefficient feedback strategy. (b) Vehicle number of each route with congestion coefficient feedback strategy. (c) Average speed of each route with congestion coefficient feedback strategy. The parameters are $L=2000$, $p=0.25$ and $S_{dyn}=0.5$. 
Fig. 10. (Color online) (a) Flux of each route with weighted congestion coefficient feedback strategy. (b) Vehicle number of each route with weighted congestion coefficient feedback strategy. (c) Average speed of each route with weighted congestion coefficient feedback strategy. The parameters are $L=2000$, $p=0.25$, $S_{d/n}=0.5$, and weight factor ($k$) is fixed at -1.98.
accommodating capacity by using CAFS is better than that of other four strategies, and at most one vehicle can drive out each time step as explained before; therefore more cars the lane has, lower speeds the vehicles have. Fortunately, flux consists of two parts, mean velocity and vehicle density. Hence, as long as the vehicle number (because vehicle density is defined as $\rho=N/L$, and $L$ is fixed at 2000, so $\rho \propto \text{vehicle number (N)}$) is large enough, the flux can also be the largest.

Fig. 12 shows the dependence of flux, vehicle number, average speed on time step when adopting PFS. The advantage of PFS is that it can predict the negative effects on the route condition caused by traffic jams happened at the end of the route, try to avoid jammed state to the best of its ability, and alleviate the negative effects as much as possible. Here, we want to stress that though PFS try to avoid the jammed state, the structure of the traffic system (one exit) and the special velocity update mechanism for the vehicle nearest to the exit still make jams happened at the end of the route occasionally. Also, this can explain the slight oscillation in Fig. 12 (a).

From Fig. 12 (b) we know that the average vehicle number is around 760 by using PFS. As to the routes’ stability, we know PFS is the optimal one, which means the vehicles should be almost uniformly distributed on each route instead of being together at the end of the routes. Furthermore, even there are 760 vehicles on each route, vehicles can still averagely occupy 2 ~ 3 sites on each route because the total length of each route is fixed at 2000 sites. This indicates there are only 1 ~ 2 sites between vehicles. Though the vehicles are almost distributed separately on each lane, the one exit structure and the special velocity update mechanism for the vehicle closest to the exit make jams still have a chance to happen at the end of the route. However, PFS can prevent jams from further expanding and alleviate the negative effects as much as possible, so that the jammed state will disappear soon. So as to analogize, even the jams happen again, the poor road condition will be relieved in a short time period.

As to the low speed shown in Fig. 12 (c), the reason is that the speed partially depends on the number of empty sites between two vehicles on the lane. The vehicle behind another vehicle can move at most the current empty sites between them which is required by NS mechanism (Nagel & Schreckenberg, 1992). The routes’ accommodating capacity is the best by using PFS, indicating the speed adopting PFS the lowest. From the stability of the velocity, we infer that the vehicles should drive at almost uniform speeds on each route. Without consider other factors, the speed should be a little more than one ($\sim 1.5$) because there are only 1 ~ 2 cells between vehicles as mentioned above. If we take the random brake effects and the occasional jams at the end of the route into account, the vehicles’ average velocity decreasing a little is possible and reasonable. Thus, the average velocity $V_{avg} \sim 1$ in this chapter could be understood. These analysis can also be applied to explain the low average velocity by using WCCFS and CAFS.

Someone may have doubts whether CAFS and PFS are really better than the other four feedback strategies due to the lower speed shown in Fig. 11 (c) & Fig. 12 (c). In order to making the readers understand more easily, we assume that the road network under study includes not only the downstream corridor section displayed in Fig. 3, but also a section upstream of the entrance where vehicles wait to enter the corridor (Dong et al., 2010c). Thus, we should take into account the total travel time ($t_{tot}$) that is the sum of driving time ($t_{driving}$) and waiting time ($t_{waiting}$) to evaluate the merits of these feedback strategies.

$$t_{tot} = t_{driving} + t_{waiting}$$ (9)
Fig. 11. (Color online) (a) Flux of each route with corresponding angle feedback strategy. (b) Vehicle number of each route with corresponding angle feedback strategy. (c) Average speed of each route with corresponding angle feedback strategy. The parameters are $L=2000$, $p=0.25$, $S_{dyn}=0.5$, and vertical distance ($H$) is fixed at 100.
Fig. 12. (Color online) (a) Flux of each route with prediction feedback strategy. (b) Vehicle number of each route with prediction feedback strategy. (c) Average speed of each route with prediction feedback strategy. The parameters are $L=2000$, $p=0.25$, $S_{dyn}=0.5$, and $T_p=60$. 
Total travel time ($t_{\text{tot}}$): $t_{\text{tot}}$ is the time period that vehicles spend on the whole traffic system that includes the upstream and downstream corridor sections. Flux is a very good proxy to evaluate the total travel time, because it can be understood as the number of vehicles passing the exit each time step. The more vehicles pass the exit during a fixed time period, the shorter time these vehicles spend on the traffic system. For example, there are totally $N$ vehicles in the section upstream of the entrance, then the average flux of the traffic system is

$$F_{\text{avg}} = N / t_{\text{tot}} (N\text{th vehicle}) \approx n / t_{\text{tot}} (n\text{th vehicle}) \quad n \in [1, N].$$  

(10)

Here, one should be aware that the average flux value is stable, thus the approximation in Eq.(10) is valid. Therefore the travel time adopting CAFS and PFS is shorter than the other four feedback strategies for the fact that the flux adopting CAFS and PFS are larger (see Fig.13).

Driving time ($t_{\text{driving}}$): $t_{\text{driving}}$ is the time period vehicles spend on the downstream route section displayed in Fig.3. It is obvious that driving time of CAFS and PFS is longer than the other four feedback strategies since the speeds $v$ adopting CAFS and PFS are lower as shown in Fig.11 (c) & Fig.12 (c) (the length of the route $L$ is fixed at 2000, thus $t_{\text{driving}} = L / v$).

Waiting time ($t_{\text{waiting}}$): $t_{\text{waiting}}$ is the time period vehicles waiting in the upstream route section. Since the total travel time ($t_{\text{tot}}$) is the sum of waiting time ($t_{\text{waiting}}$) and driving time ($t_{\text{driving}}$) as shown in Eq.(9), the waiting time adopting CAFS and PFS is shorter on the basis of above analysis.

Fig.13 shows that the average flux fluctuates feebly with a persisting increase of dynamic travelers by using six different strategies. As to the routes’ processing capacity, PFS is proved to be the best one because the flux is always the largest at each $S_{\text{dyn}}$ value and even increases with a persisting increase of dynamic travelers.

4. Conclusion

We obtain the simulation results of applying six different feedback strategies, i.e., TTFS, MVFS, CCFS, WCCFS, CAFS and PFS in a two-route scenario all with respect to flux, number of vehicles, speed, and average flux versus $S_{\text{dyn}}$. We also show the results about average flux versus weight factor ($k$) by adopting WCCFS, average flux versus position of pillar (point $T$) by adopting CAFS, and average flux versus prediction time ($T_p$) by adopting PFS. These results indicate that PFS has more advantages than the other five strategies in the two-route system with only one entrance and one exit. However, as we stress before that PFS is not easy to realize and will be invalid when the transportation system is multi-route (Dong et al., 2010d). The numerical simulations demonstrate that the weight factor $k$ (WCCFS), the position of point $T$ (CAFS) and the prediction time $T_p$ (PFS) play very important roles in improving the road conditions. In contrast with other four feedback strategies (TTFS, MVFS, CCFS and WCCFS), CAFS and PFS can significantly improve the road conditions, including increasing vehicle number and flux, reducing oscillation, and enhancing average flux with the increase of $S_{\text{dyn}}$. This can be understood because CAFS takes both the length and location of each congestion cluster into consideration; and PFS can predict the future road conditions.

With the development of science and technology, it is not difficult to realize these advanced information feedback strategies in reality. The position and velocity information of vehicles will be known through the navigation system (GPS). Then these feedback strategies can come true through computational simulation. Though the performance of PFS is the optimal one, it will cost most when adopting PFS as explained before. The rest five feedback strategies should cost almost the same. If someone can propose one feedback strategy in the near future whose
performance is better than PFS and cost is similar to CAFS, it will make great contributions to radically improve the road conditions of real-time traffic systems.

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6. References


Cellular automata make up a class of completely discrete dynamical systems, which have became a core subject in the sciences of complexity due to their conceptual simplicity, easiness of implementation for computer simulation, and their ability to exhibit a wide variety of amazingly complex behavior. The feature of simplicity behind complexity of cellular automata has attracted the researchers’ attention from a wide range of divergent fields of study of science, which extend from the exact disciplines of mathematical physics up to the social ones, and beyond. Numerous complex systems containing many discrete elements with local interactions have been and are being conveniently modelled as cellular automata. In this book, the versatility of cellular automata as models for a wide diversity of complex systems is underlined through the study of a number of outstanding problems using these innovative techniques for modelling and simulation.

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