Joint LS Estimation and ML Detection for Flat Fading MIMO Channels

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1. Introduction

In recent years, Multi-Input Multi-Output (MIMO) communications are introduced as an emerging technology to offer significant promise for high data rates and mobility required by the next generation wireless communication systems. Using multiple transmit as well as receive antennas, a MIMO system exploits spatial diversity, higher data rate, greater coverage and improved link robustness without increasing total transmission power or bandwidth (Tse & Viswanath, 2005). However, MIMO relies upon the knowledge of Channel State Information (CSI) at the receiver for data detection and decoding. It has been proved that when the channel is Rayleigh fading and perfectly known to the receiver, the performance of a MIMO system grows linearly with the number of transmit or receive antennas, whichever is less (Numan et al., 2009). Therefore, an accurate and robust channel estimation is of crucial importance for coherent demodulation in wireless MIMO systems.

Use of MIMO channels, when bandwidth is limited, has much higher spectral efficiency versus Single-Input Single-Output (SISO), Single-Input Multi-Output (SIMO), and Multi-Input Single-Output (MISO) channels. It is shown that the maximum achievable diversity gain of MIMO channels is the product of the number of transmitter and receiver antennas. Therefore, by employing MIMO channels not only the mobility of wireless communications can be increased, but also its robustness against fading that makes it efficient for the requirements of the next generation wireless services. To achieve maximum capacity and diversity gain, some optimization problems should be considered (Yatawatta et al., 2006).

The emergence of MIMO communication systems as practical high-data-rate wireless communication systems has created several technical challenges to be met. On the one hand, there is potential for enhancing system performance in terms of capacity and diversity. On the other hand, the presence of multiple transceivers at both ends has created additional cost in terms of hardware and energy consumption. For coherent detection as well as to do optimization such as water filling and beamforming, it is essential that the MIMO channel is known. However, due to the presence of multiple transceivers at both the transmitter and receiver, the channel estimation problem is more complicated and costly compared to a SISO system. Of concern, however, is the increased complexity associated with multiple transmit/receive antenna systems. First, increased hardware cost is required to implement
multiple Radio Frequency (RF) chains and adaptive equalizers. Second, increased complexity and energy is required to estimate large-size MIMO channels. Energy conservation in MIMO systems has been considered in different perspectives. For instance, hardware level optimization can be used to minimize energy. On the other hand, energy consumption can be minimized at the receiver by using low-rank equalization or/and reducing the order of MIMO systems by selection of antennas both at the receiver and transmitter, without degrading the system performance (Karami & Shiva, 2006).

In order to attain the advantages of MIMO systems and guarantee the performance of communication, effective channel estimation algorithms are needed. Many channel estimation (identification) algorithms have been developed in recent years. In the literature, three classes of methods to estimate the channel response are presented. They include Training Based Channel Estimation (TBCE) schemes relying on training sequences that are known to the receiver (Xie et al., 2007; Biguesh & Gershman, 2006; Nooralizadeh et al., 2009; Nooralizadeh & Shirvani Moghaddam, 2010), Blind Channel Estimation (BCE) methods (Sabri et al., 2009; Panahi & Venkat, 2009; Chen & Petropulu, 2001), identifying channel only from the received sequences, and Semi Blind Channel Estimation (SBCE) approaches as combination of two aforementioned procedures (Cui & Tellambura, 2007; Wo et al., 2006; Chen et al., 2007; Abuthinien et al., 2007; Khalighi & Bourennane, 2008).

One of the most usual approaches to identify MIMO CSI is TBCE. This class of estimation is attractive especially when it decouples symbol detection from channel estimation and thus simplifies the receiver implementation and relaxes the required identification conditions. In this scheme, the channel is estimated based on the received data and the knowledge of training symbols during training symbol transmit. Then, the acquired knowledge of the channel is used for data detection. TBCE schemes can be optimal at high Signal to Noise Ratios (SNRs), but they are suboptimal at low SNRs. The optimal choice of training signals is usually investigated by minimizing Mean Square Error (MSE) of the linear MIMO channel estimator. It is perceived that optimal design of training sequences is connected with the channel statistical characteristics (Hassibi & Hochwald, 2003).

Many blind channel estimation techniques can be found in the literature, and a good overview is given in (Tong & Perreau, 1998). The blind channel estimation methods can be classified into Higher-Order Statistics (HOS) based techniques (Cardoso, 1989; Comon, 1994; Chi et al., 2003) and Second Order Statistics (SOS) based techniques (Chang et al., 1997). Blind algorithms typically require longer data records and entail higher complexity.

Semi-blind channel estimation schemes, as the main core of this chapter, use a few training symbols to provide the initial MIMO channel estimation and exchange the information between the channel estimator and the data detector iteratively (Fang et al., 2007). The main steps of proposed SBCE-ML method (Shirvani Moghaddam & Saremi, 2010) are as follows:

Step 1. Initial channel estimation by using the training only;
Step 2. *Given channel knowledge, perform data detection;
*Given data decisions, perform channel estimation by taking the whole burst as a virtual training;
Step 3. Repeat step 2 until a certain stopping criterion is reached.

Several solutions have been proposed to minimize the computational cost, and hence the energy spent in channel estimation of MIMO systems. In (Yatawatta et al., 2006) authors present a novel method of minimizing the overall energy consumption. Unlike existing methods, this method considers the energy spent during the channel estimation phase which includes transmission of training symbols, storage of those symbols at the receiver,
and also channel estimation at the receiver. Also they developed a model that is independent of the hardware or software used for channel estimation, and use a divide-and-conquer strategy to minimize the overall energy consumption.

In (Numan et al., 2009), a better performance and reduced complexity channel estimation method is proposed for MIMO systems based on matrix factorization. This technique is applied on training based Least Squares (LS) channel estimation for performance improvement. Experimental results indicate that the proposed method not only alleviates the performance of MIMO channel estimation but also significantly reduces the complexity caused by matrix inversion. Simulation results show that the Bit Error Rate (BER) performance and complexity of the proposed method clearly outperforms the conventional LS channel estimation method.

In (Song & Blostein, 2004), authors focused on the achievable Symbol Error Rate (SER) performance of a MIMO link with interference. Prior results on estimation of vector channels and spatial interference statistics for Code Division Multiple Access (CDMA) SISO systems. Most studies of channel estimation and data detection for MIMO systems assume spatially and temporally white interference. For example, Maximum Likelihood (ML) estimation of the channel matrix using training sequences was presented assuming temporally white interference. Assuming perfect knowledge of the channel matrix at the receiver, ordered Zero-Forcing (ZF) and Minimum Mean Squared Error (MMSE) detection were studied for both spatially and temporally white interference. However, in cellular systems, the interference is, in general, both spatially and temporally colored. This paper proposes a new algorithm that jointly estimates the channel matrix and the spatial interference correlation matrix in an ML framework. It develops a multi-vector-symbol MMSE data detector that exploits interference correlation.

In (Zaki et al., 2009), a training-based channel estimation scheme for large non-orthogonal Space-Time Block Coded (STBC) MIMO systems is proposed. The proposed scheme employs a block transmission strategy where an $N_t \times N_t$ pilot matrix is sent (for training purposes) followed by several $N_t \times N_t$ square data STBC matrices, where $N_t$ is the number of transmit antennas. At the receiver, channel estimation (using an MMSE estimator) and detection (using a low-complexity Likelihood Ascent Search (LAS) detector) will be iterated till convergence or for a fixed number of iterations. Simulation results of this research show that good BER and high capacity are achieved by the proposed scheme at low complexities.

Joint channel estimation, data detection, and tracking are the most important issues in MIMO communications. Without joint estimation and detection, inter substream interference occurs. Joint estimation and detection algorithms used in MIMO channels are developed based on MultiUser Detection (MUD) algorithms in CDMA systems. ML is the optimum detector in these type of joint channel estimation and data detection algorithms. In (Karami & Shiva, 2006), a new approach for joint data estimation and channel tracking for MIMO channels is proposed based on the Decision-Directed Recursive Least Squares (DD-RLS) algorithm. RLS algorithm is commonly used for equalization and its application in channel estimation is a novel idea. In this paper, after defining the weighted least squares cost function it is minimized and eventually the RLS MIMO channel estimation algorithm is derived. The proposed algorithm combined with the Decision-Directed Algorithm (DDA) is then extended for the blind mode operation. From the computational complexity point of view being $O(3)$ versus the number of transmitter and receiver antennas, the proposed
algorithm is very efficient. Also, through various simulations, the MSE of the tracking of the proposed algorithm for different joint detection algorithms is compared with Kalman filtering approach which is one of the most well-known channel tracking algorithms.

The aim of (Rizogiannis et al., 2010) is to investigate receiver techniques for ML joint channel/data estimation in flat fading MIMO channels, that are both data efficient and computationally attractive. The performance of iterative LS for channel estimation combined with Sphere Decoding (SD) for data detection is examined for block fading channels, demonstrating the data efficiency provided by the semi-blind approach. The case of continuous fading channels is addressed with the aid of RLS. The observed relative robustness of the ML solution to channel variations is exploited in deriving a block QR-based RLS-SD scheme, which allows significant complexity savings with little or no performance loss. The effects on the algorithms’ performance of the existence of spatially correlated fading and Line-Of-Sight (LOS) paths are also studied. For the multi-user MIMO scenario, the gains from exploiting temporal/spatial interference color are assessed. The optimal training sequence for ML channel estimation in the presence of Co-Channel Interference (CCI) is also derived and shown to result in better channel estimation/faster convergence. The reported simulation results demonstrate the effectiveness, in terms of both data efficiency and performance gain, of the investigated schemes under realistic fading conditions. High throughput at a communication systems require high quality channel estimation at the receiver in order to provide reliable data detection, such as that performed by ML techniques. The channel estimation task is especially challenging in time varying channels, such as the one often arising in wireless communication links.

This paper (Wo et al., 2006) deals with joint data detection and channel estimation for frequency-selective MIMO systems with focus on the analysis of the channel estimator. First, it presents a scheme alternating between joint Viterbi detection and LS channel estimation and analyze its performance in terms of unbiasedness. Since in the proposed technique the channel estimator exploits both known pilot symbols (non-blind) as well as unknown information bearing symbols (blind), this channel identification scheme is referred to as semi-blind. Second, it derives the Cramer-Rao Lower Bound (CRLB) for semi-blind channel estimation of frequency selective MIMO channels, which provides a theoretical lower bound of the achievable MSE of any unbiased estimator. By simulation the MSE performance of the proposed algorithm is evaluated and compared to the CRLB. The obtained results are universal for systems with an arbitrary number of antennas and an arbitrary channel memory length. As an example, a SBCE algorithm with LS channel estimator and ML data detector will be first introduced and analyzed. It will be shown that the presented semiblind channel estimator is biased at low SNR but tends to be unbiased at high SNR. Interestingly but reasonably, the MMSE achievable by any unbiased channel estimator at high SNR will be the same as that all data symbols are a-priori known at the receiver, but only the training symbols are known at low SNR. Simulation results show that the MSE performance of the presented SBCE coincides with the CRLB at high SNRs but exceeds CRLB at low SNRs due to biasing. Of particular interest is the SNR value where a semiblind channel estimator begin to approach the CRLB, which means that a SBCE will be able to fully exploit the channel information carried by all observations for SNRs larger than this value.

Reliable coherent communication over mobile wireless channels requires accurate estimation of time-varying multipath channel parameters. Traditionally, channel estimation is achieved by sending training sequences or using pilot channels. Recently, there is a
growing interest in training or pilot-based channel estimation for Direct Sequence CDMA (DS-CDMA) systems. In (Rizanera et al., 2005), authors address the problem of mobile radio channel estimation at high channel efficiency using a small number of training symbols. A decision aided channel estimation scheme is proposed for slow fading multipath DS-CDMA channels. The approach is an extension of single-user LS channel estimation. It is demonstrated that, due to the suggested channel estimate updating algorithm, the proposed scheme improves the channel estimation accuracy significantly. An adaptive method has been considered to provide channel estimates. In this method, the received signal is correlated with the locally generated spreading code at each multipath delay for channel estimation at each symbol interval.

By using MIMO technology an increase in the system capacity and/or an improvement in the quality of service can be achieved. The key to fully utilize the MIMO capacity relies heavily on the requirement of accurate MIMO channel estimation. This chapter have a review on TBCE as well as SBCE methods and offers some comparative simulation results. Simulations are done in different cases, MIMO $2 \times 2$ with and without space-time Alamouti coding, and also MIMO $4 \times 4$ to see the effect of the number of antenna elements. In addition, performance of different estimators, LS, Linear MMSE (LMMSE), ML and Maximum A’ Posteriori (MAP) are evaluated based on BER and SER with respect to perfect channel estimator. It also proposes the proper method to estimate flat fading MIMO channels that uses LS estimator and ML detector in a joint state.

2. System model

Consider a MIMO system equipped with $N_T$ transmit antennas and $N_R$ receive antennas. The block diagram of a typical MIMO $2 \times 2$ is shown in Fig. 1.

![Fig. 1. General architecture of a MIMO $2 \times 2$.](/content/images/)

where $x_1, x_2$ are the input (transmitted) signals of time slot 1 in locations $A$ and $B$, respectively. $x'_1, x'_2$ are associated input signals of time slot 2.

It is assumed that the channel coherence bandwidth is larger than the transmitted signal bandwidth so that the channel can be considered as narrowband or flat fading. Furthermore, the channel is assumed to be stationary during the communication process of a block. Hence, by assuming the block Rayleigh fading model for flat MIMO channels, the channel response is fixed within one block and changes from one block to another one randomly. During the training period, the received signal in such a system can be written as (1)
\[ Y = HX + N \]  
where \( Y, X \) and \( N \) are the complex \( N_R \)-vector of received signals on the \( N_R \) receive antennas, the possibly complex \( N_T \)-vector of transmitted signals on the \( N_T \) transmit antennas, and the complex \( N_R \)-vector of additive receiver noise, respectively. The elements of the noise matrix are independent and identically distributed (i.i.d.) complex Gaussian random variables with zero-mean and \( \sigma^2 \) variance, and the correlation matrix of \( N \) is then given by (Ma et al., 2005): 

\[ R = E\{N^H \cdot N\} = \sigma^2 N_R \cdot I_{N_R} \]  

where \( (.)^H \) is reserved for the matrix hermitian, \( E\{\cdot\} \) is the mathematical expectation, and \( I_{N_R} \) denotes the \( N_R \times N_R \) identity matrix. \( N_R \) is the number of transmitted training symbols by each transmitter antenna. The matrix \( H \) in the model (1) is the \( N_R \times N_T \) matrix of complex fading coefficients. The \((m,n)\)-th element of the matrix \( H \) denoted by \( h_{m,n} \) represents the fading coefficient value between the \( m \)-th receiver antenna and the \( n \)-th transmitter antenna. Here, it is assumed that the MIMO system has equal transmit and receive antennas. The elements of \( H \) and noise are independent of each other. In order to estimate the channel matrix, it is required that \( N_R \geq N_T \) training symbols are transmitted by each transmitter antenna. The function of a channel estimation algorithm is to recover the channel matrix \( H \) based on the knowledge of \( Y \) and \( X \) (Shirvani Moghaddam & Saremi, 2010). As depicted in Fig. 1, output (received) signals in locations \( C \) and \( D \) are as follow: 

\[
\begin{cases}
    y_{n1} = h_{11}.x_1 + h_{21}.x_2 + n_1 \\
    y_{n2} = h_{12}.x_1 + h_{22}.x_2 + n_2 \\
    y'_{n1} = h'_{11}.x'_1 + h'_{21}.x'_2 + n'_{1} \\
    y'_{n2} = h'_{12}.x'_1 + h'_{22}.x'_2 + n'_{2}
\end{cases}
\]  

where \( y_{n1}, y_{n2} \) are the output signals of time slot 1 in locations \( C \) and \( D \), respectively. \( y'_{n1}, y'_{n2} \) are associated output signals of time slot 2. \( n_1, n_2, n'_1, n'_2 \) are independent Additive White Gaussian Noises (AWGN). In (Alamouti, 1998), Alamouti proposed the first space-time coding for a MIMO 2×2 system. The proposed matrix is as follows: 

\[ S = \begin{bmatrix} s_1 & s_2 \\ -s_2 & s_1 \end{bmatrix} \]  

which means that in the first time slot, \( s_1 \) and \( s_2 \) will be sent and in the second one, \( -s_2^* \) and \( s_1^* \) will be transmitted. Following equations can be used to decoding process:

\[
\begin{align*}
    \tilde{x}_1 &= h'_{11}.y_{11} + h_{12}.y'_{12} + h'_{21}.y_{21} + h_{22}.y'_{22} \\
    \tilde{x}_2 &= h'_{12}.y_{11} - h_{11}.y'_{12} + h'_{22}.y_{21} - h_{21}.y'_{22}
\end{align*}
\]  

This kind of coding is used in this research. Simulation results show its great effect on the performance of the channel estimators in both TBCE and SBCE-ML schemes. 

3. Channel estimators 

As illustrated in Table 1, there are many algorithms to estimate the channel response from training sequence. As shown in introduction and also (Leus & Von Der Veen, 2005: Murthy et al., 2006), LS, LMMSE, ML, and MAP are the famous and more applicable estimators. In this investigation, perfect estimator (inverse matrix) is a proper reference to compare the
estimators. This reference method offers minimum BER in the case of a Rayleigh flat fading MIMO channel or AWGN.

<table>
<thead>
<tr>
<th>Channel Estimator</th>
<th>Estimation Formula</th>
</tr>
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<tbody>
<tr>
<td>Perfect</td>
<td>( H_{\text{Perfect}} = Y \cdot X^{-1} )</td>
</tr>
<tr>
<td>LS</td>
<td>( H_{\text{LS}} = (X^H \cdot X)^{-1} \cdot X^H \cdot Y )</td>
</tr>
<tr>
<td>LMMSE</td>
<td>( H_{\text{LMMSE}} = (\sigma_n^2 \cdot C_H^{-1} + X^H \cdot X)^{-1} \cdot X^H \cdot C_H \cdot Y )</td>
</tr>
<tr>
<td>ML</td>
<td>( H_{\text{ML}} = (X^H \cdot C_H \cdot X)^{-1} \cdot X^H \cdot C_H^{-1} \cdot Y )</td>
</tr>
<tr>
<td>MAP</td>
<td>( H_{\text{MAP}} = (X^H \cdot C_H^{-1} \cdot X + C_H)^{-1} \cdot X^H \cdot C_H^{-1} \cdot Y )</td>
</tr>
</tbody>
</table>

Table 1. Different Channel Estimators

where \((\cdot)^{-1}\) is reserved for the matrix inverse, \(C_H\) and \(C_n\) denote channel and noise covariances, respectively.

3.1 Perfect estimator

Perfect estimator is the simplest algorithm to estimate the channel matrix. By setting the noise equal to zero in (1), the perfect approach estimates the channel matrix as

\[
H_{\text{Perfect}} = Y \cdot X^{-1}
\]

Using equation (6), sub-channel responses are simply obtained by

\[
\begin{align*}
  h_{11} &= \frac{y_{n1}}{x_1} - \frac{x_2}{x_1} \left( \frac{y_{n1} - y_{n2} \cdot x'_1}{x_1^2 - x_2^2} \cdot x'_1 \right) \\
  h_{12} &= \frac{y_{n2}}{x_1} - \frac{x_2}{x_1} \left( \frac{y_{n2} - y_{n1} \cdot x'_1}{x_2^2 - x_1^2} \cdot x'_1 \right) \\
  h_{21} &= \frac{y_{n1} \cdot x'_1}{x_1^2 - x_2^2} \\
  h_{22} &= \frac{y_{n2} \cdot x'_1}{x_2^2 - x_1^2}
\end{align*}
\]

Substituting (7) back into noise-free version of (3), input signals can be expressed as

\[
\begin{align*}
  x_{1\text{est}} &= \frac{y_{n1} - h_{21} \cdot (y_{n2} \cdot h_{11} - h_{12} \cdot y_{n1})}{h_{11}} \\
  x_{2\text{est}} &= \frac{y_{n2} \cdot h_{11} - h_{12} \cdot y_{n1}}{h_{11} \cdot h_{22} - h_{12} \cdot h_{21}} \\
  x'_{1\text{est}} &= \frac{y_{n1} - h_{21} \cdot (y_{n2} \cdot h_{11} - h_{12} \cdot y_{n1})}{h_{11}} \\
  x'_{2\text{est}} &= \frac{y_{n2} \cdot h_{11} - h_{12} \cdot y_{n1}}{h_{11} \cdot h_{22} - h_{12} \cdot h_{21}}
\end{align*}
\]

where \(x_{1\text{est}}, x_{2\text{est}}\) are the estimated input signals of time slot 1 in locations A and B, and \(x'_{1\text{est}}, x'_{2\text{est}}\) are associated estimated input signals of time slot 2, respectively.
3.2 LS estimator
Considering (1), LS estimator finds $H_{est}$ so that $X.H_{est} = Y$. LS Algorithm, minimizes the Euclidian distance of $X.H_{est} - Y$. For this minimization we do following steps:

$$\|X.H_{est} - Y\|^2 = (X.H_{est} - Y)^H.(X.H_{est} - Y) = (X.H_{est})^H.(X.H_{est}) - Y^H.X.H_{est} - (X.H_{est})^H.Y + Y^H.Y$$

(9)

By differentiating (9) with respect to $H_{est}$ and setting the result equal to zero, it is obtained that $H_{est}$ should satisfy the equation (10)

$$2X^H.X.H_{est} - 2X^H.Y = 0 \rightarrow X^H.X.H_{est} = X^H.Y$$

(10)

Finally, the LS channel estimation algorithm is based on (11)

$$H_{LS} = (X^H.X)^{-1}.X^H.Y$$

(11)

3.3 LMMSE estimator
For linear model (1), the MMSE and LMMSE estimators are identical. So, let us minimize the estimation MSE of $H$. It can be expressed in the following form:

$$H_{LMMSE} = \min E\{\|H - H_{est}\|^2\}$$

(12)

Assuming $E(H) = 0$ and noise is AWGN, we can obtain that (12) will be minimized as

$$H_{LMMSE} = (\sigma_n^2.C_H^{-1} + X^H.X)^{-1}.X^H.Y$$

(13)

Comparing (13) and (11), it is obvious that

$$H_{LMMSE} - H_{LS} = \sigma_n^2.C_H.X^H.Y$$

(14)

(14) shows that LMMSE needs to find an additional term compared to LS estimator. This term depends on previous data and introduces more computational complexity.

3.4 ML estimator
To identify $H$ from (1), the ML approach maximizes (15)

$$H_{ML} = \max_H p(Y|H)$$

(15)

where $p(Y|H)$ is the conditional probability of received signal respect to channel response. It is given that the ML channel estimator (15) yields

$$H_{ML} = (X^H.C_H^{-1}.X)^{-1}.X^H.C_H.Y$$

(16)

3.5 MAP estimator
In order to estimate the channel response, in addition training bits, MAP estimator needs to find channel covariance as well as noise covariance. MAP channel estimate is in accordance with previous conditional probability $p(H|Y,X)$. MAP channel estimate can be found by solving the following equation:

$$\frac{\partial \ln(p(H|Y,X))}{\partial H}|_{H=H_{MAP}} = 0$$

(17)
By using the Bay’s identity (18) and solving the equation (17), MAP channel estimate can be found as (19)

\[ p(H | Y, X) = \frac{p(Y | H, X) p(H, X)}{p(Y | X)} \]

\[ H_{MAP} = (X^H C_n^{-1} X + C_H)^{-1} X^H C_n^{-1} Y \]  

(18)  

(19)

4. Simulation results of TBCE

In order to compare the performance of LS, LMMSE, ML, and MAP estimators in TBCE for MIMO channels, three cases, MIMO 2×2 without coding, MIMO 4×4, and Alamouti coded MIMO 2×2 are simulated. Simulation results show the performance of different estimators in terms of three metrics (BER, SER, and required processing time). For the sake of simplicity and without loss of generality, we assume Rayleigh flat fading MIMO channel with AWGN, 4QAM modulation, 8 training bits for MIMO 2×2 (\( N_T = N_R = 2 \)) and 32 bits for MIMO 4×4 (\( N_T = N_R = 4 \)) which are generated randomly and followed by 400 data bits. It is notable that when each point in our figures is obtained by averaging over 1000 independent simulation runs, the numerical and analytical results are almost identical.

Fig. 2 shows the BER as well as SER of different estimators in the case of TBCE. As depicted, LS estimator has the better performance (Lower BER and SER) rather than LMMSE, ML and MAP estimators and its performance is close to the perfect one.

As shown in Fig. 3, increasing the number of transmit antennas leads to increase the performance estimators, but it is highlighted in LS. It means, the performance of LS algorithm in a MIMO 4×4 system is improved respect to MIMO 2×2. As before, increasing the SNR is the reason for decreasing BER and SER of all estimators but it is more effective for LS one.

The BER and SER of TBCE versus SNR for various channel estimators in the case of MIMO 2×2 with Alamouti coding, are shown in Fig. 4. Comparing Fig. 4 and Fig. 2, it is observed that the BER and SER of all estimators are decreased using Alamouti coding especially at low SNRs.

Considering the processing time of TBCE equipped with prefect estimator equal to 100, Fig. 5 shows the processing time for other estimators respect to the perfect one. As expected, minimum processing time belongs to LS estimator.

![Fig. 2. Performance metrics (BER, SER) versus SNR for a MIMO 2×2 (TBCE).](image-url)
5. Simulation results of SBCE

For pure TBCE schemes, a long training is necessary in order to obtain a reliable MIMO channel estimate which reduces the system bandwidth efficiency considerably. SBCE-ML schemes require less computational complexity than blind methods and fewer training symbols than training-based methods, making them attractive for practical implementation. TBCE algorithms use only the training sequences to perform channel estimation, while a SBCE algorithm takes the data symbols also into account. Since the data symbols are practically unknown, before they can be used for channel estimation, the receiver has to perform detection in advance. Thus, the task of channel estimation changes into joint estimation of channel and data symbols.

By refining the channel estimate and the data decisions in a recursive manner, considerable performance gain can be achieved step by step. As depicted in Fig. 6, in an iterative structure, output of estimator is applied to detector for detecting data bits and also output of detector is applied to the estimator as virtual bits and to estimate the channel again. This iterative procedure runs until a criterion is achieved [Shirvani Moghaddam & Saremi, 2010].

For example, difference of estimation for two successive iterations is lower than a level. LS, LMMSE, ML and MAP estimators may be used in estimation part but ML detector is more attractive in semi-blind joint estimation and detection schemes. In the first step, channel response is estimated considering short training bits. Then, by using the ML detector, symbols are detected according to (20):

$$X_{est} = \arg \min_{\hat{X} \in \mathbb{X}} \left\{ \| Y - H \cdot \hat{X} \|_F^2 \right\}$$  \hspace{1cm} (20)
where $H_{\text{est}}$ is used for detecting $X_{\text{est}}$ and previous detected data is the virtual training sequence to next estimation. $\| \cdot \|_F$ denotes the Frobenius norm. This process will be continued until (21) be satisfied.

$$
(H_{\text{est},i}, X_{\text{est},i}) = (H_{\text{est},i-1}, X_{\text{est},i-1}) \quad (21)
$$

The proposed method can be summarized as follow:

1. $i = 0$: $H_0$ (i denotes the iteration index);
2. $i = i + 1$;
   a. ML Data Detection
   b. Channel Estimation
3. Repeat step 2 until $(H_{\text{est},i}, X_{\text{est},i}) = (H_{\text{est},i-1}, X_{\text{est},i-1})$

In the next subsections, simulation results of SBCE-ML method for a Rayleigh flat fading MIMO system in three cases, MIMO 2×2 (with and without Alamouti coding) and MIMO 4×4 are presented. For this type of channel estimation, 8 and 32 training bits are used for MIMO 2×2 and MIMO 4×4, respectively followed by 40000 data bits. Simulation results of SBCE scheme are presented to find the efficient estimator with good performance (BER as well as SER) and lower processing time.
Fig. 7 illustrates the BER as well as SER of SBCE-ML using various estimators versus different SNR for a Rayleigh flat fading MIMO 2×2 channel. It is obvious that, increasing SNR is the reason for decreasing both BER and SER. As depicted, not only the performance of LS algorithm is better than other estimators but also is close to the perfect one.

Fig. 7. Performance metrics (BER, SER) versus SNR for a MIMO 2×2 (SBCE-ML).

Increasing the number of transmit antennas leads to decreasing the performance estimators, except LS. As shown in Fig. 8, the performance of LS algorithm in a MIMO 4×4 system is improved respect to MIMO 2×2. In the other hand, a power gain or SNR improvement will be achieved. For example in SBCE-ML, transmitting power will be saved about 3 dB, if BER equals to 0.3.

Fig. 8. Performance metrics (BER, SER) versus SNR for a MIMO 4×4 (SBCE-ML).

The BER and SER of SBCE-ML method versus SNR for various channel estimators in the case of MIMO 2×2 with Alamouti coding, are shown in Fig. 9. it is observed that the LS estimator outperforms the other estimators especially at low SNRs.

Fig. 9. Performance metrics (BER, SER) versus SNR for an Alamouti coded MIMO 2×2 (SBCE-ML).
Fig. 10 shows the processing time for different estimators (LS, LMMSE, ML, MAP) with respect to the perfect estimator in SBCE-ML scheme. In this figure, required time for perfect one is considered as 100 and other estimators’ processing time is evaluated based on the perfect one. It is obvious that minimum processing time belongs to LS estimator.

![Fig. 10. Relative processing time of different estimators with respect to perfect one in a MIMO 2×2 (SBCE).](image)


Simulation results of TBCE and SBCE-ML methods show that the required processing time and both BER and SER of LS estimator compared with other estimators is much better. In this section by focusing on LS estimator, LS-based TBCE and LS-based SBCE-ML are compared in a MIMO 2 × 2 (with and without Alamouti coding) and a MIMO 4×4, for different SNRs based on BER, SER, required channel estimation processing time and relative length of training bits.

Fig. 11 indicates the BER and SER metrics of LS-based TBCE and LS-based SBCE-ML schemes for different SNRs. As shown, for both TBCE and SBCE-ML methods, increasing SNR is the reason for decreasing both BER and SER. As depicted in this figure, SBCE-ML offers a bit better performance rather than TBCE.

![Fig. 11. Performance metrics (BER, SER) of LS-based TBCE and SBCE-ML schemes in different SNRs for a MIMO 2×2.](image)
As shown in Fig. 12, the performance of both LS-based TBCE and SBCE-ML schemes in a MIMO 4×4 system is improved respect to MIMO 2×2. In the other hand, a power gain or SNR improvement will be achieved. For example in SBCE-ML, transmitting power will be saved about 3 dB, if BER equals to 0.3. In TBCE method, for BER equals to 0.2, transmitting power will be saved about 0.5 dB. It is worthwhile to note that the excess of transmit or/and receive antennas in MIMO systems leads to a higher capacity.

Fig. 12. Performance metrics (BER, SER) of LS-based TBCE and SBCE-ML schemes in different SNRs for a MIMO 4×4.

The BER and SER of both LS-based TBCE and SBCE-ML schemes versus SNR in the case of MIMO 2×2 with Alamouti coding, are shown in Fig. 13. As shown in this figure, when SNR equals to 0.25 dB, BER is 0.0130 for SBCE-ML and 0.0386 for TBCE. It means 3 times better performance in lowest SNRs for SBCE-ML method rather than TBCE one. At higher SNRs, the performance of LS estimator in both channel estimation schemes is analogous.

By considering the required processing time of LS-based TBCE and SBCE-ML schemes related to perfect estimator, Fig. 14 shows that SBCE-ML method needs 25 percent more processing time to estimate the channel than TBCE method. It is due to joint LS estimation and ML detection of SBCE method.

Fig. 15, 16 show the required training sequences in each frame of data for TBCE and SBCE-ML schemes, respectively. As depicted in Fig. 15, in TBCE method, transmitter sends 8 training bits before 400 information bits in each burst for a MIMO 2×2 system and 32 bits for a MIMO 4×4 system. Figure 16, illustrates the required number of training and information bits in SBCE-ML method for both MIMO 2×2 and MIMO 4×4. Considering the same training bits, 400 information bits in the case of TBCE method are changed to 40000 bits in SBCE-ML. As mentioned before, TBCE method needs more bits to estimate the channel because training sequences should be transmitted periodically. On the other word, SBCE-ML

Fig. 13. Performance metrics (BER, SER) of LS-based TBCE and SBCE-ML schemes in different SNRs for an Alamouti coded MIMO 2×2.
Fig. 14. Relative processing time of LS-based TBCE and SBCE-ML schemes in a MIMO 2×2.

Fig. 15. The burst of LS-based TBCE. A) MIMO 2×2, B) MIMO 4×4.

Fig. 16. The burst of LS-based SBCE-ML. A) MIMO 2×2, B) MIMO 4×4.

Method needs to transmit just one training sequence. Therefore, redundancies of TBCE
method are 2% and 8% for MIMO 2×2 and MIMO 4×4 systems, respectively. In the case of
SBCE-ML method, redundancies are 0.02% and 0.08%, respectively. It means 100 times
lower training bits for SBCE-ML respect to TBCE.

7. Conclusion

MIMO systems play a vital role in fourth generation wireless systems to provide advanced
data rate. In order to attain the advantages of MIMO systems, it is necessary that the receiver
and/or transmitter have access CSI. The time-varying nature of the channel typically requires
the use of frequent channel retraining, which in turn increases the data overhead due to
training signals, thus reducing the system’s overall spectral efficiency. Hence, effective channel
estimation algorithms are needed to guarantee the performance of communication.

In this chapter, training based as well as semi-blind channel estimation schemes in Rayleigh
flat fading MIMO systems are investigated. After introducing LS, LMMSE, ML and MAP
estimators, they are simulated in a Rayleigh flat fading MIMO channel considering AWGN.
Simulation results show that LS estimator is the best choice in both TBCE and SBCE-ML
schemes. This selection is due to faster processing and lower BER as well as SER of LS
estimator with respect to other estimators. In addition, it is illustrated that when the number
of transmitter or/and receiver antennas increases, the performance of both TBCE and SBCE-ML schemes significantly improves. Moreover, Alamouti coding has more effect on the performance of SBCE-ML rather than TBCE. Comparing LS-based TBCE and LS-based SBCE-ML methods based on BER, SER, required training bits, and processing time, simulation results introduce most appropriate channel estimation method that uses an iterative algorithm. This new proposed method is based on LS estimator and ML detector. According to simulation results, LS-based SBCE-ML method compared to LS-based TBCE method in different SNRs offers lower BER and also SER, 25 percent higher processing time, and 100 times lower training bits. Some new research works and simulations can be considered to extend the above mentioned results and techniques as follow:
1. Considering the TBCE and SBCE-ML methods for Rician flat fading MIMO channels and extending the results of (Shirvani Moghaddam & Saremi, 2010) for these channels;
2. Applying the new versions of LS algorithm, Scaled LS (SLS) and Shifted SLS (SSLS) proposed in (Nooralizadeh & Shirvani Moghaddam, 2010), for SBCE-ML scheme;
3. Considering the effect of type of training sequence, orthogonal as well as optimum (Nooralizadeh et al., 2009), in channel estimation performance;
4. Finding the channel estimation results based on MSE (or Normalized MSE) criteria;
5. Extending the results of (Nooralizadeh & Shirvani Moghaddam, 2011) and comparing TBCE and SBCE-ML schemes in frequency selective fading MIMO channels;
6. Extending the analytical and simulation results of (Wo et al., 2006) considering the BER and SER performance metrics instead of MSE one.

8. References


In recent years, it was realized that the MIMO communication systems seem to be inevitable in accelerated evolution of high data rates applications due to their potential to dramatically increase the spectral efficiency and simultaneously sending individual information to the corresponding users in wireless systems. This book, intends to provide highlights of the current research topics in the field of MIMO system, to offer a snapshot of the recent advances and major issues faced today by the researchers in the MIMO related areas. The book is written by specialists working in universities and research centers all over the world to cover the fundamental principles and main advanced topics on high data rates wireless communications systems over MIMO channels. Moreover, the book has the advantage of providing a collection of applications that are completely independent and self-contained; thus, the interested reader can choose any chapter and skip to another without losing continuity.

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