Electromagnetic Waves Propagation Characteristics in Superconducting Photonic Crystals

Arafa H Aly

Physics department, Faculty of Sciences, Beni-suef University
Egypt

1. Introduction

Photonic crystals (PCs) are structures with periodically modulated dielectric constants whose distribution follows a periodicity of the order of a fraction of the optical wavelength. Since the first pioneering work in this field, many new interesting ideas have been developed dealing with one-dimensional (1D), two-dimensional (2D), and three-dimensional (3D) PCs. Researchers have proposed many new and unique applications of photonic devices which may revolutionize the field of photonics in much the same way as semiconductors revolutionized electronics. They can generate spectral regions named photonic band gaps (PBGs) where light cannot propagate in a manner analogous to the formation of electronic band gaps in semiconductors [1,2]. There are several studies of metallic [3-7] and superconducting photonic crystals [7,8] which are mostly concentrated at microwave, millimeterwave, and far-infrared frequencies. In those frequencies, metals act like nearly perfect reflectors with no significant absorption problems.

Yablonovitch [1] main motivation was to engineer the photonic density of states in order to control the spontaneous emission of materials embedded with photonic crystal while John’s idea was to use photonic crystals to affect the localization and control of light. However due to the difficulty of actually fabricating the structures at optical scales early studies were either theoretical or in the microwave regime where photonic crystals can be built on the far more reading accessible centimeter scale. This fact is due to the property of the electromagnetic fields known as scale invariance in essence, the electromagnetic fields as the solutions to Maxwell’s equations has no natural length scale and so solutions for centimeter scale structure at microwave frequencies as the same for nanometer scale structures at optical frequencies.

The optical analogue of light is the photonic crystals in which atoms or molecules are replaced by macroscopic media with different dielectric constants and the periodic potential is replaced by a periodic dielectric function. If the dielectric constants of the materials is sufficiently different and also if the absorption of light by the material is minimal then the refractions and reflections of light from all various interfaces can produce many of the same phenomena for photons like that the atomic potential produced for electrons[9].

The previous details can guide us to the meaning of photonic crystals that can control the propagation of light since it can simply defined as a dielectric media with a periodic
modulation of refractive index in which the dielectric constant varies periodically in a
specific directions. Also it can be constructed at least from two component materials with
different refractive index due to the dielectric contrast between the component materials of
the crystal. It’s characterized by the existence of photonic band gap (PBG) in which the
electromagnetic radiation is forbidden from the propagation through it.
Optical properties of low dimensional metallic structures have also been examined recently.
For example, the optical transmission through a nanoslit collection structure shaped on a
metal layer with thin film thickness was analyzed in Refs. [10,11]. The photonic band
structures of a square lattice array of metal or semiconductor cylinders, and of an array of
metal or semiconductor spheres, were enumerated numerically in Ref. [12]. In addition,
superconducting (SC) photonic crystals also attract much attention recently [13,14]. In new
experiments superconducting metals (in exact, Nb) have been used as components in optical
transmission nanomaterials. Dielectric losses are substantially reduced in the SC metals
relative to analogous structures made of normal metals. The dielectric losses of such a SC
nanomaterial are reduced by a factor of 6 upon penetrating into the SC state [15]. Indeed,
studies of the optical properties of superconductor metal/dielectric multilayers are not
numerous, may be the results have been used in the design of high reflection mirrors, beam
splitters, and bandpass filters [16]. The superiority of a photonic crystal with
superconducting particles is that the scattering of the incident electromagnetic wave due to
the imaginary part of the dielectric function is much less than for normal metallic particles at
frequencies smaller than the superconducting gap. The loss caused by a superconducting
photonic crystal is thus expected to be much less than that by a metallic photonic crystal. For
a one-dimensional superconductor–dielectric photonic crystal (SuperDPC), it is seen like in
an MDPC that there exists a low-frequency photonic band gap (PBG). This low frequency
gap is not seen in a usual DDPC. This low frequency PBG is found to be about one third of
the threshold frequency of a bulk superconducting material [12]. In this paper, based on the
transfer matrix method, two fluid models, we have investigated the effect of the different
parameters on transmittance and PBG in a one-dimensional superconductor-dielectric
photonic crystals.

2. Numerical methods

We will explain in brief a mathematical treatment with a simple one dimensional photonic
crystal structure (1DPC) (see fig.1) which is composed of two materials with thicknesses (\(d_2\)
and \(d_3\)) and refractive indices (\(n_2\) and \(n_3\)) respectively. The analysis of the incident
electromagnetic radiation on this structure will be performed using the transfer matrix
method (TMM).

A one-dimensional nonmagnetic conventional and high temperature superconductor-
dielectric photonic crystal will be modelled as a periodic superconductor-dielectric
multilayer structure with a large number of periods \(N \gg 1\). Such an \(N\)-period superlattice is
shown in Fig. 1, where \(d = d_2 + d_3\) is the spatial periodicity, where \(d_2\) is the thickness of the
superconducting layer and \(d_3\) denotes the thickness of the dielectric layer. We consider that
the electromagnetic wave is incident from the top medium which is taken to be free space
with a refractive index, \(n_1 = 1\). The index of refraction of the lossless dielectric is given by
\(n_3 = \sqrt{\varepsilon_3}\), \(n_2\) the index of refraction of the superconductor material, which can be described
Fig. 1. A superconductor–dielectric structure. The thicknesses of superconducting and dielectric are denoted by $d_2$ and $d_3$, respectively, and the corresponding refractive indices are separately indicated by $n_1$, $n_2$, $n_3$, where $n_1=1$ and $n_4$ is the index of substrate layer.

on the basis of the conventional two-fluid model \cite{18}. Accordingly to the two-fluid model the electromagnetic response of a superconductor can be described in terms of the complex conductivity, $\sigma = \sigma_1 - i\sigma_2$, where the real part indicating the loss contributed by normal electrons, and the imaginary part is due to superelectrons, the imaginary part is expressed as $\sigma_2 = 1 / \omega \mu_0 \lambda_i^2$, where the temperature-dependent penetration depth is given by $\lambda_i = \lambda_i(T) = \lambda_0 / \sqrt{1 - f(T)}$, where Gorter-Casimir expression for $f(T)$ is given for low and conventional superconductor by $f(T)=\left(T / T_c\right)^4$, and for high temperature superconductor $f(T)=\left(T / T_c\right)^2$ \cite{13,18}.

We shall consider the lossless case, meaning that the real part of the complex conductivity of the superconductor can be neglected and consequently it becomes $\sigma = -i\sigma_2 = -(1 / \omega \mu_0 \lambda_i^2)$. The relative permittivity as well as its associated index of refraction can be obtained by,

$$\varepsilon_{r_2} = 1 - \frac{c^2}{\omega^2 \lambda_i^2} \quad \text{and} \quad n_2 = \sqrt{\varepsilon_{r_2}} = \sqrt{1 - \frac{c^2}{\omega^2 \lambda_i^2}}$$

(1)

We will go to mention the mathematical form of the dynamical matrices and for the propagation matrix to obtain an expressions for the reflection and transmission, the dynamical matrices take the form \cite{17}:

$$D_a = \begin{pmatrix} \frac{1}{n_a \cos \theta_a} & 1 \\ -\frac{1}{n_a \cos \theta_a} & \frac{1}{n_a \cos \theta_a} \end{pmatrix} \text{for } S\text{-wave}$$

(2)

$$D_a = \begin{pmatrix} \cos \theta_a & \cos \theta_a \\ n_a & -n_a \end{pmatrix} \text{for } P\text{-wave}$$

(3)

with

$$\beta = n_a \frac{\omega}{c} \sin \theta_a \quad \text{and} \quad k_{a\pm} = n_a \frac{\omega}{c} \cos \theta_a$$
while the propagation matrix take the form:-

\[
P_\eta = \begin{pmatrix}
\exp(i\phi_\eta) & 0 \\
0 & \exp(-i\phi_\eta)
\end{pmatrix}
\]  
(4)

Since the number of the propagation matrix depend on the number of materials which build our structure [17]. Finally the transfer matrix method can take the form:-

\[
M = \begin{pmatrix}
M_{11} & M_{12} \\
M_{21} & M_{22}
\end{pmatrix}
\]  
(5)

\[
P_2 = \begin{pmatrix}
\cos\phi_1 + i\sin\phi_1 & 0 \\
0 & \cos\phi_1 - i\sin\phi_1
\end{pmatrix}
\]

\[
P_3 = \begin{pmatrix}
\cos\phi_2 + i\sin\phi_2 & 0 \\
0 & \cos\phi_2 - i\sin\phi_2
\end{pmatrix}
\]

Since;

\[
\phi_1 = \frac{2\pi d_2}{\lambda} n_2 \cos\theta_2, \text{ and } \phi_2 = \frac{2\pi d_3}{\lambda} n_3 \cos\theta_3.
\]

The components of the transfer matrix method can be written in a detailed form for an S-wave as:-

\[
M_{11} = \frac{1}{2} \begin{pmatrix}
1 + \frac{n_4 \cos \theta_4}{n_1 \cos \theta_1} \\
\frac{n_2 \cos \theta_2 + \frac{n_4 \cos \theta_4}{n_2 \cos \theta_2}}{\frac{n_2 \cos \theta_2 + \frac{n_4 \cos \theta_4}{n_2 \cos \theta_2}}}
\end{pmatrix} \cos \phi_2 + \begin{pmatrix}
\frac{n_3 \cos \theta_3 + \frac{n_4 \cos \theta_4}{n_3 \cos \theta_3}}{\frac{n_3 \cos \theta_3 + \frac{n_4 \cos \theta_4}{n_3 \cos \theta_3}}}
\end{pmatrix} (i\sin \phi_2) \cos \phi_1 + 
\]

\[
M_{12} = \frac{1}{2} \begin{pmatrix}
1 - \frac{n_4 \cos \theta_4}{n_1 \cos \theta_1} \\
\frac{n_2 \cos \theta_2 - \frac{n_4 \cos \theta_4}{n_2 \cos \theta_2}}{\frac{n_2 \cos \theta_2 - \frac{n_4 \cos \theta_4}{n_2 \cos \theta_2}}}
\end{pmatrix} \cos \phi_2 + \begin{pmatrix}
\frac{n_3 \cos \theta_3 - \frac{n_4 \cos \theta_4}{n_3 \cos \theta_3}}{\frac{n_3 \cos \theta_3 - \frac{n_4 \cos \theta_4}{n_3 \cos \theta_3}}}
\end{pmatrix} (i\sin \phi_2) \cos \phi_1 + 
\]

\[
M_{21} = \frac{1}{2} \begin{pmatrix}
1 - \frac{n_4 \cos \theta_4}{n_1 \cos \theta_1} \\
\frac{n_2 \cos \theta_2 - \frac{n_4 \cos \theta_4}{n_2 \cos \theta_2}}{\frac{n_2 \cos \theta_2 - \frac{n_4 \cos \theta_4}{n_2 \cos \theta_2}}}
\end{pmatrix} \cos \phi_2 - \begin{pmatrix}
\frac{n_3 \cos \theta_3 - \frac{n_4 \cos \theta_4}{n_3 \cos \theta_3}}{\frac{n_3 \cos \theta_3 - \frac{n_4 \cos \theta_4}{n_3 \cos \theta_3}}}
\end{pmatrix} (i\sin \phi_2) \cos \phi_1 - 
\]

\[
M_{22} = \frac{1}{2} \begin{pmatrix}
1 + \frac{n_4 \cos \theta_4}{n_1 \cos \theta_1} \\
\frac{n_2 \cos \theta_2 + \frac{n_4 \cos \theta_4}{n_2 \cos \theta_2}}{\frac{n_2 \cos \theta_2 + \frac{n_4 \cos \theta_4}{n_2 \cos \theta_2}}}
\end{pmatrix} \cos \phi_2 - \begin{pmatrix}
\frac{n_3 \cos \theta_3 + \frac{n_4 \cos \theta_4}{n_3 \cos \theta_3}}{\frac{n_3 \cos \theta_3 + \frac{n_4 \cos \theta_4}{n_3 \cos \theta_3}}}
\end{pmatrix} (i\sin \phi_2) \cos \phi_1 - 
\]

(6) (7) (8)
Electromagnetic Waves Propagation Characteristics in Superconducting Photonic Crystals

\[ M_{22} = \frac{1}{2} \left[ 1 + \frac{n_4 \cos \theta_4}{n_1 \cos \theta_1} \right] \cos \phi_2 - \frac{n_3 \cos \theta_3 \left( \frac{n_2 \cos \theta_2 + n_4 \cos \theta_4}{n_1 \cos \theta_1} \right) \sin \phi_2 + \frac{n_3 \cos \theta_3}{n_1 \cos \theta_1} \cos \phi_1 - \frac{1}{2} \left( \frac{n_2 \cos \theta_2 + n_4 \cos \theta_4}{n_1 \cos \theta_1} \right) \cos \phi_2 - \frac{n_2 \cos \theta_2 + n_4 \cos \theta_4}{n_1 \cos \theta_1} \cos \phi_2 - \frac{n_2 n_4 \cos \theta_2 \cos \theta_4}{n_1 n_3 \cos \theta_1 \cos \theta_3} \left( i \sin \phi_1 \right) \]

Where the reflectance and transmittance can be written as:

\[ R = \left| r \right|^2 = \left| \frac{M_{21}}{M_{11}} \right|^2 \] (10)

\[ T = \frac{n_4 \cos \theta_4}{n_1 \cos \theta_1} \left| h \right|^2 = \frac{n_4 \cos \theta_4}{n_1 \cos \theta_1} \left| \frac{1}{M_{11}} \right|^2 \] (11)

Where r and t is the reflection and transmission and we can also obtain by the same method the components of the transfer matrix method (TMM) for P-wave.

### 3. Results and discussions

The periodicity of the permittivity plays the same role for the photons that propagate inside the structure than the atomic potential for the electrons. Leading further this analogy, the thicknesses and the index contrast of the photonic crystal determinate many of its optical properties as it does for conduction properties of semiconductors. Playing on these two parameters, we can obtain frequency ranges for which light propagation is forbidden in the material and others ranges for which light can propagate. These frequency ranges are also scale dependent. Reducing the size of the elementary cell of the periodic lattice shifts the whole frequency range to higher values. The consequence of this property is the possibility to transpose a photonic crystal design from the microwave domain to infrared or visible wavelengths. In our results we have studied one dimensional superconducting (Super)/dielectric (Na3AIF6) photonic crystals (SuperDPC’s). In all our figures we have used the thickness of Na3AIF6 layer is 320nm and the thickness layer of superconducting material is 80nm. Also we have used different periods equal to 7 and the incidence of angle is 48° for the all our results (Fig’s 2 and 3).

In fig. 2a we have examined the transmittance in the case of s-polarized depend on the wavelengths in the range of ultraviolet (UV), visible (VI) and near infrared (NIR). We have obtained the magnitude of transmittance 100% from 100nm to 350nm UV range) and we have obtained the PBG from 600nm to 1050nm. At the 730 nm we have got unique peak explaining as a defect localized mode which can be used as Fabry-Port micro cavity, this is a good application. In the case of p-polarized (fig.2b), we can show different results and there are about seven PBG’s in the range from 100nm to 900nm. The width of each PBG is widest at the long wavelength as from 700nm to 900nm and is narrowest at the short wavelengths as at 100 nm. Also we have examined the angle dependence on wavelengths to Super/Nas3AIF6 structure (fig.3).
Fig. 2. The transmittance spectra in Super (80nm)/Na3AIF6 (320nm) structure, N=7, a) p-pol and b) s-pol
Fig. 3. The incident angle dependence on wavelength to Super/Na3AIF6 structure

4. Conclusion

We performed numerical analyses to investigate the wave propagation characteristics of a simple-one dimensional superconducting(Super)-dielectric Na3AIF6 structure. The advantage of a photonic crystal with superconducting particles is that the dissipation of the incident electromagnetic wave due to the imaginary part of the dielectric function is much greater for normal metallic than for superconducting particles, because the imaginary part of the dielectric function for superconducting particles is negligible in comparison with the imaginary part of the dielectric function for normal metal particles at frequencies smaller than the superconducting gap. We have obtained good applications at the 730 nm and we have got unique peak explaining as a defect localized mode which can be used as Fabry-Port micro cavity.

5. References

The book collects original and innovative research studies of the experienced and actively working scientists in the field of wave propagation which produced new methods in this area of research and obtained new and important results. Every chapter of this book is the result of the authors achieved in the particular field of research. The themes of the studies vary from investigation on modern applications such as metamaterials, photonic crystals and nanofocusing of light to the traditional engineering applications of electrodynamics such as antennas, waveguides and radar investigations.

How to reference

In order to correctly reference this scholarly work, feel free to copy and paste the following:
