Comparison among Different Sale-Bidding Strategies to Hedge against Risk in a Multi-Market Environment

Daniele Menniti, Nadia Scordino, Nicola Sorrentino and Antonio Violi
University of Calabria, Dep. of Electronic, Computer and System Science
via Pietro Bucci, 87036 Arcavacata di Rende (CS), Italy

1. Introduction
With deregulation in the electricity market industry, competition was introduced among Generation Companies (GenCos), which no longer enjoyed guaranteed rates of return, like in old regulated environment, since price of electricity was no more fixed but varying according to market conditions. The price of electricity GenCos receive in the new competitive market depends on many factors of uncertainty: other GenCos bidding strategies, congestion in transmission, power demand, volatility of spot price of electricity (Liu & Wu, 2006). Scheduling decisions of GenCos are then a determinant factor of their own profitability, which nonetheless depends on either how much a GenCo is able to evaluate market risks or how it can manage such risks. Risk management is the process of achieving a desired return/profit through a particular strategy, which should take into account all the aforementioned factors of uncertainty. However, the complexity of the problem is so high that only strategies taking into account a subset of the above uncertainty factors were proposed in the literature. In order to reduce spot price volatility, diversification and portfolio optimization in physical trading markets were proposed (see e.g. (Liu & Wu, 2006)). Other GenCos bidding strategies and congestion management were conversely embedded in several algorithms based on game theory and evolutionary programming (see e.g. (Byde, 2003) through (Jia et al., 2007)). Nonetheless the problem of demand forecasting was faced from different points of view (Darbellay, 2001), (Kirschen, 2003) but only in very few cases it was introduced in a risk management formulation (Zhou Ming, 2003) through (Conejo et al., 2008). In particular (Menniti et al. 2007) formulates a stochastic optimization problem with recourse as a tool to decide how much energy to bid in a multi-session market, with the aim to maximize the overall profit and minimize the risk of achieving revenues lower than a given threshold, with risk measured by the Conditional Value at Risk. (Conejo et al., 2008) utilizes a stochastic optimization problem with recourse very similar to the one in (Menniti et al., 2007), which nonetheless addresses the problem of a power producer facing the possibility of signing forward contracts as a form of protection against pool price volatility but at the cost of lower expected profits and considering only one market session over three of the present proposal.
This paper proposes a comparison among different sale-bidding strategies embedding risk due to daily-price volatility and to uncertainty typical of a process of bids acceptance, as well as delivery risk due to transmission congestion, taking into account zonal spot prices. The GenCo was modeled as a price taker, its price bids coincide with marginal costs and thus only energy bids were represented as decision variables. Each sale-bidding strategy then consists of the hourly energy quantities to bid for the 24 hours of the next day in a multi-session market, with the aim of maximizing overall profits and minimizing risk exposure. The convenience of a strategy was evaluated in terms of efficient frontier (Liu & Wu, 2006), that is the set of non-dominated solutions, in terms of maximum expected profit and minimum risk of profit variation, for varying values of risk aversion, whereas risk of profit variation was modelled using a discrete formulation of the Conditional Value at Risk (Rockafellar & Uryasev, 2000). The efficient frontiers relating to different sale-bidding strategies were produced by means of an enhanced formulation of the proposal in (Menniti et al. 2007) of a mixed-integer multi-stage stochastic programming problem with recourse. The problem is stochastic since it takes into account volatility of spot prices, modeled with a set of discrete variables, whereas a set of relating outcomes of these discrete variables is called scenario (Birge & Louveaux, 1997).

Besides the stochastic nature of the proposed optimization problem, it is worth to underline the need for a multi-stage formulation. Bidding strategies in energy and reserve markets are consecutive: the decision on the quantity of energy to bid in reserve market is a consequence of the clearing of previous markets, from which different levels (multi-stage) of decisions. The possibility to dynamically decide the quantity of energy to bid (multi-stage decision strategy), depending on the acceptances in preceding markets, allows to reduce risk in comparison to other strategies, such as fixed-mix and greedy (Dempster et al., 2002), (Fleten et al. (2002). In fact, according to a fixed-mix strategy, bids are percentages of the available capacity and a priori decided. Nonetheless a greedy strategy is a particular fixed-mix chance in which the whole production capability is devoted to the forecasted most convenient market session. Simulations were carried out in order to set up the efficient frontiers for multi-stage, fixed-mix and greedy strategies, applying the enhanced multi-stage stochastic programming problem to the Italian Power Exchange (IPEX) framework, and using field data of historical trends in the Italian market.

2. Italian market structure overview

In this section a description is provided of the basic Italian market structure which, like most of electricity markets, presents two alternatives to trade energy: Power Exchange and (physical) forward market.

2.1 Power exchange

Power Exchange is managed by a market operator, GME (www.mercatoelettrico.org), which determines the generation units to be deployed and how much energy each selected unit should produce to meet power demand. From a GenCo’s point of view, selling energy in the Power Exchange (PEx) means to submit a bid (price and quantity) and get either of the two alternative results: (1) PEx accepts the bid and pays the Market Clearing Price (MCP) for the actual energy output of the GenCo; (2) PEx rejects the bid, and the GenCo sells nothing in the spot market. MCP depends on bids of all market participants, as well as on demand of energy, and is therefore uncertain. Unlike other European energy markets, e.g. Powernext in
France or EEX in Germany, GME Power Exchange is not a merely financial market with the sole purpose of determining prices and quantities, but an actually physical market, where physical injection and consumption schedules of energy are defined as a result of a clearing process.

To clear the market, a zonal model is used to manage network congestions, thus zonal prices will value producer bids.

Moreover, the Italian Power Exchange is made up of three sessions: the **Day-Ahead Market**, the **Intraday Market** and the **Ancillary Services Market**, which are described in detail below for completeness sake.

The **Day-Ahead Market** (DAM) takes place in the morning of the “day-ahead” of the day of delivery. At the end of the offer/bid submission sitting, GME activates the market solution process. For each hour of the following day, the market algorithm accepts offers/bids so as to maximize the value of transactions, while satisfying transmission limits on capacity between zones. DAM clearing energy quantities and prices define the injection and consumption schedules for each hour of the following day.

The **Intraday Market** (IM) is not configured as a trading market, since participants submit demand offers or supply bids only to revise schedules resulting from the Day-Ahead Market. This market takes place immediately after the Day-Ahead Market, usually in the afternoon. The process of acceptance of offers/bids in the Intraday Market is similar to that described for the Day-Ahead Market. However, in the Intraday Market, also accepted offers/bids referring to consumption points are remunerated at zonal clearing prices and not at unique market clearing prices of the IM, like in the DAM.

The need for an Intraday Market after the Day-Ahead Market arises because of the use of simple offers/bids: since the 24 hourly schedules of injection or consumption are determined independently of each other, they are not guaranteed to be jointly consistent with constraints of production units. As an example, suppose a unit with a start-up time of 2 hours submits 24 supply bids for 100 MWh at given prices for the 24 hours of the next day, and all bids are accepted but one, at 7 a.m. The daily generation schedule resulting from the market would be then unfeasible for such a unit, whereas it cannot be shut-down at 7:00 and started-up at 8:00. The availability of an Intraday Market will thus allow that unit to submit appropriate demand/supply bids in order to revise previous unfeasible schedules.

The **Ancillary Services Market** (ASM) is the session within which market participants submit offers/bids to increase or decrease energy injection or consumption. The Italian TSO, TERNA (ex GRTN), uses these offers/bids a) as-planned, to correct any schedule violating transmission limits and to create reserve margins for the following day; b) in real-time, to re-establish an equilibrium between demand and production of energy, in the case of deviations from schedules. Unlike what happens in energy markets, offers/bids in the Ancillary Services Market are remunerated at the offered price rather than at the corresponding hourly zonal price.

The more the number of units in a GenCo ownership and of instances of market sessions, the higher the complexity level of the decision problem to be solved and the more risky the bidding operation. A valid means to control and reduce risk is diversification. Diversification is to engage in a wide variety of markets so that the exposure to the risk of any particular market is limited (Liu & Wu, 2006). Applying this concept to energy trading in an electricity market, diversification means to trade energy through different physical trading approaches, in which actual physical energy is traded, such as spot and contract markets.
2.2 Contract market

In a contract market, GenCos trade energy by way of signing physical forward contracts with their counterparties (e.g., energy consumers). Specific details, such as trading quantity (MW), trading duration (h), trading price (€/MWh), and delivery point are bilaterally negotiated between GenCo and consumer. Bilateral contracts are signed before the actual trading period, which means that trading quantity and price are set in advance, however they are embedded within the DAM session. In fact, when supply bids and consumption offers are checked for compliance with transmission constraints in the DAM, also bilateral contracts are embedded, with maximum priority of price, i.e. respectively zero-price supply bids and price-independent demand offers. If at least one transmission limit is violated, i.e. there is scarcity of transmission capacity, market is split into two or more zones. In this case, each zone $z$ has a different clearing price $P_z$ and this implies that: i) there is a value of transmission right between zones $x$ and $y$, equal to $P_y - P_x$, i.e. the bilateral contract is required to pay/receive such a fee to/from Terna for flows which contribute to congestion/congestion-relief on the grid. Transmission rights are assigned to bilateral contracts until exhaustion of transmission capacity and thus to the most competitive offers/bids submitted in the market. The bilateral contract will pay a fee for the transmission right, $P_y - P_x$, for the quantity of electricity quoted in the contract. Congestion and the resulting zonal prices are thus uncertain and unpredictable, and this makes risky inter-zonal bilateral contracts, whereas only intra-zonal contracts are risk-free in such a market.

3. Decision approach for the formulation of sale-bidding strategies

The tool used in this paper, already proposed by the authors elsewhere (Menniti et al. 2007), and here enhanced by the introduction of new decision variables and relating constraints, is to be used on a daily basis, the day ahead the DAM, IM and ASM sessions take place, by a GenCo which decides to recur to bidding diversification in order to maximize overall profits and minimize risk exposure. The GenCo is also supposed to honor a physical forward contract, in the remainder bilateral contract, according to a daily load profile at a given price. As an improvement of the stochastic programming problem presented in (Menniti et al. 2007), the GenCo can decide to which production units refer the bilateral contract, given that a number of units in its ownership are located in different zones and that the price cleared in a zone may differ from that of another zone because of delivery risk due to transmission congestions. As a result of the optimality of the adopted strategy, the bilateral contract will be then honored by units placed in the zones where zonal prices result the lowest, producing energy where it is more convenient and thus minimizing delivery risk. The interested reader is referred to Appendix C for a more detailed treatment of the constraints of the problem (equations (16)-(33)), whereas the objective function which drives the optimal choice of energy to bid in a multi-session market is formulated in the following section.

3.1 Objective function of the problem

As said above, the aim of the paper is proposing a way to define a sale-bidding strategy for a GenCo who wants to maximize overall expected profits over the operating day and conversely minimize risk exposure. For this reason, the authors considered in (Menniti et al. 2007) a risk-reward structure for the objective function, which is a choice of modeling widely used in many applicative contexts characterized by a high level of uncertainty,
Comparison among Different Sale-Bidding Strategies to Hedge against Risk in a Multi-Market Environment

This choice consists in a weighted sum of two terms: the expected overall profit and the Conditional Value at Risk on possible losses occurring in the entire planning horizon of one day:

$$\text{max } \mathbb{E}[\text{Profit}]-\kappa \text{CVaR}$$  \hspace{1cm} (1)

where $\kappa$ is a user-defined trade-off value, called in the remainder the risk aversion parameter, which models how much the GenCo is averse to risk, whereas high values of $\kappa$ model a conservative approach, i.e. low propensity to risk. For each scenario, i.e. for each likely realization of the discrete variables modeled by an intuitive scenario tree, the overall profit for the entire planning horizon of one day is defined as the difference between revenues and costs. Revenues and costs depend on prices and on the energy actually cleared, thus not known in advance and modeled as expected values. Let $\eta_{it}^s$, $\pi_{it}^l$, and $\theta_{it}^\nu$ denote probabilities of occurrence of outcomes $s$, $l$ and $\nu$, respectively related to the DAM, IM and ASM sessions, for each period $t$ and for the zone which generation unit $i$ belongs to. The expected value of overall profits can then be defined as:

$$\mathbb{E}[\text{Profit}]=\sum_{i=1}^{I} \sum_{t=1}^{T} \sum_{\nu=1}^{V} \theta_{it}^\nu (R_{it}^\nu - C_{it}^\nu) + \sum_{t=1}^{T} R_{it}^{Bil}$$  \hspace{1cm} (2)

with $R_{it}^{Bil}$ being the revenue due to bilateral contract at time $t$, a constant quantity not dependent on the decision variables.

Moreover, let $\lambda_{it}^s$, $\mu_{it}^l$, and $\zeta_{it}^\nu$ denote outcomes of random clearing prices in the DAM, IM and ASM at the $t$-th hour for the zone which generation unit $i$ belongs to. Overall revenues for generation unit $i$ at time $t$ according to outcome $\nu$ are then:

$$R_{it}^\nu = R_{it}^{p(p(\nu))} + R_{it}^{p(\nu)} + R_{it}^{ASM} \quad \forall \nu, \forall i, \forall t$$  \hspace{1cm} (3)

Where:

$$R_{it}^{p(p(\nu))} = \lambda_{it}^{s(\nu)} x_{it} \quad \forall s, \forall i, \forall t$$  \hspace{1cm} (4)

$$R_{it}^{p(\nu)} = \mu_{it}^{l(\nu)} y_{it}^{\nu} \quad \forall l, \forall i, \forall t$$  \hspace{1cm} (5)

$$R_{it}^{ASM} = \zeta_{it}^{\nu} \rho_{it}^{\nu} z_{it}^{\nu} \quad \forall \nu, \forall i, \forall t$$  \hspace{1cm} (6)

Overall cost related to generator $i$ at time $t$ according to outcome $\nu$ is instead:

$$C_{it}^\nu = C_{it}^{\text{Prod}} + C_{it}^{\text{SU}} + C_{it}^{\text{SD}} + C_{it}^{p(\nu)} \quad \forall \nu, \forall i, \forall t$$  \hspace{1cm} (7)

where:

$$C_{it}^{\text{Prod}} = (\alpha_{it} U_{it} + \beta_{it} Q_{it}^\nu) \quad \forall \nu, \forall i, \forall t$$  \hspace{1cm} (8)

is the production cost of unit $i$ at time $t$, whereas $\alpha_i$ and $\beta_i$ are coefficients of the production-cost function of unit $i$, constant for each hour $t$. It should be noted that power output $Q_{it}^\nu$ becomes zero whereas unit $i$ is not committed, i.e. binary variable $U_{it}$ is equal to zero, as
stated by constraints (16) and (25) provided in the Appendix C. Production-cost function was approximated with such a linear function since we are dealing with marginal costs.

\[ C_{itSU} = SU_i \Delta_{it} \quad \forall i, \forall t \]  
\[ C_{itSD} = SD_i \Delta_{it} \quad \forall i, \forall t \]

with \( C_{itSU} \) and \( C_{itSD} \) the start-up and shut-down costs incurred if unit \( i \) is going to be started-up/shut-down at the \( t \)-th hour, whereas \( SU_i \) and \( SD_i \) are linear cost coefficients. Finally, \( \Delta_{it} \) and \( \Delta_{it}^{-} \) are binary variables, respectively equal to 1 if unit \( i \) is going to be started-up/shut-down at the \( t \)-th hour, 0 otherwise;

\[ C_{itAM} = \mu_{it} \delta_{it} y_{it}^{p(0)} \quad \forall i, \forall t \]

When the GenCo buys energy on the IM, the purchase cost (11) has an expression similar to IM revenues (5): the bid of energy accepted in the IM, \( \delta_{it}^{+} y_{it}^{p(0)} \), will be remunerated at the zonal clearing price, \( \mu_{it} \) (depending on the zone unit \( i \) belongs to). As to the term CVaR in (1), a discrete formulation is given in Appendix B.

The solution of the stochastic optimization programming problem (1), (16)-(33) for different values of the risk aversion parameter \( \kappa \), provides the efficient frontier for any adopted bidding strategy, which represents how the expected profit augments as CVaR increases (see Fig. 10 of the case study).

4. Numerical results

This section presents the results obtained simulating different bid strategies adopted by a small GenCo operating in the Italian PEx, which was supposed to own 3 thermo-electrical units, with operational features provided in Tab. 1, whose last row indicates where units are located. Note that the minimum power output of all units, \( Q_{i}^{\text{min}} \), is 0 MW. The zonal clearing prices of the Italian Power Exchange (IPEx) in January 2005 were considered for simulation (www.mercatoelettrico.org). Moreover, the GenCo serves a bilateral contract

<table>
<thead>
<tr>
<th></th>
<th>Unit 1</th>
<th>Unit 2</th>
<th>Unit 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( SU_i ) [€]</td>
<td>805</td>
<td>805</td>
<td>805</td>
</tr>
<tr>
<td>( SD_i ) [€]</td>
<td>43</td>
<td>43</td>
<td>43</td>
</tr>
<tr>
<td>( UT_i ) [h]</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>( DT_i ) [h]</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>( \alpha_i ) [€]</td>
<td>892</td>
<td>892</td>
<td>892</td>
</tr>
<tr>
<td>( \beta_i ) [€/MWh]</td>
<td>14</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>( Q_i^{\text{max}} ) [MW]</td>
<td>500</td>
<td>400</td>
<td>280</td>
</tr>
<tr>
<td>( Q_i^{\text{min}} ) [MW]</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

| Location Zone | North | Middle North | Sardinia |

Table 1. Units data
which is supposed to absorb 40% of the daily power production of the GenCo, according to the daily consumption profile shown in Fig. 1.

The remainder of the section is organized as follows: first, field data of historical trends in the Italian PEX during January 2005 were reported and analyzed. Using the heuristic procedure for scenario generation explained in Appendix A, 300 scenarios for prices and percentages of acceptance were generated. A comparison among three different strategies, multi-stage, fixed-mix and greedy, was then proposed and commented, in terms of efficient frontiers, resulting from the use of the proposed optimization tool.

Fig. 1. Reference load profile (Qbil)

**4.1 Italian market data**

Mean hourly prices averaged over all zones in the Day-Ahead Market during January 2005 were plotted in Fig. 2, which highlights a high inter hour volatility, with values of prices ranging from a minimum of €/MWh 31.32 (4 a.m.) to a maximum of €/MWh 114.21 (7 p.m.).

Fig. 2. Mean hourly prices [€/MWh] in the Italian Day-Ahead Market over January 2005

Fig. 3 compares mean monthly prices, always averaged over all zones, in the Day-Ahead Market and Intraday Market over 2005, from which it can be noticed how prices were quite similar in the DAM and IM, whereas during January DAM mean price resulted 2.27% higher than IM mean price, and this feature was taken into account within the scenario tree generation, as described in Appendix A. Conversely, Fig. 4 depicts mean hourly prices of DAM and ASM over January 2005. Mean hourly prices of ASM were derived as the average over zones of “the last” zonal hourly bids accepted in the ASM (that is bids with the highest prices).
4.2 Comparison among multi-stage, fixed-mix, and greedy strategies

The enhanced stochastic optimization problem was implemented on a Pentium 4, 1.8 GHz with 1056 GB of RAM using AIMMS (www.aimms.com) as modeling environment and ILOG CPLEX 10 (www.ilog.com) as optimization solver. The size of the mixed-integer linear programming problem (1), (16-33) expressed as the number of continuous variables, binary variables and constraints is provided in Tab. 2. More in detail, \( I, T, S, L \) and \( V \) are respectively the number of units, the number of time intervals, and the number of likely outcomes of the Day-Ahead Market, of the Intraday Market and of the Ancillary Services Market. The CPU time required to solve the stochastic problem was of 1175.38 sec per strategy, therefore suitable for practical implementations.

Starting from all the previous assumptions, the purpose of the following simulation was twofold: \( i) \) showing effects of risk aversion on a sale-bidding strategy by a GenCo operating in different market sessions; \( ii) \) comparing the effectiveness of the proposed multi-stage decisional approach with other realistic classic strategies, such as fixed-mix and greedy. These two classic strategies are similar since both decide a priori the offers of energy in each market session as predetermined percentages of the available capacity of production. However, according to a greedy strategy, values assumed for these percentages were chosen so as to concentrate profits in a “greedy” way in one of the three markets of IPEx, and

Fig. 3. Mean monthly prices [€/MWh] in the Italian Day-Ahead Market and Intraday Market over 2005

Fig. 4. Mean hourly prices [€/MWh] in the Italian Day-Ahead Market and Ancillary Service Market over January 2005
Comparison among Different Sale-Bidding Strategies to Hedge against Risk in a Multi-Market Environment

Table 2. Problem size

<table>
<thead>
<tr>
<th>Continuos decision variables</th>
<th>I T(2+2S+L+V)=29 664</th>
</tr>
</thead>
<tbody>
<tr>
<td>Binary variables</td>
<td>I T(3+2S)=1656</td>
</tr>
<tr>
<td>Constraints</td>
<td>I T(7+5S+L+2V)+I+T=54 531</td>
</tr>
</tbody>
</table>

assumed in the paper equal to 100% for offers in the DAM, and 0% in the IM and ASM. Differently, for a fixed-mix strategy, offers were “distributed” in the DAM, IM and ASM according to different predetermined percentage, assumed equal to 80%, 5% and 15% respectively, these percentage reflecting the real trend of the Italian Market over 2005. Moreover, a fixed-mix and a greedy strategy also differ for the way in which offers are managed in the IM and ASM. In fact, with a fixed-mix strategy, when an offer is refused by a generic market session, the residual capacity of production may be offered in other subsequent market sessions, always according to the programmed percentages, which for a greedy strategy are instead 0%.

According to the above assumptions, offers in the DAM, IM, and ASM obviously vary as a function of the particular strategy chosen, as Fig. 5 shows for a generic value of the risk aversion parameter (κ=0.5). For κ=0.5 most of offers are committed in the DAM if a greedy strategy is adopted, whilst correspondingly offers decrease if we move from a greedy towards a multi-stage strategy, in favor of offers in the more remunerative Ancillary Service Market (Stage 3).

Fig. 5. Energy bids in the DAM in MWh as a function of greedy, fixed-mix, and multi-stage strategies, for κ=0.5
Moreover, observing the generic strategy, the decision of which units to commit is a function of the geographic position of units themselves, whereas unit 2 belongs to Middle-North and presents offers higher than unit 3, which belongs to Sardinia, where prices were higher than in Middle-North.

Obviously, how offers are distributed over sessions and over 24 hours also depends on the aversion to risk modeled by the $\kappa$ parameter. Simulations for varying values of the risk aversion parameter (ranging from 0 (full acceptance of risk) to 1 (maximum aversion to risk) with a step of 0.1) were performed. Calling efficient frontier the set of solutions for different values of risk aversion (Liu & Wu, 2006), a comparison among the efficient frontiers obtained adopting multi-stage, fixed-mix and greedy strategies was reported in Tab. 3, and Fig. 6. Each point of Fig. 6 represents the most profitable sale-bidding strategy as a function of a given risk level or, equivalently, the less risky sale-bidding strategy as a function of a given value of profitability. Moreover, focusing attention on a generic strategy, it is evident that a non-conservative approach, which corresponds to low values of $\kappa$, allows higher potential gains, although a higher risk value as well.

With the purpose of highlighting the validity of a multi-stage decisional approach versus other “non-recursive” approaches, Fig. 6 proves that the efficient frontier of a multi-stage strategy dominates the others, in terms of both profitability and risk. With non-recursive approach we denote the impossibility to “recur” to successive decisions to “correct” likely losses due to undesired realizations of variables of previous stages. This result can be attributed to the possibility actually offered by a multi-stage stochastic strategy, i.e. to take into account different likely scenarios and to dynamically correct previous decisions according to the observed outcomes of the market clearing process.

<table>
<thead>
<tr>
<th>$\kappa$</th>
<th>Multistage $E[\text{Profit}]$</th>
<th>CVaR $E[\text{Profit}]$</th>
<th>Fixed-Mix $E[\text{Profit}]$</th>
<th>CVaR $E[\text{Profit}]$</th>
<th>Greedy $E[\text{Profit}]$</th>
<th>CVaR $E[\text{Profit}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>621978</td>
<td>51754.8</td>
<td>282471.3</td>
<td>64916.5</td>
<td>208544.6</td>
<td>44870.86</td>
</tr>
<tr>
<td>0.10</td>
<td>621745.1</td>
<td>39608.43</td>
<td>282471.3</td>
<td>39623.87</td>
<td>208420.9</td>
<td>28721.51</td>
</tr>
<tr>
<td>0.20</td>
<td>621332.1</td>
<td>37463.89</td>
<td>282471.3</td>
<td>39623.87</td>
<td>208416.8</td>
<td>28685.03</td>
</tr>
<tr>
<td>0.30</td>
<td>621085.8</td>
<td>36311.77</td>
<td>282471.3</td>
<td>39623.87</td>
<td>208416</td>
<td>28681.49</td>
</tr>
<tr>
<td>0.40</td>
<td>621085.8</td>
<td>36311.77</td>
<td>282471.3</td>
<td>39623.87</td>
<td>208416</td>
<td>28681.49</td>
</tr>
<tr>
<td>0.50</td>
<td>620916</td>
<td>35910.5</td>
<td>282464.9</td>
<td>39608.43</td>
<td>208416</td>
<td>28681.41</td>
</tr>
<tr>
<td>0.60</td>
<td>620916</td>
<td>35910.5</td>
<td>282464.9</td>
<td>39608.43</td>
<td>208416</td>
<td>28681.41</td>
</tr>
<tr>
<td>0.70</td>
<td>620179.7</td>
<td>34835.55</td>
<td>282464.9</td>
<td>39608.43</td>
<td>208416</td>
<td>28681.41</td>
</tr>
<tr>
<td>0.80</td>
<td>617893.5</td>
<td>31747.6</td>
<td>282464.9</td>
<td>39608.43</td>
<td>208407</td>
<td>28668.94</td>
</tr>
<tr>
<td>0.90</td>
<td>613996.5</td>
<td>27249.12</td>
<td>282464.9</td>
<td>39608.43</td>
<td>208372.8</td>
<td>28626.79</td>
</tr>
<tr>
<td>1</td>
<td>613996.5</td>
<td>27249.12</td>
<td>282464.9</td>
<td>39608.43</td>
<td>207258.6</td>
<td>27495.31</td>
</tr>
</tbody>
</table>

Table 3. Expected profit and CVaR as a function of varying values of the risk aversion parameter according to multistage, fixed-mix and greedy strategies

Simulations for intermediate values of $\kappa$, not reported here for brevity sake, obviously produced different intermediate schedules, thus a clear conclusion can be drawn: the GenCo should make a decision on its desired level of risk before solving its scheduling problem and then bidding in the electric energy market.
Fig. 6. Efficient frontiers for multistage, fixed-mix and greedy strategies

5. Conclusions

A multi-stage strategy represents the best trade-off in terms of maximum expected profit and minimum risk of profit volatility in comparison with other strategies, such as fixed-mix or greedy, which do not allow to “recur” to successive decisions to “correct” likely losses due to undesired previous outcomes. The appropriate behaviour of the proposed multi-stage strategy is demonstrated through a case study based on field record of the Italian PEx, in which it was shown that the efficient frontier of the multi-stage strategy dominates the efficient frontiers of the other two strategies analyzed.

6. Appendix A: Heuristic procedure for scenario generation

A GenCo which recurs to bidding diversification in the DAM, IM and ASM must decide within different succeeding time horizons “how much energy must be bid” and “at which price”. The first decision, “how much energy must be bid” in each market session, was modeled by means of continuous decision variables, whereas price bids in the DAM and IM were not considered as decision variables, having assumed that the GenCo is a price-taker and has no capability of altering the electricity price. In particular, the evolution of the random clearing prices of DAM and IM was modeled by an intuitive scenario tree (see Fig. 7), and a zonal model was adopted thus taking into account also delivery risk due to transmission congestion (Liu & Wu, 2006). Differently from DAM and IM, according to the “pay as bid” mechanism, bids accepted in the ASM are remunerated at the price bid, which clearly represents a decision variable. For the sake of simplicity, as a first step of this research activity, ASM price bids were assumed in the paper as input data, within nodes of the third stage, but they were not considered stochastic variables such as DAM and IM clearing prices, thus they do not vary with the scenario, but only with hours and zones. This means that a unit belonging to zone A is supposed to bid at a different price in comparison with a unit located in zone B at the same $t$-th hour.
Besides uncertainty of the clearing zonal prices of DAM and IM, the above mentioned scenario also captures the uncertainty relating the acceptance of energy bids, respectively, $\gamma_{it}^s$, ($\delta_{it}^{l+}$, $\delta_{it}^{l-}$), and $\rho_{it}^v$, over the three different time horizons associated to DAM, IM and ASM sessions.

Fig. 7. Scenario tree formulation.

Root-node stands for stage 0, and embeds no uncertainty. Stages 1-3 model the three market sessions of Power Exchange and are thus associated with the sequential decision process. Each generic node $k$ has a unique immediate predecessor $p(k)$ in the preceding stage and a finite number of successors in the next stage, but the root-node. Nodes without any successor are called the leaves of the tree. They are in a one-to-one correspondence with each scenario, whereas a scenario is a path from root-node to a leaf and represents a joint outcome over all market sessions. At each hour $t=1..T$ and for each zone, each node captures the evolution of random decision variables by which uncertainty is represented, i.e. percentages of bids accepted in the corresponding marker session, clearing prices and relating probabilities of occurrence.

The heuristic procedure given below receives historical time series data as input and generates the scenario tree used for simulation as output.

Step 1  For each $t$-th hour and zone $z$.

1.1 Observe $P_{z_i}^{DAM\min}$ and $P_{z_i}^{DAM\max}$.

1.2 Devide the corresponding range into 10 sub-ranges of equal amplitude, as depicted in Fig. 8.

1.3 Observe, for each $s$-th sub-range, $P_{z_i}^{s\min},P_{z_i}^{s\max}$, and calculate $P_{z_i}^{s\text{mean}}$.

1.4 Compute the probability of occurrence of $P_{z_i}^{s\text{mean}}$.

Step 2  Assign $\gamma_{it}^s=0$ for $s=1-4$, $\gamma_{it}^s=0.5$ for $s=5-6$, $\gamma_{it}^s=1$ for $s=7-10$.

Step 3  For each $s$-th node of DAM, $t$-th hour and zone $z$.

3.1 scale $P_{z_i}^{s\text{mean}}$ by ten scaling factors opportunely chosen.

3.2 Compute the occurrence probability of the $l$-th hourly zonal IM clearing price as 1/10 of the occurrence probability of the price of the predecessor node $p(l)$, for the same hour and the same zone.

3.3 Assign $\delta_{it}^{l+}$, $\delta_{it}^{l-}=0$ for $l$=the first 4 sons of p(l), $\delta_{it}^{l+}$, $\delta_{it}^{l-}=0.5$ for $l$=sons 5-6 of p(l), $\delta_{it}^{l+}$, $\delta_{it}^{l-}=1$ for the remaining sons of p(l).

Step 4  For each $t$-th hour and zone $z$.

4.1 Observe the last price bid accepted in the ASM and store it in each $v$-th node of the scenario tree.
4.2 Compute the occurrence probability of the \( v \)-th hourly ASM price as \( 4/10, 2/10 \) and \( 4/10 \) of the occurrence probability of the price of the predecessor node \( p(v) \), for the same hour and the same zone.

4.3 Assign to each triplet of nodes in ASM with the same predecessor \( p(v) \) \( \rho^v_0 = 0 \) for \( v = \) the first son of \( p(v) \), \( \rho^v_1 = 0.5 \) for the second son, and \( \rho^v_1 = 1 \) for the last son.

With reference to Stage 1 (DAM session) and focusing attention on each hour \( t \) and zone \( z \), excluding Sundays and Saturday evening, we observed which minimum and maximum zonal prices, \( P^\text{DAM}_{zt}^{\text{min}} \) and \( P^\text{DAM}_{zt}^{\text{max}} \), occurred over the days of January 2005 in the Italian DAM (Step 1.1). We divided the corresponding range into 10 sub-ranges of equal amplitude, each with a lower and upper bound, respectively \( \min_s P^s_{zt} \) and \( \max_s P^s_{zt} \) (Step 1.2). For each \( s \)-th sub-range, the relating mean price, \( P^s_{zt}^{\text{mean}} \), was calculated (Step 1.3), having thus generated \( s = 10 \) likely hourly zonal clearing prices, as depicted in Fig. 8, each corresponding to a node of Stage 1.

The probability of occurrence of \( P^s_{zt}^{\text{mean}} \) was calculated as the number of prices at the \( t \)-th hour belonged to interval \( s \) over all the observed days, divided by the number of observed days (Step 1.4). Step 1 was iterated for each hour \( t \), with \( t = 1..24 \), and for each of the 7 zones of the Italian PEx. Step 2 gives value to percentages of bid acceptance for DAM, \( \gamma^s_{it} \), at each \( t \)-th hour, and for each zone unit \( i \) belongs to. In particular, \( \gamma^s_{it} = 0 \) for \( s = 1-4 \), \( \gamma^s_{it} = 0.5 \) for \( s = 5-6 \), and \( \gamma^s_{it} = 1 \) for \( s = 7-10 \).

Fig. 8. Formulation of scenario tree for prices.

Generation of 10 outcomes of zonal clearing prices for the DAM.

Each node belonging to Stage 1 has also 10 sons, belonging to Stage 2, and, for these last, prices were derived as follows. From the monthly trading report of January 2005 ([www.mercatoelettrico.org](http://www.mercatoelettrico.org)), it was observed that clearing prices of the DAM averaged greater than the corresponding IM clearing prices (2.27%). This behavior was replicated in the generation of the \( L = 100 \) IM clearing prices by scaling the \( P^s_{zt}^{\text{mean}} \) clearing prices of each \( s \)-th father node (DAM) by ten different factors (assumed less than 1 for seven over ten sons and greater than 1 for the remaining nodes, (Step 3.1)). For each \( l \)-th hourly zonal IM clearing price the occurrence probability was computed as \( 1/10 \) of the occurrence probability of the price of the predecessor node \( p(l) \), for the same hour and the same zone (Step 3.2). Percentage of bid acceptance for IM, \( \delta^s_{it}^{+} \) and \( \delta^s_{it}^{-} \), were assumed equal to 0 for 4 nodes, 0.5 for 2 nodes, 1 for the rest of the nodes, all sons of the same predecessor, \( p(l) \) (Step 3.3).

Finally, Stage 3 models the ASM session. Each \( l \)-th predecessor node belonging to the IM (Stage 2) has 3 sons in the ASM and this because the only differentiation among ASM nodes
was simulated by the different percentage of acceptance of a bid, $\rho^v_{it}$. In fact, $\rho^v_{it}$ was assumed equal to 0, 0.5 or 1, respectively meaning bid is refused, unit is marginal, or bid price is less than the highest forecasted accepted price bid. Hourly prices vary with hours and zones, whereas they do not vary with nodes and, for a given hour $t$, and a given zone $z_i$, they were assumed equal to the value of the average, over all the days of January 2005, of the last (i.e. the highest) accepted price bid in the ASM (Step 4.1). For each triplet of nodes in the ASM, probabilities of occurrence were assumed equal to 0.4, 0.2, and 0.4 respectively of the probabilities of the price in the predecessor node $p(v)$ of the IM, for the same hour and the same zone (Step 4.2). Moreover, each triplet of nodes in ASM with the same predecessor $p(v)$ will have percentage of bid acceptance $\rho^v_{it} = 0$ for $v =$ the first son of $p(v)$, $\rho^v_{it} = 0.5$ for $l=$second son of $p(v)$, $\rho^v_{it} = 1$ for $p(v)$ last son.

The overall scenario tree has then $V=300$ leaves, each corresponding to a scenario, that is a likely evolution of uncertain outcomes of the multi-session market.

7. Appendix B: CVaR as risk measure in energy trading

With an evaluation on a 24-hour time-frame, CVaR was here chosen to detect the risk of loss or, at least, the risk of achieving revenues lower than a given threshold for a GenCo which trades energy in both Power Exchange and Contract market. A discrete version (12) for CVaR was formulated elsewhere (Menniti et al., 2007), considering a confidence level $\varepsilon$ equal to 0.95. As demonstrated elsewhere (Ahmed, 2006), CVaR is a risk measure which preserves convexity, and its linear relaxation still maintains this convexity feature.

$$CVaR_\varepsilon = VaR_\varepsilon + \frac{1}{1 - \varepsilon} E \left[ \max \left( Loss^v - VaR_\varepsilon, 0 \right) \right]$$ (12)

where $Loss^v$ is a loss function, here assumed as the opposite of the profit function, $Profit^v$. Since uncertainty of market prices was represented by means of a finite set $V$ of likely scenarios, (12) can be linearized using a set of auxiliary variables and constraints as follows:

$$CVaR_\varepsilon = VaR_\varepsilon + \frac{1}{1 - \varepsilon} \sum_{v=1}^{V} \theta^v \sigma^v$$ (13)

with $\theta^v$ the probability that scenario $v$ can occur, and:

$$\sigma^v \geq Loss^v - VaR_\varepsilon \quad \forall v = 1..V$$ (14)

$$\sigma^v \geq 0 \quad \forall v = 1..V$$ (15)


Classical multi-period problems with unit commitment include provisions for modeling restrictions on the operation of thermal generation units. These restrictions include most notably minimum/maximum power output limits, ramping limitations, and minimum up- and down-time constraints (Shahidehpour et al., 2002), (Conejo et al., 2002). Because we do not implement the presence of hydro generation units and do note devote our attention at
the process of scheduling of tertiary reserves and its later deployment through generation re-dispatch, we do not thus model ramping limitations.

Constraints of the stochastic programming problem modeling the decision problem faced by the GenCo are formulated below as.

\[ x_{it}^h + x_{it}^b \leq Q_i^{\text{max}} U_{it} \quad \forall i, \forall t \]  
(16)

\[ \sum_{i=1}^{J} x_{it}^b = Q_i^{b} \quad \forall t \]  
(17)

\[ y_{it}^{s+} \leq Q_i^{\text{max}} U_{it} - \gamma_{it}^s x_{it} - x_{it}^b \quad \forall i, \forall t, \forall s \]  
(18)

\[ y_{it}^{s-} \leq \gamma_{it}^s x_{it} \quad \forall i, \forall t, \forall s \]  
(19)

\[ y_{it}^{p+} \leq M \phi_{it}^{s+} \quad \forall i, \forall t, \forall s \]  
(20)

\[ y_{it}^{p-} \leq M \phi_{it}^{s-} \quad \forall i, \forall t, \forall s \]  
(21)

\[ \phi_{it}^{s+} + \phi_{it}^{s-} \leq 1 \quad \forall i, \forall t, \forall s \]  
(22)

\[ z_{it}^l \leq Q_i^{\text{max}} U_{it} - \gamma_{it}^{p} x_{it} - x_{it}^b - \delta_{it}^{s+} y_{it}^{p+(l)} + \delta_{it}^{s-} y_{it}^{p-(l)} \quad \forall i, \forall t, \forall l \]  
(23)

\[ Q_i^v = x_{it}^b + \gamma_{it}^{p} x_{it} + \delta_{it}^{p+} y_{it}^{p+(v)} - \delta_{it}^{p-} y_{it}^{p-(v)} + \rho_{it}^v z_{it}^v \quad \forall i, \forall t, \forall v \]  
(24)

\[ Q_i^{\text{min}} U_{it} \leq Q_i^v \quad \forall i, \forall t, \forall v \]  
(25)

\[ \sum_{t=1}^{G_i} (1-U_{it}) = 0 \quad \forall i \]  
(26)

\[ \sum_{l=t}^{t+UT_i-1} U_{il} \geq UT_i \Delta^+_i \quad \forall i, \forall t = G_i + 1..T-UT_i + 1 \]  
(27)

\[ \sum_{l=t}^{t+UT_i-1} U_{il} \geq UT_i \Delta^+_i \quad \forall i, \forall t = G_i + 1..T-UT_i + 1 \]  
(28)

where \( G_i = \min[T, (UT_i-R^0_i)U_{id}] \).

\[ \sum_{t=1}^{F_i} U_{it} = 0 \quad \forall i \]  
(29)

\[ \sum_{l=t}^{t+DT_i-1} U_{il} \geq DT_i \Delta^-_i \quad \forall i, \forall t = F_i + 1..T-DT_i + 1 \]  
(30)
\[
\sum_{i=t}^{T} (1 - U_{it} - \Delta_{it}) \geq 0 \quad \forall i, \forall t = T- DT_{i} + 2..T
\]
(31)

where \( F_i = \min [T, (DT_{i} - S_{i}^0)(1 - U_{i0})] \).

\[
\Delta_{it}^+ - \Delta_{it}^- = U_{it+1} - U_{it} \quad \forall i, \forall t
\]
(32)

\[
\Delta_{it}^+ + \Delta_{it}^- \leq 1 \quad \forall i, \forall t
\]
(33)

### 7.1 First Stage constraints: bidding in the Day-Ahead Market

Constraints (16) require that the sum of a bid of unit \( i \) in the DAM, \( x_{it} \), and of the production to satisfy the bilateral contract, \( x_{it}^{bil} \), must not be exceeded the unit maximum power output, \( Q_{i}^{\max} \) at the \( t \)-th hour.

Constraints (17) guarantee the satisfaction of the bilateral contract by means of zero-price bids in the DAM (\( x_{it}^{bil} \)).

### 7.2 Second Stage constraints: bidding in the Intraday Market

Constraints (18) say that unit \( i \) can sell in the IM at most its maximum capacity, \( Q_{i}^{\max} \), decreased of the accepted bid in the previous DAM, \( \gamma_{it}^x x_{it} + x_{it}^{bil} \).

Constraints (19) limit the purchase bid of unit \( i \) to the only quantity already accepted in the DAM, \( \gamma_{it}^s x_{it} \). Moreover, in order to avoid buying and selling bids in the IM at the same hour by the same unit, we introduced additional binary variables, \( \phi_{it}^+ \) and \( \phi_{it}^- \), and sets of constraints (20)-(22).

### 7.3 Third Stage constraints: bidding in the Ancillary Services Market

Also in the ASM session, the GenCo may commit still unused units or increase the output of one or more units already committed for other sessions, under constraints (23) which impose to respect unit maximum output.

### 7.4 Other constraints

Constraints (24) express the energy produced by each unit \( i \), \( Q_{it}^v \), at each period \( t \) and for each likely outcome \( v \), as the sum of all the energy effectively sold over the three market sessions and by bilateral contract.

Production \( Q_{it}^v \) is also constrained (25) by the minimum power output of unit \( i \), whereas unit \( i \) is committed (\( U_{it} = 1 \)) for period \( t \) under scenario \( v \).

Constraints (26)–(28) represent linear expressions of minimum up-time constraints (Conejo et al., 2002). In particular, set of equations (26) is related to the initial status of the units. Set of equations (27) ensure the satisfaction of minimum up-time constraints during all likely sets of consecutive periods of size \( UT_{i} \). Finally, set of equations (28) is essential for the last \( UT_{i}^{-1} \) periods, i.e. if a unit is started-up in one of these periods, it must still remain on-line during next periods.

Similarly to (26)-(28), constraints (29)–(31) formulate the minimum down-time constraints (Conejo et al., 2002). Equations (29)–(31) are identical to (26)-(28) just changing \( U_{it} \), \( \Delta_{it}^- \), \( DT_{i} \), and \( S_{i}^0 \) with (1- \( U_{it} \)), \( \Delta_{it}^+ \), \( UT_{i} \), and \( R_{i}^0 \), respectively.
Finally, constraints (32) and (33) are necessary to model the start-up and shut-down status of units and to avoid the simultaneous commitment and decommitment of a unit (Conejo et al., 2002).

9. References


Birge JR. & Louveaux FV. (1997), *Introduction to stochastic programming*, 0387982175, New York Springer


www.mercatoelettrico.org


Stochastic Optimization Algorithms have become essential tools in solving a wide range of difficult and critical optimization problems. Such methods are able to find the optimum solution of a problem with uncertain elements or to algorithmically incorporate uncertainty to solve a deterministic problem. They even succeed in "fighting uncertainty with uncertainty." This book discusses theoretical aspects of many such algorithms and covers their application in various scientific fields.

How to reference
In order to correctly reference this scholarly work, feel free to copy and paste the following:
