A Study on Facial Expression Recognition Model using an Adaptive Learning Capability

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1. Introduction

The study of facial expression recognition for the purpose of man-machine emotional communication is attracting attention lately (Akamatsu, 2002a; Akamatsu, 2002b; Akamatsu, 2002c; Akamatsu, 2003; Fasel & Luettin, 2003; Pantic & Rothkrantz, 2000; Tian et al., 2001). Most facial expression recognition models that have been proposed eventually create some classifier based on the expression images taken during a short period of time and using them as base data for learning (Pantic et al., 2005; Gross, 2005). However, because so many facial expression patterns exist that a human being cannot make representations of all of them, it is difficult to obtain and retain all available patterns and use them as learning data in a short time. The actual facial expressions that change from one time to another would show the other patterns at other times that are not contained in the learning data. For that reason, it is thought to be difficult to maintain and recognize those facial expressions just as they are without changing them continuously for a long time using the same classifier that was created at the initial stage.

For a facial expression recognition model to retain its high robustness along the time axis continuously for a long time, the classifier created at the initial stage should be evolved to be adaptive gradually over time. In other words, what is necessary for the model is that it retains existing knowledge (i.e. past facial patterns) and simultaneously learns to keep adding newly available knowledge (i.e. new facial patterns) as it becomes available.

As described in this chapter, we propose a method of creating a facial expression recognition model that can offer the adaptive learning capability described above. In addition, its degree of usefulness is described. We will show it from results of experiments made for evaluation of the incremental learning capability that the model has. Thereby, we will examine that point specifically.

2. Previous studies

Earlier reports (Ishii et al., 2008a; Ishii et al., 2008b) have presented a generation method of a subject-specific emotional feature space using the Self-Organizing Maps (SOM) (Kohonen, 1995) and the Counter Propagation Networks (CPN) (Nielsen, 1987). The feature space expresses the correspondence relationship between the change of facial expression pattern and the strength of emotion on the two-dimensional space centering on “pleasantness” and “arousal”. Practically speaking, we created two kinds of feature space, Facial Expression...
Map and Emotion Map, by learning the facial images using the CPN. The CPN is a supervised learning algorithm that combines the Grossberg learning rule with the SOM. With a facial image fed into the CPN after some learning process, the Facial Expression Map can determine the unique emotional category for the image that is fed in. Furthermore, the Emotion Map can quantize the level of the emotion of the image based on the level of facial pattern changes that occur.

Figures 1 and 2 respectively present the Facial Expression Map and Emotion Map generated using the proposed method. Figure 3 shows the recognition result for expression of “fear” and “surprise”, which reveals pleasantness value and arousal value gradually change with the change of facial expression pattern. Moreover, the change of pleasantness value and arousal value is similar, although facial expression patterns of two subjects are different.

Figure 4 depicts the procedures of the previous method. The method consists of following three steps.

Step 1. Extraction of subject-specific facial expression categories using the SOM.
Step 2. Generation of Facial Expression Map using the CPN.
Step 3. Generation of Emotion Map using the CPN.

Details of target facial expression images and above three steps are explained below.
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Fig. 4. Flow chart of proposal method in previous studies

Step 1: SOM (Extraction of facial expression categories)

- Facial Expression Images
- SOM Learning
- Facial Expression Categories
- Representative Images
- Assignment of the emotion category (Six Basic Emotions and Neutral) by visual check.
- Coordinate value on Circumplex Model
- Teaching Signals
- Input Images
- Teaching Signals
- CPN Learning
- Facial Expression Map
- Emotion Map

Step 2: CPN (Generation of Facial Expression Map)

Step 3: CPN (Generation of Emotion Map)

Fig. 3. Recognition result for “fear” and “surprise” of Subject A and B
2.1 Target facial expression images
Open facial expression databases are generally used in conventional studies (Pantic et al., 2005; Gross, 2005). These databases contain a few images per expression and subject. For this study, we obtained facial expression images of ourselves because the proposed method extracts subject-specific facial expression categories and the representative images of each category from large quantities of data.

This section presents a discussion of six basic facial expressions and a neutral facial expression that two subjects manifested intentionally. Basic facial expressions were obtained as motion videos including a process in which a neutral facial expression and facial expressions were manifested five times respectively by turns for each facial expression. Neutral facial expressions were obtained as a motion video for about 20 s. The motion videos were converted into static images (30 frame/s, 8 bit gray, 320 × 240 pixels) and used as training data. A region containing facial components was processed in this chapter; extraction and normalization of a face region image were performed according to the following procedures. Figure 5 shows an example of face region images after extraction and normalization.

1. A face was detected using Haar-like features (Lienhart & Maydt, 2002); a face region image normalized into a size of 80 × 96 pixels was extracted.
2. The image was processed using a median filter for noise removal. Then smoothing processing was performed after dimension reduction of the image using coarse grain processing (40 × 48 pixels).
3. A pseudo outline that is common to all the subjects was generated; the face region containing facial components was extracted.
4. Histogram linear transformation was performed for brightness value correction.

![Fig. 5. Examples of facial expression images](image_url)

2.2 Extraction of facial expression category
The proposed method was used in an attempt to extract a subject-specific facial expression category hierarchically using a SOM with a narrow mapping space. A SOM is an unsupervised learning algorithm and classifies given facial expression images self-organizedly based on their topological characteristics. For that reason, it is suitable for a classification problem with an unknown number of categories. Moreover, a SOM compresses the topological information of facial expression images using a narrow mapping space and performs classification based on features that roughly divide the training data.
We speculate that repeating these hierarchically renders the classified amount of change of facial expression patterns comparable; thereby, a subject-specific facial expression category can be extracted. Figure 6 depicts the extraction procedure of a facial expression category. Details of the process are explained below.

(a) SOM architecture.

(b) Learning with SOM and setup of new training data.

Fig. 6. Extraction procedure of a facial expression category

1. Expression images described in Section 2.1 were used as training data. The following processing was performed for each facial expression. The number of training data is assumed as \( N \) frames.
2. Learning was conducted using a SOM with a Kohonen layer of five units and an input layer of 40 \( \times \) 48 units (Fig. 6 (a)), where the number of learning sessions as set as 10,000 times.
3. The weight of the Kohonen layer \( W_{ij} \) (0 \( \leq \) \( W_{ij} \) \( \leq \) 1) was converted to a value of 0 - 255 after the end of learning, and a visualized images were generated (Fig. 6 (b)), where \( n_1 \) - \( n_5 \) are the number of training data classified into each unit.
4. Five visualized images can be considered as representative vectors of the training data classified into each unit (\( n_1 \) - \( n_5 \)). Therefore, whether a visualized image was suitable as a representative vector was judged using a threshold process. Specifically, for the upper and lower faces presented in Fig. 6 (c), a correlation coefficient between a visualized image and classified training data was determined for each unit. The standard deviation of those values was computed. When the standard deviation of both regions was 0.005 or less in all five units, the visualized image was considered to represent training data and the subsequent hierarchization processing was cancelled.
5. The correlation coefficient of weight \( W_{ij} \) between each adjacent unit in the Kohonen layer was computed. The Kohonen layer was divided into two bordering on between the units of the minimum (Fig. 6 (b)).
6. The training data \((N_1 \text{ and } N_2)\) classified into both sides of the border were used as new training data; processing described above was repeated recursively. Consequently, the hierarchic structure of a SOM was generated.

7. The lowermost hierarchy of the hierarchic structure was defined as a facial expression category. Five visualized images were defined as representative images of each category after learning completion. Then the photographer of the facial expression images performed visual confirmation to each facial expression category and conducted implication in emotion categories.

2.3 Generation of facial expression map

It is considered that recognition to a natural facial expression requires generation of a facial expression pattern (mixed facial expression) that interpolates each emotion category. The proposed method used the representative image obtained in Section 2.2 as training data and carried out data expansion of facial expression patterns between each emotion category using CPN with a large mapping space. The reason for adopting CPN, a supervised learning algorithm, is that the teaching signal of training data is known by processing in Section 2.2. The mapping space of CPN has a greater number of units than the number of training data, and has a torus structure because it is presumed that a large mapping space allows CPN to perform data expansion based on the similarity and continuity of training data. Figure 7 depicts the CPN architecture to generate Facial Expression Map. The details of processing are described below.

Fig. 7. CPN architecture for generation of Facial Expression Map

1. In fact, CPN has a structure comprising an input layer of \(40 \times 48\) units and a two-dimensional Kohonen layer of \(30 \times 30\) units. In addition, the Grossberg layer 1 of seven units was prepared, to which the teaching signal of six basic facial expressions and a neutral facial expression were input.

2. Representative images obtained in Section 2.2 were used as training data, and learning was carried out for each subject. As the teaching signal to the Grossberg layer 1, 1 was
input into units that mean emotion categories of representative images, otherwise 0. The number of learning was set to 20,000 times.

3. The weights \( W_{g1} \) of the Grossberg layer 1 were compared for each unit of the Kohonen layer after learning completion; an emotion category of the greatest value was used as the label of the unit. A category map generated by the processing described above was defined as a subject-specific Facial Expression Map.

### 2.4 Generation of emotion map

Even if the facial expression pattern appearing on a face is peculiar to an individual, the internal emotion that humans express on the face and the emotion that humans recognize from the facial expression are considered to be person-independent and universal. Therefore, it is presumed necessary to match the grade of emotion based on a common index for each subject to the grade of change in facial expression patterns extended in Section 2.3. The proposed method is centered upon the Circumplex model of Russell (Russell & Bullock, 1985) as a common index. Specifically, the coordinate values based on the Circumplex model were input as teaching signals of CPN, in parallel to processing in Section 2.3. Then generation of an emotion feature space was tried, which matches the grade of change of facial expression patterns and the grade of emotion. Figure 8 depicts the generation procedure of Emotion Map. The details of processing are described as follows.

1. The Grossberg layer 2 of one unit that inputs the coordinate values of the Circumplex model was added to the CPN structure (Fig. 8 (a)).

2. Each facial expression stimulus is arranged in a circle on a space centering on “pleasantness” and “arousal” in the Circumplex model (Fig. 8 (b)). The proposed method expresses this circular space as the complex plane depicted in Fig. 8 (c), and complex number based on the figure were input to the Grossberg layer 2 as teaching signals. For example, when an inputted training data is an emotion category of happiness, a teaching signal for Grossberg layer 2 is \( \cos (\pi/4) + i \sin (\pi/4) \).

3. This processing was repeated to the maximum learning number.

4. Each unit of the Kohonen layer was plotted onto the complex plane after learning completion based on the values of the real and imaginary parts of the weight \( W_{g2} \) in Grossberg layer 2. Then this complex plane was defined as a subject-specific Emotion Map.

### 3. Proposed method

The facial expression feature space described in Section 2 above, Facial Expression map and Emotion Map, has generalization capability for facial expression images that have not been learned, but it has no learning capability for the facial expression images that are being added continually. From this perspective, we examined, specifically in our study, the algorithm of incremental learning capability, called Adaptive Resonance Theory (ART), which has characteristics of both stability and plasticity. ART is an unsupervised learning algorithm. When the matching level between the input data and the existing category data is lower than the vigilance parameter value provided in advance, it takes the input data to add as a new category of data. Actually, the input data used in the method we propose are the intensity of the facial expression images. For that reason, we used Fuzzy ART (Carpenter et al., 1991) in our study because it can accept analog inputs.
Fig. 8. CPN architecture for generation of Emotion Map
3.1 Fuzzy ART

Figure 9 shows the Fuzzy ART architecture. The Fuzzy ART is formed of two layers, the input layer F_1 and the output layer F_2. The quantities of neurons of the F_1 and F_2 layer are, respectively, M and N. Input I is an M-dimensional vector (I_1, ... , I_M), where each component I_i is in the interval [0, 1]. A neuron of layer F_2 represents one category and is characterized by its weight vector W_j ≡ (W_j1, ... , W_jM). Fuzzy ART dynamics are determined by a choice parameter α (α > 0); a learning rate parameter β (0 ≤ β ≤ 1); and a vigilance parameter ρ (0 ≤ ρ ≤ 1). The learning algorithm is described below.

1. Initially, each category is uncommitted,

\[ w_{j1} = ... = w_{jM} = 1 \]  \hspace{1cm} (1)

2. For each input I and each category j, the choice function T_i is defined by

\[ T_j(I) = \frac{|I \wedge w_j|}{\alpha + |w_j|}, \]  \hspace{1cm} (2)

where the fuzzy AND operator are defined by

\[ (x \wedge y)_i = \min(x_i, y_i), \]  \hspace{1cm} (3)

and where the norm is defined by

\[ |x| = \sum_{i=1}^{M} |x_i|. \]  \hspace{1cm} (4)

3. The category choice is indexed by J, where

\[ T_J = \max \left\{ T_j : j = 1...N \right\}. \]  \hspace{1cm} (5)

If more than one T_j is maximal, the smallest index is chosen.

4. Resonance occurs if the match function of the chosen category meets the vigilance criterion. Resonance occurs when

\[ \frac{|I \wedge w_j|}{|I|} \geq \rho. \]  \hspace{1cm} (6)

Reset occurs when

\[ \frac{|I \wedge w_j|}{|I|} < \rho. \]  \hspace{1cm} (7)

5. Next layer F_2 winning nodes, T_j is inhibited for the duration of the input representation to prevent it from competing further. A new index j is then chosen by (5). The search
process continues until the chosen \( J \) satisfies (6). Once equation (6) is fulfilled, the weight vector \( W_j \) is updated according to the equation

\[
\begin{align*}
\text{new } & = \beta \left( I \wedge \text{old} \right) + (1 - \beta) \text{old}
\end{align*}
\]

\[ (8) \]

Fig. 9. Fuzzy ART architecture

### 3.2 Improvement to incremental learning capability of ART

Fuzzy ART creates a new category without fail when the matching function value between the input data and the existing category data is lower than vigilance parameter. In other words, when a facial expression that differs greatly from any existing one is fed in, a new category is created, which results in addition of further redundant knowledge in the \( F_2 \) layer. Consequently, in our study, the results of which are described in Fig. 10, we set two different vigilance parameters in Fuzzy ART, \( \delta_1 \) and, \( \delta_2 (\delta_1 > \delta_2) \). Thereby, the judgment made by the Fuzzy ART can be branched to any of the following three:

a. Update of the existing category data (Matching function value \( \geq \delta_1 \)): When the input data are part of the existing category, weight of existing category gets updated (i.e. an existing knowledge update).

b. Addition of a new category (\( \delta_2 \leq \text{Matching function value} < \delta_1 \)): When the input data are similar to the existing category, they are newly added into that category as new facial pattern data (i.e. a new knowledge addition).

c. Rejection (Matching function value \( < \delta_2 \)): When the input data differ from any existing category to a great degree, no incremental learning process is accomplished (as described in (a) or (b) above); rather, it is simply rejected.

Through the processing described above, the Fuzzy ART additionally learns only the facial expression that is similar to existing category data. Consequently, it can expand the level of the knowledge it holds little by little. It also suppresses creation of any redundant knowledge by itself.
3.3 Facial expression recognition model with adaptive learning capability

Figure 11 depicts the procedures of the proposed method. The method consists of a facial expression feature space that is created by the CPN and Fuzzy ART, which are linked with each unit of the feature space (Kohonen layer of CPN). Details of proposed method are explained below.

1. The processing described in Section 2 above creates a facial expression feature space using the initially created training data (CPN learning).

2. The weight $W_{CPN}$ for each unit of the facial expression feature space is set after the learning process progresses, with it as the weight $W_{ART}$ of the initial category of the Fuzzy ART.

3. Using the test data fed into the facial expression feature space, it searches the feature space for the winner unit whose Euclid distance to the weight of each unit is the least of all.

4. With the test data fed into the Fuzzy ART linked with the winner unit, it performs the processes described in 3.1 and 3.2 above and moves on to determine which process to take, choosing from either (a) an existing knowledge update, (b) a new knowledge addition, or (c) rejection. If either (a) or (b) is chosen, then that associated emotion category of the facial expression feature space becomes the finally determined recognition resulting from the test data that were entered. If it is (c), then the test data differ greatly from the feature of the existing category data so that the recognition is undetermined.

5. The method repeats the processes of (3) and (4) for some period. Then it moves on and sets, as new training data, the weight of the associated category unit that the Fuzzy ART holds, and performs re-learning for the facial expression feature space similarly to process (1). Through this series of facial expression recognition processes, the facial expression feature space can acquire new knowledge after the re-learning process is accomplished.

Repeating the processes of (3), (4), and (5) above, the facial expression feature space can additionally learn new knowledge in parallel to the process of recognizing the facial expression in the feature space while it holds existing pieces of knowledge continuously as they are.
4. Experiment for evaluation

In our study, we conducted an experiment to evaluate the method we proposed, particularly addressing the incremental learning capability of the method. In practice, we had emotion categories of two kinds, which included joyful expression (happiness) and no facial expression (neutral); we also performed an experiment to evaluate the additional learning ability of the method with respect to joyful expression patterns. The feature space size is 30 units (one dimension). The vigilance parameters of the Fuzzy ART were $\rho_1 = 0.98$, and $\rho_2 = 0.96$, each of which was set empirically. Details of experiments are explained below.

1. We first created a facial expression feature space using initial training data of as many facial expression image pictures as given on 100 (which included 50 images of “joyful expression of a person with his mouth shut” and another 50 of “no facial expression”).

2. We accomplished the facial expression recognition and the incremental learning using 200 images of the additional data that were fed in (which included joyful expressions with the person’s mouth opening gradually). Subsequently, we conducted the re-learning process for those associated facial expression feature spaces. We repeated this series of processes 10 times.

3. We conducted facial expression recognition for as many test data as on the 2635 images fed in (which were of the additionally obtained joyful expressions) using the 10 feature spaces created in (2) above.
During the experiment, we used a USB camera to capture facial expression images. We used those images after applying a normalizing process to each described in Section 2.1 above.

5. Experimental results and discussion

5.1 Incremental learning capability of facial expression feature space

We next examine the experimental results we obtained from (2) of Section 4 above. Figure 12 portrays the number of occurrences of the additional learning process made for each re-learning of the associated facial expression feature space (which include existing category updates, new category additions, and rejections). For the feature space created at the initial stage of the whole process, we had existing category updates 62 times, and new category addition 25 times out of 200 images of additional learning data, all of which indicate that the facial expression recognition rate we obtained was 44%. These values of category update and addition increased gradually as the re-learning processes were repeated five times. Continuing that, the recognition rate we obtained at the 10th time of re-learning process improved to 88% (with 131 updates and 45 new additions). Conversely, the number of rejections we had decreased. The rejection was processed about 20 times serially from the 5th time to the 10th time of re-learning. One reason is that the face portions of the image data we used were slightly shifted in position, or that the facial expression data that had been entered differed greatly from the existing knowledge we had (i.e., from those facial expression patterns that had been held in the facial expression feature space).

Figure 13 presents occurrences in terms of the Euclidean distance between the winner unit of the facial expression feature space and the additional learning data that were fed in. The shorter this distance, the more precisely the level of recognition was proved to have been gained. Similarly, Table 1 shows the associated average and variance values of the Euclidean distances given on Fig. 13. Although the distances and the variances are all showing large values in the feature space at its initial stage, they are decreasing and converging as the re-learning process progresses.

![Figure 12](www.intechopen.com)
Fig. 13. Frequency of the Euclidean distance between the weight of winner unit on the facial expression feature space and the additional learning data (200 images)

![Graphs showing frequency of Euclidean distance](image)

Table 1. Average and variance values of the Euclidean distance shown Fig. 13

<table>
<thead>
<tr>
<th>Number of re-learning process</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Average value</strong></td>
<td>3.63</td>
<td>2.33</td>
<td>2.18</td>
<td>2.19</td>
<td>1.44</td>
<td>1.15</td>
<td>1.16</td>
<td>1.16</td>
<td>1.19</td>
<td>1.17</td>
<td>1.17</td>
</tr>
<tr>
<td><strong>Variance value</strong></td>
<td>1.16</td>
<td>1.60</td>
<td>1.44</td>
<td>1.54</td>
<td>0.54</td>
<td>0.33</td>
<td>0.34</td>
<td>0.35</td>
<td>0.33</td>
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</tr>
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</table>

Figure 14 portrays results of visualizing the weight of the facial expression feature space for up to the fifth iteration of the re-learning process. Looking at the visualized images of the facial expression feature space in Fig. 14, we recognize that the greater number of facial patterns of the person with his mouth gradually opening is certainly captured as the number of re-learning processes increases. In addition, it is readily understood is that the person’s joyful expression with his mouth shut and the patterns of his lack of facial expression contained in the initial learning data are retained as existing knowledge just as they are, without changing, even as the re-learning process progresses.

Those results given and described above show that the facial expression feature space is learning to add new knowledge while the facial expression recognition process progresses with existing knowledge kept as it is without changing.
5.2 Experiment for evaluation using unlearning data

Figure 15 presents results of facial expression recognition when as many as 2635 images are fed into the facial expression feature space at each re-learning step. Figure 16 shows occurrences of each Euclidean distance taken by each winner unit of the feature space when the test data are fed in. Table 2 shows the average and variance values of the Euclidean distance. Examination of Fig. 15 reveals that the recognition rate for the initial feature space is 44% (with 870 times of update and 300 times of new addition), but it increases to 59% (with 1120 times of update and 430 times of new addition) at the 7th re-learning step of the feature space with the re-learning process examined. Investigation of Fig. 16 and Table 2 shows that the average and variance values of the Euclidean distance tend to decrease and converge as the re-learning process progresses in the same way as that shown in the experiment of Section 5.1.
All of the points described above reveal that the facial expression feature space is capturing new knowledge without fail as it proceeds in parallel with the re-learning process. Results demonstrate that the method we propose is practical for use as an adaptive facial expression recognition model that has robustness along the time axis.

![Frequency of the Euclidean distance between the weight of winner unit on the facial expression feature space and the test data (2635 images)](image)

**Fig. 16.** Frequency of the Euclidean distance between the weight of winner unit on the facial expression feature space and the test data (2635 images)

<table>
<thead>
<tr>
<th>Number of re-learning process</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
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<tbody>
<tr>
<td>Average value</td>
<td>3.20</td>
<td>2.72</td>
<td>2.62</td>
<td>2.66</td>
<td>2.42</td>
<td>2.15</td>
<td>2.20</td>
<td>2.21</td>
<td>2.22</td>
<td>2.20</td>
<td>2.21</td>
</tr>
<tr>
<td>Variance value</td>
<td>1.45</td>
<td>1.26</td>
<td>1.13</td>
<td>1.14</td>
<td>0.69</td>
<td>0.51</td>
<td>0.54</td>
<td>0.54</td>
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</tbody>
</table>

Table 2. Average and variance values of the Euclidean distance shown Fig. 16

### 6. Conclusion

In this chapter, we proposed a facial expression recognition model with the adaptive learning capability. To demonstrate how useful it can be, we conducted some basic experiments for evaluation purposes with both joyful expressions and no facial expressions examined. Results show clearly that the facial expression feature space that we created using the method we proposed was capable of capturing additional knowledge while it maintained the existing knowledge as it was without alteration. The results we obtained demonstrate that the proposed method can be useful for an adaptive facial expression
recognition model that has a robustness feature along the time axis. We plan to conduct additional evaluative experiments in the future and examine the six basic facial expressions available. Moreover, we are planning to continue our work based on those facial expression images that we have accumulated during the long period of time we have continued this study.

7. Acknowledgment

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8. References


Kohonen Self Organizing Maps (SOM) has found application in practical all fields, especially those which tend to handle high dimensional data. SOM can be used for the clustering of genes in the medical field, the study of multi-media and web based contents and in the transportation industry, just to name a few. Apart from the aforementioned areas this book also covers the study of complex data found in meteorological and remotely sensed images acquired using satellite sensing. Data management and envelopment analysis has also been covered. The application of SOM in mechanical and manufacturing engineering forms another important area of this book. The final section of this book, addresses the design and application of novel variants of SOM algorithms.

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