Measurement and Assessment Methods of Forest Aboveground Biomass: A Literature Review and the Challenges Ahead

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1. Introduction

The measurement and assessment of aboveground tree biomass (bole, branches, and foliage), or M, plays a key role in the management of forest resources. Estimates are required for evaluating: a) the stocks and fluxes of several biogeochemical elements and b) the amount of primary energy obtainable from forests as an alternative to fossil fuels. Moreover, biomass is a fundamental state variable in several ecological and eco-physiological models (Brown, 1997; Chavé et al., 2005; Návar, 2009a,b; Richardson et al., 2002). The development and use of allometric equations is the standard methodology for the estimation of tree, plot, and regional aboveground biomass (Brown, 1997). Dry weight measurements conducted on harvested trees, fresh and dry weights of biomass components and recording independent tree variables are required to construct allometric equations at the species, stands or tree community levels. Alternate M assessment methods include the multiplication of bole volume by its wood specific gravity; with branch and foliage biomass integrated using other approaches. Standing bole volume, V, can be also multiplied by biomass expansion factors, BEF, at the tree level or stand scale to compute M. Allometric biomass equations can be classified according to the parameter estimation method as empirical, semi-empirical and process, theoretical models. Using three meta-analysis datasets, empirical equations are reported in log-linear (82.6%), non-linear (12.0%), seemingly un-related (3.9%), linear (0.7%), and non-linear seemingly un-related (0.6%) regression. Diameter at breast height, D, and at the bole base, Db, canopy height, H, canopy area, CA, and wood specific gravity, \( \rho_w \), are common exogenous variables that individually or in combination explain M with deviations larger than 16% of the mean measured tree aboveground biomass value (Chavé et al., 2005). A fully theoretical, physically parameterized model is available (West et al., 1997), although preliminary evaluations demonstrate that it requires further refinement before can be recommended as a non-destructive M assessment methodology. More flexible, restrictive models that make use of only a small number of harvested trees and fit available allometric equations result in good M approximations (Zianis & Mencuccini, 2004). Semi-empirical non-destructive models based on shape-dimensional analysis and assuming a constant exponent value are being tested for simple and complex forests with compatible preliminary M assessments (Návar, 2010a,b). This wealth of information on biomass allometry necessitates be properly describing, organizing, and classifying in order to better
understand weakness and robustness of available methods to compute tree and eventually plot and regional aboveground biomass. For places deprived of tree allometry, a combination of a wide range of allometric equations developed off site appears to improve tree M evaluations according to the Central Limit Theorem. Biomass stocks and their spatial distribution remain poorly evaluated at the plot scale regardless of the wealth of information on tree biomass allometry (Chavé et al., 2003; Houghton et al., 2001, 20015; Návar et al., 2010). The conventional methodology that expands tree M to sample inventory stands stands is: a) a grid of sampling plots and b) allometric equations fit tree data recorded in the forest inventory, since there is scarce information on allometric equations that straightforward calculate plot or stand M. New approaches that employ timber volume are named BEF and at the present they require calibration to appraise local plot M (Brown, 2002). Uncertainties of more than two orders of magnitude are identified when calculating plot M by applying different off site allometric models to forest inventory datasets and main sources of variation are: a) the error due to tree measurements, b) ground sampling uncertainty, and above all, c) the error due to the choice of an allometric model relating M to other tree dimensions (Chavé et al., 2003; Návar et al., 2010). Tree or plot M interpolates at larger spatial scales, AGB, by a variety of field measurements, environmental gradients and remote sensing techniques (Houghton, 2005a,b). A diversity of remote sensing techniques, spatial resolutions, tree and forest attributes, and interpolation methodologies make AGB assessment highly variable, with uncertainties as large as three orders of magnitude. Main sources of variation are attributable to: a) the precision of estimated tree or stand M, b) the interpolation method applied, c) the lack of a good correlation between ground and remote sensing data, d) the correct location of ground data, e) the representativeness of plots across the landscape, f) temporal variations in the satellite image, g) the correct area of each forest class, and h) others. Combining remote field data collection techniques (LIDAR) with locally-derived tree allometry and the semi-empirical shape-dimensional non-destructive model of tree M assessment would eventually improve AGB at the spatial scale of interest.

Given this brief literature review, the reliable M estimation of trees, plots, stands or tree communities remains a key challenge for the successful implementation of sustainable forest management plans. This paper deals with the description of available tree allometry, how they contrast to provide tree, plot and regional M assessments and what are the future challenges ahead. Preliminary observations point towards the combination of available conventional allometric models with restrictive, semi-empirical and theoretical non-destructive methods of tree or plot M evaluation while universally-applied functions emerge. In addition, the interpolation of improved tree or plot M appraisals to regional scales with a combination of field techniques, environmental gradient approaches and remote sensing methods must eventually improve AGB assessments at regional and national spatial scales.

Key words: Measuring and assessing aboveground biomass, empirical, semi-empirical, theoretic models, tree, stand and regional scales.

2. Aboveground tree biomass allometry

2.1 Introduction

Aboveground tree and forest biomass is the living and dead matter in standing trees and shrubs and can be classified in foliage, branches, and boles. Bark, hardwood and softwood
are timber biomass components. The evaluation of conventional goods and environmental services furnished by forests entails the assessments of tree, stand and regional M. The stocks and fluxes of several biogeochemicals are calculated with the evaluation of M (Brown, 1997; Houghton, 2005). So is the amount of primary energy obtainable from forests as an alternative to fossil fuels (Richardson et al., 2002). In addition, standing aboveground biomass is a fundamental state variable in several ecological and eco-physiological models (Zianis & Mencuccini, 2004).

The development and application of allometric equations is the standard methodology for aboveground tree biomass estimation (Brown et al., 1989; Chavé et al., 2001; 2003; Návar, 2009a). A simple classification of allometric equations based on methods of parameter estimation is: empirical, semi-empirical and theoretical models. Meta-analysis studies report examples of empirical functions (Ter Mikaelian & Korzukhin, 1997; Jenkins et al., 2003; Zianis & Mencuccini, 2004; Zianis et al., 2005; Návar, 2009b). Non-destructive models such as the empirical reductionist (Zianis & Mencuccini, 2004); the semi-empirical shape-dimensional analysis (Návar, 2010a), the constant B-slope approach (Návar, 2010b) and process, theoretical methods (West et al., 1999) are also available in the scientific literature. Empirical allometric equations are statistically parameterized with measured, weighted and recorded field and laboratory tree biomass data. The conventional allometric biomass model \( \text{Ln}(M) = \text{Ln}(a) + BL\text{Ln}(D) \pm e_i; \) where M and D are log transformed and the a and B the scalar coefficients estimated by least square techniques in linear regression, is the most commonly fitted and reported equation. Other parameter-fitting techniques and mathematical forms of biomass equations are classified as: non-linear, seemingly un-related linear, linear and non-linear seemingly un-related regression, power and exponential functions. Tree diameter recorded at breast height, basal diameter, canopy cover, canopy height and wood specific gravity commonly explain individually or in conjunction tree M with deviations larger than 16% of the mean measured tree M.

Semi-empirical non-destructive methods of tree M computations that focus on independent and easy ways to calculate the conventional allometric scalar coefficients had been recently proposed. They require both physical and statistical parameters. The fractal methodology coupled with shape-dimensional relations was preliminary explored with good degree of precision for temperate trees of northwestern Mexico (Návar, 2010a) and for Mexican tropical forests (Návar et al., 2010). This procedure assumes that bole volume and top height allometric relations suffice to calculate the a and B scalar coefficients. When contrasted with the conventional allometric model, this method results in compatible tree M assessments. A reduced semi-empirical, non-destructive model that assumes the B-scalar exponent is a constant value and the a-scalar intercept is a function of the standard wood specific gravity value is also under close mathematical advancement with good preliminary precision for North American temperate trees (Návar, 2010b).

The classic theoretical allometric model, WBE, was developed with the use of fractal techniques (West et al., 1999). Two variables, a C-scalar coefficient and the entire tree specific gravity, \( \rho \), suffice to calculate tree M; since it assumes the B-scalar exponent is a fixed value of 8/3. The WBE equation is physically parameterized but, at the present, it needs further refinement before can be recommended as non-destructive method of M assessment. Discussions regarding its application are ongoing and they center on the right value of the B-slope scalar coefficient that it has been shown to be smaller than 2.67 (Zianis & Mencuccini, 2004; Pilli et al., 2006; Návar, 2009a,b; 2010b).
Other approaches involve the bole volume estimation and then multiplied by the standard wood specific gravity value (Mohren & Klein Goldewijkt, 1990). A dimensionless biomass expansion factor, BEF, escalates bole volume to total tree M (Brown, 1997). Gracia et al. (2004); Lehtonen (2005); Návar-Cháidez (2009); Silva-Arredondo and Návar-Cháidez, (2009) reported independent BEF at the tree level or plot scales, which are developed by employing biomass of the entire tree in conjunction with bole volume allometry.

Tree M assessments are variable regardless of the wealth of information on biomass allometry. For harvested trees, deviations have been reduced to close to 16% with the use of D, H and $\rho_w$, (Chavé et al., 2005). However, the expansion of these equations to trees with other dimensions or outside the forest area where the equation was developed deserves more attention.

2.2 The need for tree allometry
A great number of allometric equations have been reported for North American and European tree species and forests (Ter Mikaelian & Korzukhin, 1997; Jenkins et al., 2003; Zianis & Mencuccini, 2004; Zianis et al., 2005; Fehrmann & Klein, 2006; Chojnacky, 2009; Návar, 2009a). Tree allometry for complex tropical (Brown, 1997; Chavé et al., 2001; 2003; 2005) and semi-arid, sub-tropical tree species (Návar et al., 2002a; 2004; Návar, 2009b) and forest plots (Martínes-Yrizar et al., 1992; Návar et al., 2002b) are less represented. At regional scales, current allometric data for complex, diverse tropical forests are almost entirely based on Southeast Asian (Brown, 1997; Ketterings et al., 2001) and South American measurements (Overmann et al., 1994; Araujo et al., 1999; Chavé et al., 2001; 2005; Chambers et al., 2001; Brandeis et al., 2006; Feldpausch et al., 2006). Brown (1997) and Chave et al. (2005) reported a set of allometric equations for tropical world forests; however, several sites were not well typified in this data set. For example, with the exception of the report published by Cairns et al. (2000), most Mexican tropical forests remains with limited information on tree and stand M development, analysis and comparisons.

2.3 Development of tree allometry
The development of conventional biomass allometry compels that trees are harvested. Measurements of diameter at breast height and at the bole base are carried out on each standing tree. Top height is better measured once the tree is felled down. Tree dissection into the main biomass components: stem or bole, foliage, and branches are performed on felled trees. Boles are logged into smaller sizes to facilitate weighting. Foliage, branches, and dissected logs are fresh weighted separately per tree. The total fresh weight of each component for each tree is obtained in the field using scales. Samples of each component of each tree are fresh weighted and oven-dried in the laboratory (to constant weight at 70°C). Sample fresh and dry weights must be precisely recorded, since dry to fresh weight ratios for each sample of each component multiplied by the total fresh weight of each biomass component calculate total dry biomass per each tree component. Deviations of this methodology have been proposed where only small portions of each biomass component are weighted and the remaining is calculated by dimensional analysis.

2.4 Fitting allometric equations
A data matrix of exogenous, independent variables (D, Db, H, CA, $\rho_w$, etc.) and dependent variables (dry foliage, branch, bole, and/or total aboveground biomass) for n, number of
harvested trees are available for fitting tree biomass equations. That is, allometry relates one measurement of an organism to another. Easily measured variables such as diameter and top height relates to volume, biomass, etc. which are more difficult to make. A wide range of empirical allometric models are available in the scientific literature to fit collected biomass data using the independent variables described above. They can be classified as simple log-linear, simple linear, simple non-linear, multiple linear and non-linear, seemingly un-related linear and non-linear regression equations. Power or exponential functions also projects tree M, although they are scarcely reported in the scientific literature. Allometric equations quite often fit each individual biomass component (i.e., see for example the biomass equations compiled by Ter Mikaelian & Korzukhin, 1997; and Návar, 2009b). However, Cunnia and Briggs (1964) showed that when summing the equations for boles, branches, and foliage, results would often deviate from the recorded total aboveground biomass. Therefore, Cunnia & Briggs (1984; 1985) and Parresol (1999; 2001) developed advanced regression techniques and computer programs for estimating coefficient values for endogenous variables that simultaneously calculate individual equation parameters and restrict scalar coefficients to add total tree M. Biomass datasets are also a vital source of information to fit theoretical, semi-empirical non-destructive and restrictive methods of tree M assessment but sometimes other independent variables must be collected. Biomass datasets should be split into: a) fitting and b) validating models. However, biomass studies are expensive and quite often data is not sufficient to calculate scalar coefficients with small variance that are consistent with population mean parameters. These issues addressed further below must be the center of future allometric studies.

**The Log-linear equation.** The most commonly reported mathematical model for biomass allometry takes the form of the Log linear-transformed function:

\[
\ln(M) = \ln(a) + B\ln(D) \pm e_i
\]  

Equation [1] and [2] are similar but not mathematically equivalent:

\[
\begin{align*}
\ln(M) &= \ln(a) + B\ln(D) \pm e_i \\
M &= \exp(\ln(a) + B\ln(D)) \pm e_i \\
M &= aD^B \pm e_i
\end{align*}
\]  

Where \( \ln = \) the logarithmic transformation function; \( e_i = \) error. The scalar coefficients \( a \) and \( B \) of equations [1] and [2] are calculated by least square techniques in linear regression. Before conducting this statistical test; \( M \) and \( D \) data is log transformed. The transformation improves parameter estimation by reducing variability and heteroscedasticity. This technique frequently named the intrinsic linear regression entails a weighting parameter to further reduce heterogeneous variance since the logarithmic transformation compresses the data in both axes. When the biomass units are re-transformed back to the original units, the largest data values are often underestimated. Beskersville (1965) recommended to multiply equation [1] by a correction factor, \( CF \), that is calculated as \( CF = \exp(MSE/2) \), where: \( MSE = \) mean square error of the regression analysis of variance. Equation [1] is the standard, classical allometric biomass model reported in compiled equations by Ter Mikaelian & Korzukhin (1997); Jenkins et al. (2003); Zianis & Mencuccini (2004); and Návar (2009b). The standard error, \( Sy \), of equation [1] is in logarithmic M units and consequently it is not equivalent to: \( Sy = \sqrt{MSE} \) where \( MSE = \) mean...
square error. Therefore, equation [1] has to be fitted to the original tree data to evaluate M and with measured and estimated M, Sy can be calculated in conventional M dimensions. **The Linear equation.** Linear equations frequently reported in allometric studies take the following form:

\[ M = a + BX \pm e_i \cdot M = BX \pm e_i \]  

(3)

Where \( X = D^2H \) (m\(^3\)), BA (m\(^2\)), Canopy Cover (%).

Least square techniques in linear regression conventionally estimates the scalar coefficients, \( a \) and \( B \). Basal area, the combined variable, \( D^2H \), or canopy cover are the explanatory, exogenous variables of equation [3]. The allometric function that entails basal area was originally calculated by measuring M in plots and it has the advantage that can be escalated down to the individual tree level. Examples of this equation are found in Martínez-Yrizar et al. (1992). When using \( D^2H \) as independent variable, examples are reported in Padron and Navarro (2004) and in Návar-Cháidez et al. (2004a). Flombaum and Sala (2007) found canopy cover (%) predicted better shrub M for Argentinean semi-arid shrublands. The standard error, Sy, of equation [3] is evaluated in conventional M units. **The Non Linear equation.** The non-linear equation takes the form of the end portion of model [2], although the error is multiplicative:

\[ M = aD^B \cdot e_i \]  

(4)

Equation [4] is similar but mathematically not equivalent to equation [2], since scalar coefficients are estimated using one of the several non-linear parameter-fitting techniques available such as Newton, Gauss-Newton, Marquardt, etc. That is, scalar coefficient values differ if estimated in linear or non-linear regression techniques. Návar (2009a) reported several examples for temperate tree species of northwestern Mexico. Non-linear models report the analysis of variance in conventional M units and therefore Sy can be straightforwardly computed. **The multiple linear or non linear equations.** The multiple linear or intrinsically linear equations take the form:

\[ M = a + BX_1 + CX_2 + DX_3 + \ldots + ZX_n \pm e_i \]  

(5)

Or

\[ M = (\exp^{(a+BX_1+CX_2+DX_3+\ldots+ZX_n)}) \pm e_i \]  

(6)

Or a combination of both. Where: \( X_1, X_2, X_3,\ldots, X_n = D, D^2, D^2H, \rho_w, \rho_wD^2H \ldots \).

Least square techniques in linear or intrinsically linear multiple regression calculates scalar coefficients \( a, B, C, D, Z \). Brown (1997) and Chavé et al. (2005) reported classical examples for world dry, moist and rain tropical forests that use \( D, H, \) and \( \rho_w \) as exogenous variables. Multiple linear models supply the standard error of M as the root mean square. Intrinsically linear multiple regression models require a similar procedure to that described in model [1] to calculate Sy in standard M units. **Seemingly un-related linear regression.** Seemingly un-related regression is the recommended statistical technique to develop tree allometry for endogenous variables, since biomass components are related each other; i.e., leaf biomass relates to branch biomass, these associates to bole biomass, and all these components make total aboveground biomass
(Cunnia & Briggs, 1984; 1985; Parresol, 1999; 2001). Therefore, a simple example of a set of biomass component equations that are linearly related takes the following forms:

\[
\begin{align*}
M &= M_l + M_{br} + M_{bo} + \varepsilon_i \\
M_l &= a_l + B_l(D^2H) + \varepsilon_i \\
M_{br} &= a_{br} + B_{br}(D^2H) + \varepsilon_i \\
M_{bo} &= a_{bo} + B_{bo}(D^2H) + \varepsilon_i \\
M &= a_l + B_l(D^2H) + a_{br} + B_{br}(D^2H) + a_{bo} + B_{bo}(D^2H) + \varepsilon_i
\end{align*}
\]

(7)

Where: \( l = \) leaf or foliage, \( br = \) branch, \( bo = \) bole or stem. All six scalar coefficients, \( a_l, a_{br}, a_{bo}, B_l, B_{br} \), and \( B_{bo} \) are independently and simultaneously estimated by least square techniques in linear regression with the constraint that \( M = M_l + M_{br} + M_{bo} \). That is, the sum of each component equals the total tree \( M \).

Parresol (1999) developed and reported this regression technique in computer programs using examples for \( P. eliottii \) trees. Návar et al. (2004a) fitted this technique for young pine trees of Durango, Mexico and Návar et al. (2004b) did it for semi-arid, sub-tropical shrub species of northeastern Mexico.

**Seemingly un-related non-linear regression.** For un-related non-linear regression, a set of equations written in a simple format are:

\[
\begin{align*}
M &= (M_l + M_{br} + M_{bo}) \cdot \varepsilon_i \\
M_l &= a_l D^{B_l} \cdot \varepsilon_i \\
M_{br} &= a_{br} D^{B_{br}} \cdot \varepsilon_i \\
M_{bo} &= a_{bo} D^{B_{bo}} \cdot \varepsilon_i \\
M &= (a_l D^{B_l} + a_{br} D^{B_{br}} + a_{bo} D^{B_{bo}}) \cdot \varepsilon_i
\end{align*}
\]

(8)

Where: \( M_l = \) foliage biomass, \( M_{br} = \) branch biomass; \( M_{bo} = \) bole biomass; and \( a \) and \( B \) are statistical parameters that are independently and simultaneously estimated by least square techniques in non-linear regression and restricted to provide total aboveground biomass.

Parresol (2001) reported the mathematical development and computer programs for this technique and empirical examples can be found in Návar (2009b).

**2.5 Examples of empirical equations fitted to an independent dataset**

Empirical allometric equations should be cautiously fitted since they may significantly deviate from tree M records. For example, Návar (2009b) reported applications of these empirical equations to a biomass dataset taken for complex semi-arid, sub-tropical shrub species of northeastern Mexico (Figure 1).

All statistical techniques previously described converge into a single equation for more compact biomass datasets as it was shown for young pine trees of northwestern Mexico by Návar (2009b). Note that all equations mimic well the non-linear nature of this M-Db relationship even though multiple linear and seemingly un-related linear equations are fitted to this data.
Fig. 1. Five empirical allometric equations fitted to aboveground biomass for 913 shrubs and small trees harvested in northeastern Mexico (NL = Non linear regression, SUR = Seemingly un-related non-linear regression, LR = Linear regression, WLR = weighted linear regression, NSUR = non-linear seemingly un-related regression).

2.6 Other allometric models reported in the scientific literature

Other empirical allometric equations reported in the scientific literature were compiled for European tree species by Zianis et al. (2005) but they fall within these major power and exponential classifications: $M = a + bD^c$; $M = a + b[D/(D+cf)] + dX1 + \ldots + nXn$; $M = a(D+1)[b+c \cdot \log(D)] H D$; $M = a(1-\exp(-bD))^c$.

Where: cf is a standard coefficient; a, b, c are statistical coefficients to be estimated; X1, .. Xn are the independent variables described by D, D^2, DH, DH^2, etc.

Multicollinearity problems arise when several related exogenous variables explain M making the model unstable in the correct coefficient values.

2.7 Examples of the application of empirical tree allometry to biomass data sets

The application of several available allometric equations to independent biomass datasets often results in M assessments with large deviations. Figure 2 shows examples for tropical dry and rain forests as well as for the IAN 710 Hevea brasiliensis hybrid trees. Tree allometry is frequently developed with sample data that does not meet the probabilistic sampling requirements. Therefore, local tree allometry improves tree M predictions in contrast to biomass equations developed for tree species with a wide spatial distribution range (Návar-Cháidez, 2010). As a consequence fitting off-site allometric equations often show large tree M uncertainties, which are addressed in the following section of this chapter. Deviations have also been explained by changes in wood specific gravity values and shifts in bole tapering and slenderness. Local, specific tree M allometry has been recommended by Návar-
Cháidez (2010) but further contrasting studies are required in order to understand variations between on-site and off-site equations.

![Graph showing biomass estimates for different forest types and species](image)

Fig. 2. Aboveground biomass estimates for tropical dry and rain forests as well as for the IAN 710 Hevea brasiliensis Muell. hybrid trees of Mexico (Data Sources: Návar, 2009a; Brown, 1997 and Monroy & Návar, 2005).
2.8 Randomness of scalar coefficients with sample size

Empirical allometric equations pose scalar coefficients that are statistically calculated. For example, the conventional model [1] has the $a$-intercept and the $B$-slope scalar values. The scalar coefficients vary with the species, diameter structure, structural tree diversity, the parameter-fitting techniques, sample size, etc. Indeed, scalar parameters show large variations in meta-analysis studies even though they are calculated with the same regression technique. For example, the $B$-scalar exponent has a mean (standard deviation) value of 2.37 (0.27), 2.38 (0.23), and 2.40 (0.22) reported by Zianis & Mencuccini (2004), Návar (2009b), and Fehrmann & Klein (2006), respectively. Návar-Cháidez (2010), in a biomass simulation study with 600 trees, found that the $a$ and $B$ scalar values randomly oscillate with sample sizes of less than 60 trees pointing out at the need for harvesting sufficient trees to calculate parameter values that are consistent with population means and that have the least variance (Figure 3). Tree M calculations with this set of scalar parameters produce deviations as large as 30%.

2.9 Alternate tree allometry models

2.9.1 Reduced number of harvested trees to develop M assessment models

Zianis and Mencuccini (2004) proposed the small tree sampling scheme to simplify allometric analysis irrespective of tree species and forest site. The methodology harvests only two small trees that quite often are $D < 25$ cm. With recorded $D$ and $M$, available allometric equations for similar tree species and for similar data ranges found in the forest inventory are fitted. Those equations that predict tree M as close as to the measured values are selected and an average of scalar coefficients $a$ and $B$ values are estimated to come up with an individual allometric model. This approach was tested with a good balance between acceptable biomass predictions and low data requirements (Návar-Cháidez, 2010). Contrasting results are reported in Figure 6.

2.9.2 The empirical non-destructive model of M assessment

Zianis and Mencuccini (2004) developed de reductionist model [9] by fitting empirical relationships between the $B$-scalar exponent and the slope of the power relationship between $H-D$, $B^*$. The resulting equation was $B = 1.9262 + 0.6972B^*; r^2=0.42$. In a similar fashion, the scalar intercept was projected with the calculated $B$-scalar exponent taking advantage of the good relationship between these coefficients; $a = 7.0281B^{-4.7558}; r^2=0.70$. These two equations empirically describe the conventional model [1] scalar coefficients, as follows:

$$M = aD^B = \left[\left(7.0281 * B^{-4.7558}\right)^{1.9262+0.6972B^*}\right]$$

Where: $B^*$ is the scalar exponent of the H-D power relationship; $B$ = is the scalar exponent derived from the empirical equation that relates $B$ vs. $B^*$.

2.9.3 The theoretical model of tree M assessment

The West et al. (1999) theoretical model, WBE, was developed using the fractal geometry analysis that applies to natural occurring networks that carry sustaining fluids in organisms, in which each small part of the network is a self-similar replicate of the whole. Hence the fractal model offers much proportionality relating components of structure and function. The WBE framework describes aboveground biomass with the following equation:
Fig. 3. Randomness of the scalar coefficients $a$ and $B$ as function of the percentage of data sampled for fitting the conventional allometric model.
The scalar exponent, $B$, is fixed to $8/3$ (2.67); $\rho$ is the wood specific gravity that is referred as the total tree specific gravity (an average of the specific gravity for wood, bark, branches and leaves); and $C =$ is a proportionality constant. Note that $B = 2.67$.

Comparisons between measurements and predictions by the WBE and other empirical equations were carried out for several biomass data sets. In general, empirical models approximated better recorded tree $M$ values than the WBE one (Zianis and Mencuccini, 2004). Pilli et al. (2006) suggested that $M$ could be estimated by using universal $B$ parameters that change with the forest age. Návar (2009b) found evidence that $B$ is a function of diameter at breast height and Návar (2010b) successfully tested the hypothesis that $B$ is a function of the place where diameter is measured.

### 2.9.4 Semi-empirical non-destructive models of tree $M$ assessment

**a) The shape-dimensional relationships derived from fractal geometry.** Návar (2010a) proposed according to the classical physics equation, that mass is a function of volume $x$ specific gravity. Analogous, the aboveground biomass components are linearly and positively related to stem volume, $V$, and the entire bole wood specific gravity, $\rho$; $M = (V^*\rho)$. A simple dimensional analysis shows that the volume of a tree bole is $V = (a_vD^2H)$; where $a_v = 0.7854$ if the bole volume is a perfect cylinder. For temperate tree species of northwestern Mexico, mean $a_v$ values of 0.55 have been calculated demonstrating that tree boles or pieces of stems have a non-standard shape that is only approximated by ideal objects. Therefore, the description of natural items falls beyond the principles provided by Euclidean geometry. Mandelbrot (1983) introduced the neologism of fractal geometry to facilitate the understanding of the form and shape of such objects. A positive number between two and three is a better estimation of the tree’s crown dimension, and it is assumed that the overall shape of a tree (stem and crown) may possess a similar fractal dimension. In mathematical terms:

$$V = f(a_vD^dH^h)$$

Where: $a_v$ is a positive number that describes the taper and $d$ and $h$ are positive numbers with $2 \leq d+h \leq 3$.

Since $2 \leq d+h \leq 3$, tree shapes can be described as hybrid objects of surface and volume because they are neither three-dimensional solids, nor two-dimensional photosynthetic surfaces and indentations and gaps are the main characteristics of their structure (Zeide, 1998).

The scaling of $H$ with respect to $D$ has been examined in terms of stress and elastic similarity models following biomechanical principles. When stress-similarity for self-loading dictates the mechanical design of a tree, $H$ is predicted to scale as the $\frac{1}{2}$ power of $D$ (McMahon, 1973) and a final steady state $H$ is attained in old trees that reflects an evolutionary balance between the costs and benefits of stature (King, 1990). Empirical data found that $H$ scales to the 0.535 power of $D$ for a wide range of plant sizes, supporting this hypothesis (Niklas, 1994). However, for local biomass studies, the $B^*$ coefficient diverges from the $\frac{1}{2}$ power and it is a function of several variables. Hence, if $H = f(a_hD^{B^*})$ with $0 < B^* \leq 1 \approx 1/2$, then Eq. (2) becomes
Furthermore, if tree biomass is assumed to be proportional to V (with the tree specific gravity as the proportionality constant), then \( M = f(a_v a_h D^{d+HB^*} \times \rho) \) and in conjunction with Eq. (1), the \( B \)-scalar exponent, \( B_{theoric} \), is:

\[
B_{theoric} = d + hB^* 
\] (13)

And the \( a \)-scalar intercept, \( a_{theoric} \), is:

\[
a_{theoric} = (a_v \times a_h) 
\] (14)

Finally, a fully theoretical model that requires the following relationships \( V = f(D, H) \) and \( H = f(D) \), in addition to the wood specific gravity of the entire aboveground biomass is:

\[
M = \rho(a_v a_h)D^{d+HB^*} 
\] (15)

Model [15] was described as the shape-dimensional analysis approach (Návar, 2010a). In the absence of total aboveground tree \( \rho \) and \( a_h \) data, the intercept coefficient can be preliminarily derived taking advantage of the good relationship between the scalar coefficients, as follows;

\[
a_{theoric} = (a_v \times a_h)\rho = f(B_{theoric} = d + hB^*) 
\] (16)

With this empirical relationship, a final non-destructive semi-empirical model of aboveground biomass assessment is;

\[
M = a_{theoric} = (a_v \times a_h)\rho = f(B_{theoric} = d + hB^*) 
\] (17)

Meta-analysis studies noted that the scalar coefficients \( a \) and \( B \) are negatively related to one another in a power fashion because high values of both \( a \) and \( B \) would result in large values of \( M \) for large diameters that possibly approach the safety limits imposed by mechanical self loading (Zianis & Mencuccini, 2004; Pilli et al., 2006; and Návar, 2009a; 2009b). This mathematical artifact offers the basic tool for simplifying the allometric analysis of forest biomass in this approach.

In the meantime tree \( \rho \) and \( a_h \) data is collected, model [17] is a preliminary non-destructive semi-empirical method for assessing \( M \) for trees of any size. The procedure can be applied as long as volume allometry is available in addition to the relationship between \( a-B \) that has to be developed preferentially on-site. The methodology is flexible and provides compatible tree \( M \) evaluations since large estimated \( B \) values would have small \( a \) figures and vice versa. Site-specific allometry can be derived with this model that may improve tree \( M \) estimates in contrast to conventional biomass equations developed off-site. Three major disadvantages of this non-destructive approach are: a) the inherent collinearity problems of estimating \( a \) with \( B \), b) the log-relationships between \( V = f(D, H) \) and \( D = f(H) \) are required in order to estimate \( B \), and c) an empirical equation that relates \( a \) to \( B \) should be developed on site or alternatively use preliminary reported functions by Zianis & Mencuccini (2004) and by Návar (2009a; 2010a). All these three equations estimate compatible \( a \)-intercept values with
an estimated $B$ slope coefficient. Examples of the application of this semi-empirical model are reported in Figure 6.

**b) Reducing the dimensionality of the conventional allometric equation by assuming a constant $B$ slope coefficient value.** The development of a model that is consistent with the WBE (model [10]) and the conventional log-transformed, most popular equation (model [1]) was proposed by Návar (2010b). Models [1] and [10] have the following common properties: $a = C p; B_{WBE} \neq B; B_{WBE} = 2.67$ and $B$ is a variable that it is a function of several tree and forest attributes, including sample size; they both feed on diameter at breast height as the only independent variable. The main WBE model assumption is that the $B_{WBE}$-scalar slope coefficient is a constant value. This assumption has spurred recent research on semi-empirical allometric models. Hence, Ketterings et al. (2003) and Chavé et al. (2005) reduced the dimensionality of model [1] by proposing a constant $B$-slope coefficient, as well. Tree geometry analysis and assuming that $D$ scales to $2.0H$; where $H$ is the slope value of the $H = f(D)$ relationship; i.e., $D^{2.0H}$ are some methods justified for finding this constant. In this report, I hypothesized, according to the Central Limit Theorem, that compilations and Meta analysis studies on biomass equations should shed light onto the population mean $B$-scalar slope coefficient value.

Návar (2010b) summarized several Meta analysis studies on aboveground biomass. Table 1 shows statistical results of these studies compiled from the work conducted by Jenkins et al. (2003); Zianis and Mencuccini (2004); Pilli et al. (2006); Fehrmann and Kleinn (2006); Návar (2009a,b) where there is increasing evidence that the population mean $B$-value is around 2.38. This coefficient differs from the WBE scaling exponent. The Návar (2010b) equation,

<table>
<thead>
<tr>
<th>Model</th>
<th>$N$</th>
<th>$\bar{a}$</th>
<th>$\sigma$</th>
<th>$\text{CI}$</th>
<th>$\bar{a}$</th>
<th>$\sigma$</th>
<th>$\text{CI}$</th>
<th>$\bar{B}$</th>
<th>$\sigma$</th>
<th>$\text{CI}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jenkins et al. (2003)</td>
<td>10(2456)</td>
<td>0.11</td>
<td>0.03</td>
<td>0.02</td>
<td>0.12</td>
<td>0.03</td>
<td>0.02</td>
<td>2.40</td>
<td>0.07</td>
<td>0.05</td>
</tr>
<tr>
<td>Ter Mikaelian and Korzukhin (1997)</td>
<td>41</td>
<td>0.15</td>
<td>0.08</td>
<td>0.03</td>
<td>0.11</td>
<td>0.04</td>
<td>0.01</td>
<td>2.33</td>
<td>0.17</td>
<td>0.05</td>
</tr>
<tr>
<td>Fehrmann and Kleinn (2006)</td>
<td>28</td>
<td>0.17</td>
<td>0.16</td>
<td>0.06</td>
<td>0.12</td>
<td>0.02</td>
<td>0.01</td>
<td>2.40</td>
<td>0.25</td>
<td>0.09</td>
</tr>
<tr>
<td>Návar (2009b)</td>
<td>78</td>
<td>0.16</td>
<td>0.15</td>
<td>0.03</td>
<td>0.14</td>
<td>0.09</td>
<td>0.02</td>
<td>2.38</td>
<td>0.23</td>
<td>0.05</td>
</tr>
<tr>
<td>Návar (2010a)</td>
<td>34</td>
<td>0.10</td>
<td>0.11</td>
<td>0.04</td>
<td>0.12</td>
<td>0.05</td>
<td>0.02</td>
<td>2.42</td>
<td>0.25</td>
<td>0.08</td>
</tr>
<tr>
<td>Zianis and Mencuccini (2004)</td>
<td>277</td>
<td>0.15</td>
<td>0.13</td>
<td>0.01</td>
<td>0.12</td>
<td>0.04</td>
<td>0.01</td>
<td>2.37</td>
<td>0.28</td>
<td>0.03</td>
</tr>
<tr>
<td>$\mu$</td>
<td></td>
<td>0.14</td>
<td>0.11</td>
<td>0.03</td>
<td>0.12</td>
<td>0.05</td>
<td>0.01</td>
<td>2.38</td>
<td>0.21</td>
<td>0.06</td>
</tr>
</tbody>
</table>

$N$ = number of biomass equations; $\bar{a}$ = average coefficient value; $\sigma$ = Standard deviation; CI = confidence interval values ($\alpha = 0.05$; D.F = $n-1$); $\mu$ = population mean. Jenkins et al. (2003) compiled 2456 grouped in 10 biomass equations for temperate North American clusters of tree species. Ter Mikaelian and Korzukhin (1997) reported equations for 67 North American tree species but I employed only 41 equations that describe total aboveground biomass. Návar (2009b) reported a Meta-analysis for 229 allometric equations for Latin American tree species but only 78 fitted the conventional model for aboveground biomass. Návar (2010) reported $B$-scalar exponent values for 34 biomass equations calculated from shape-dimensional analysis. Zianis and Mencuccini (2004) reported equations for 279 worldwide species. It is recognized that several studies report the equations that were compiled by Jenkins et al. (2003).

Table 1. Scalar coefficients of the allometric conventional model and re-calculated $a$-scalar intercept values assuming that $B = 2.38$ for six meta-analysis studies.
consistent with the work conducted by Burrows (2000) and Fehrmann and Kleinn (2006), shows that the scaling exponent of the WBE model is correct as long as $D_{0.10}$ m is reported in the allometric model. Enquist et al. (1998) and West et al. (1999) defined that the WBE approach was derived on the assumption that the relationship between diameter and tree height, $H$, scales with the assumed exponent value of $2/3$. This coefficient has been found to be close to $1/2$ as it was discussed above.

The assumption of a constant $B$-scaling exponent value necessitates the re-calculation of the $a$-scalar intercept value for available allometric equations. A graphical example for this approach is shown in Figure 4 for 41 total aboveground biomass equations reported by Ter Mikaelian & Korzukhin (1997).

![Graph showing total aboveground biomass equations for 41 North American tree species reported by Ter Mikaelian and Korzukhin (1997) overlapped with allometric equations that assume a constant $B$-slope value of 2.38 and re-calculating the $a$-intercept scalar coefficients. Note the suitability of the reduced semi-empirical, non-destructive model of tree biomass assessment.](image)

The re-calculation of the $a$-intercept is not straightforward. That is, the mathematical solution for the $a$-scaling intercept is not unique. For a reported biomass equation, it is a function of $D$, as it is described in the following example:

$$M = a_{kn}D^{b_{kn}} \therefore M = a_{kn}D^{2.38}$$

$$a_{kn} = \frac{a_{kn}D^{b_{kn}}}{D^{2.38}} = a_{kn}D^{(b_{kn} - 2.38)}$$

(18)
Using the example for the *Alnus rugosa*, Ter Mikaelian and Korzukhin (1997) reported the following equation: \( \ln(M) = 0.2612 + 2.2087 \ln(D) \). Then, by assuming that the B-scalar exponent value is 2.38 instead of 2.2087, the new \( a_{unk} \)-intercept figure is mathematically solved as follows:

\[
a_{unk} = \frac{0.2612D^{2.2087}}{D^{2.38}} = 0.2612D^{(2.2087 - 2.38)}
\]

if \( D = 10 \text{ cm} \); \( a_{unk} = 0.1760 \)

if \( D = 70 \text{ cm} \); \( a_{unk} = 0.1261 \)

Using simulated M-D data, the statistical \( a_{unk} \)-intercept value would be 0.1229. Therefore, the mathematical method of finding the value of \( a_{unk} \) is skewed. In the absence of a statistical program, it is recommended to estimate the \( a \)-scaling intercept by mathematically solving equation [19] with the largest D value recorded in the biomass study or in the forest inventory. The re-calculation of the \( a \)-scalar intercept can also be derived with the assumption that \( B = 2.67 \) or any other \( B \)-constant coefficient and produce similar goodness of fit. For 41 allometric aboveground equations reported by Ter Mikaelian and Korzukhin (1994), the mean (confidence interval) \( a \)-scalar intercept value is 0.1458 (0.026). Re-calculated values with the assumption that \( B = 2.38 \) and that \( B = 2.67 \) result in mean values of 0.1174 (0.012) and 0.042 (0.0045), respectively. The recalculated \( a \)-value with the assumption that \( B = 2.38 \) outcome consistent and unbiased \( a \)-intercept figures, statistically similar to those of the original equations (Table 1). The assumption that \( B = 2.67 \) deviates notoriously the intercept coefficient values by 3.5 orders of magnitude. That is, the WBE model has to be re-defined in either the \( B \)-scalar exponent to 2.38 or the \( C \) coefficient to a higher value.

A set of biomass equations would have a constant \( B \)-scalar exponent, a set of re-calculated \( a_{unk} \) figures and standard \( \rho_w \) values, a data source sufficient to construct the reduced semi-empirical, non-destructive method of M assessment. This methodology assumes: a) that the bole wood specific gravity, \( \rho_w \), is similar to the entire tree specific gravity, \( \rho \), value; and b) that \( a_{unk} \) and \( \rho_w \) are linearly related with a 0 intercept, and a slope coefficient that describes the \( C \) proportionality constant of the WBE model. Návar (2010b) derived the following relationship: \( M = (0.2457(\pm0.0152))\rho_w^*D^{2.38} \) for 39 biomass equations for temperate North American tree species. That is, the equation within brackets computes the \( a \)-scalar intercept with only \( \rho_w \) values. This mathematical function is called the Návar (2010b) reduced equation and it is expected to vary between forests and between forest stands. Therefore, this relationship must be locally developed when information is available. Brown (1997) and Chavé et al. (2005) for worldwide tropical species and Miles and Smith (2009) for North American tree species reported comprehensive lists of \( \rho_w \) values. If for one moment, it is again assumed that \( \rho_w = \rho \), and that \( B = 2.38 \), then the \( C \) coefficient of the Návar (2010) model would have confidence bounds of 0.2304 and 0.2609 for North American temperate tree species, respectively. The application of this model to 10 clusters of species reported by Jenkins et al., (2003) is reported in Figure 5. The Návar (2010b) reduced model deviates notoriously for the woodland tree species showing that it is specific in nature.
Fig. 5. Contrasts between the reduced semi-empirical, non-destructive model of Návar (2010b) and empirical equations for 10 clusters of tree species reported by Jenkins et al., (2003) for North American tree species.
2.10 Examples of semi-empirical methods of tree M assessment

Projected tree M values by the restrictive, the reduced, and the shape-dimensional non-destructive, semi-empirical models reside within the confidence bounds of the conventional model for most allometric equations tested for northern Mexico (Figure 6). The shape-dimensional non-destructive model proposed by Návar (2010a) fits better biomass datasets than the reduced or the restrictive models, since the later model estimates the \( a \)-intercept coefficient with an \( r^2 = 0.65 \). Equations reported to estimate \( a \) with \( B \), instead of with \( \rho_w \), has an \( r^2 \) value > than 0.70 (Zianis and Mencuccini, 2004; Fehrmann and Klein, 2006; Pilli et al., 2006; Návar, 2010a). Indeed, Návar (2010a), in a simulation study, observed that \( r^2 > 0.90 \) for relationships derived for temperate tree species of northwestern Mexico.

The reduced non-destructive, semi-empirical model reported in here can be additionally employed for checking the consistency of available conventional allometric models. That is, equations that trespass \( a \)-intercept lines would biased M estimates. The limits of most empirical allometric equations can be easily determined using this non-destructive approach. The limits of biomass equations can be found just before they trespass a lines. Hence, this technique is handy for finding the right equations, their limits and as a consequence M estimates would be improved for any forest.

2.11 Future directions in the development of semi-empirical methods of M assessment

The tendency of semi-empirical and theoretical process studies to derive constant values that easily describe the mass of trees has become the center of current allometric studies. The methodology proposed by the theoretical and semi-empirical models is the basis for further development and improvement of mixed, process models. Full process models that deterministically assess tree M could never be developed since the variance in aboveground biomass data is hard to be fully explained by conventionally measured tree variables. Therefore, the need for semi-empirical techniques that convey physiological basis such as those proposed by West et al. (1999) and by Návar (2010a,b) derived from fractals, reduced and shape-dimensional analysis.

The empiricism of any non-destructive techniques of tree M assessment would arise early in the bole volume estimation. For example, the Schumacher and Hall (1933) allometric bole volume equation, i.e., \( \ln (V) = \ln(a_v) + d\ln(D)+ h\ln(H); a_vD^dH^h \), may have also constant \( d \) and \( h \) scaling exponents for most trees and the \( a_v \) intercept scaling coefficient varies within trees and in trees between forests. If so, the \( a_v \) intercept scaling coefficient of the Schumacher and Hall (1933) volume equation would improve the description of the third dimension of timber by incorporating its shape that is intrinsically related to the taper. Just as the \( a \)-scalar intercept coefficient of the allometric biomass equations describe the fourth dimension of timber, its \( \rho \), the \( h \) scaling exponent partially explains the first dimension of timber, its slenderness. These arguments physically suggest that M of a tree with diameter recorded at breast height, \( D \), should be proportional to the product of \( \rho \) times volume (\( V \)), and that volume is a function of basal area \( a \) height; as follows:

\[
M = V \rho
\]

When model [20] is further developed by coupling the Schumacher and Hall (1933) volume equation and the power function that relates \( H \) to \( D \) it would result in model [15].
Fig. 6. Testing the restrictive, the reduced and the shape-dimensional, semi-empirical, non-destructive model performance for 10 independent allometric studies collected from northwestern Mexico. The regression lines, raw data and confidence bands on the B-value of the conventional allometric model are also depicted.
Equation [15] is similar to the theoretical WBE model by assuming that $C = (av \times ah)$ and that $B = 2.67 = d + hH^*$. Empirical contrasts of the $B$-scalar exponent values calculated from shape-dimensional analysis and the constant value of the WBE model show that they are statistically different for 34 allometric studies conducted in northern Mexico. The semi-empirical non-destructive model [15] is not different either to those equations proposed by Chave et al. (2005) or by Ketterings et al. (2001), which are reported as models [21] and [22], respectively.

$$M = \rho(C) \cdot D^{2+H^*}$$  \hspace{5cm} (21)

$$M = \rho_w(0.11) \cdot D^{2+H^*}$$  \hspace{5cm} (22)

Where: $H^*$ is the scaling exponent of the power function of the H-D relationship and $C$ is a proportionality constant. Note that Ketterings et al. (2001) proposed that $C = 0.11$ for tropical trees of south East Asia. The $C$ coefficient values calculated by Návar (2010d) are different than the one proposed by Ketterings et al. (2001), since it had a mean (confidence bound) value of 0.2457 ($\pm$0.0152) for North American tree species.

The $B$-scalar exponent $2+H^*$ reported in equations [21] and [22] differs from the empirical value noted in meta-analysis and shape-dimensional studies as 2.38 by Návar (2010b) and the exponent coefficient proposed by West et al. (1999) as 2.67. The $H^*$ coefficient has an approximate mean value of 0.53 (McMahon, 1973; Niklas, 1994; Návar, 2010a) and the mean scalar exponent, according to model [21] and [22], is consequently $B = 2.53$. Models [21] and [22] assume that the volume equation has an exponent of $D^{2.0}$. Návar (2010a) using the shape-dimensional analysis coupled with fractal geometry noted that $d = 1.93$ (0.066) and $h = 0.917$ (0.079) for 12 volume equations for temperate trees of northwestern Mexico. Therefore, an exponent value $d \sim 1.9$ (0.07) would be appropriate for these forests. That is, boles are neither two dimensional photosynthetic surfaces ($D^{2.0}$) nor three dimensional geometric solids ($D^{3.0}$); hence, if $d \sim 1.9$, then $B = 2.43$ in the Ketterings et al. (2001) or Chavé et al. (2005) semi-empirical models. This new slope value falls within the confidence bounds of the mean $B$-value found in Meta analysis studies ($2.38 \pm 0.06$).

The major finding of this brief review is that most current semi-empirical and theoretical studies assume a constant $B$-scalar exponent value. That is: $B_{\text{Návar}} \leq B_{\text{Chave}} = B_{\text{Ketterings}} \leq B_{\text{West}}$; $2.38 \leq 2.53 = 2.59 \leq 2.67$. Further empirical and theoretical studies are required before the constant $B$-scalar exponent value finally emerges.

2.12 Implications of reduced non-destructive models of $M$ assessment

Reduced non-destructive models that assume a constant $B$-scalar exponent easily calculates $M$ for each individual tree as well as for any set of trees since it depends upon the $a$-scalar intercept value that is a function of the wood specific gravity value. The major implicit hypothesis of a reduced model such as the WEB or the Návar (2010b) equations would then be that trees add mass, volume, area or length at a rate per unit of diameter growth that is a function of the $a$-scalar intercept, which is a function of the $\rho_w$ values. Návar (2010b) found a positive relationship between $a$ and $\rho_w$, consistent with the explicitly statement described in the theoretical and semi-empirical models. If so, then trees with large $\rho_w$ figures would grow diametrically (as well as to any other dimension) at a small rate and vice versa, since $D = \frac{\sqrt{M}}{\rho_a}$. A preliminary analysis of diameter increment and $\rho_w$ values for 15 tropical species
fitted well with a negative linear relationship with the following equation:

$$\frac{\partial D}{\partial t} = 4.23 - 5.38\rho_w; \, r^2=0.50;$$

further empirically supporting the evidence that a reduced non-destructive, semi-empirical or the theoretical model that assumes a constant $B$-scalar exponent is also physiologically and metabolically correct.

The selection of a constant $B$-scalar exponent value in a reduced semi-empirical model has several consequences. Statistically, the $B$ and $a$ scalar coefficients are related with negative power or logarithmic equations (Zianis & Mencuccini, 2004; Pilli et al., 2006; Návar, 2009a,b). Hence, the $a$-scalar intercept would deviate from values reported in most allometric studies by assuming a different $B$-scalar exponent. For example, Table 1 reports mean (confidence bound) population values for the $a$-scalar intercept as: $0.14 \pm 0.03$. Therefore, when assuming a different scalar exponent values either the taper factors (C) or the basic specific gravity ($\rho$) for the entire tree would also change. Since $\rho$ is assumed to be a fixed value for any tree, then the $a$-scalar intercept must have a fixed value as well that is only dependent upon the $C$ coefficient.

The $C$ values would be later more precisely and physically evaluated as long as new information and data analysis comes up. In the meantime, Návar (2010b) and Návar (2010d) have noted that the $C$ empirically-estimated value when plotting $\rho_w$ vs. $a$ varies between 0.2457 to 0.2687 for biomass equations reported for temperate North American and for tropical tree species, respectively. When assuming that $B = 2.38$, good tree $M$ approximations are found for temperate and some tropical but not for dry land tree species. If further assuming that $B = 2.67$, tree $M$ is overestimated for both temperate and for tropical forest communities. Whence, a $C$ coefficient value should be further calculated with this later assumption by $C_{B=2.67} = (0.2457D^{2.38})/(D^{2.67})$. Again the $C$ value is a function of $D$ and it can go from 0.18 in trees with $D = 5$ cm to 0.076 in trees with $D = 100$ cm; following a power function of $C_{B=2.67} = 0.2457D^{(2.38-2.67)} = 0.2457D^{(-0.29)}$.

An independent technique to estimate the $C$ coefficient figure was preliminarily proposed by Návar (2010b) by developing the shape-dimensional analysis as $C = (a_v*a_h)$. Mean (standard deviation) $a_v$ values of 0.55 (0.0185) were found when fitting the statistical coefficients of the Schumacher and Hall (1933) volume equation to 12 temperate tree species of northern Mexico. By assuming a mean $a$-re-calculated scalar intercept value of 0.12 (Table 1) and the mean (standard deviation) of the taper values by solving for the $a_h$ values since they are hard to find at this time, the $C$ coefficient would attain a range of 0.2104-0.2249 for 68% or 0.2037-0.2330 for 95% of the individual biomass equations, assuming the proportionality coefficient is normally distributed. The $C_{B=2.38}$ (0.2457±0.0152) for temperate North American tree species is found within this range. For tropical tree species (0.2687±0.1078), it appears to be slightly overestimated. On the other side, the $C_{B=2.67}$ (0.076-0.180) values are a tone with the $C$ coefficient (0.11) proposed by Ketterings et al. (2001) but both are underestimated when contrasting them with $C$ range values proposed by the shape-dimensional analysis. The $C$ coefficient value proposed by Ketterings et al. (2001) is dependent upon $\rho_w$ since it was calculated as: $C = \rho_w/a$. From the shape-dimensional or the fractal analysis, $C = a/\rho_w$. New approaches on how to analyze biomass data will eventually elucidate the value of $C$ and $a_h$. One way to go is to analyze backwards biomass data to solve for $C$ or by $a_h$ when applying the empirical conventional allometric model [1]. For example; when fitting the WBE model, the $C$ coefficient could be evaluated by: $C = M/(\rho_wD^{2.67})$ or when developing the semi-empirical model derived from the shape-
dimensional analysis; $a_h = M/(a_v \times \rho_w D^{2.38})$ and then $C = (a_h \times a_v)$; or by evaluating $C$ mathematically or iteratively until finding the right solution for each allometric biomass equation and then solving backwards for $a_h$. This research would eventually find the right semi-empirical, non-destructive methodology for the simple estimation of the scalar coefficients that facilitate tree M assessments. The slenderness $a_h$ parameter is related to the $D/H$ relationship. However, further mathematical evidence is required to better calculate this parameter.

Independent preliminary $C$ coefficient estimates approximated by $C = M/(\rho_w D^{2.38})$ or by $C = M/(\rho_w D^B)$ for several clusters of tree species are reported in Figure 7. The mean $C$ coefficient values are 0.27 (0.09) and 0.25 (0.04) for the conventional and Návar (2010b) models and this preliminarily analysis demonstrate that these figures are also consistent with previously reported values for temperate ($C = 0.2457 \pm 0.0152$) and for tropical forests ($C = 0.2687 \pm 0.1078$). Statistical differences are noted between clusters of species, with $C$ values smaller in Cedar/Larch, Pine, and Fir/Hemlock than in Maple/Hickory or Douglas Fir.

Fig. 7. $C$ coefficient values of the conventional and Návar (2010b) reduced models for eight groups of species. Mean and confidence bounds ($p = 0.05$) are also depicted.
2.13 Calculating the entire tree specific gravity value

Several recent allometric studies include the wood specific gravity value as an exogenous variable (Brown, 1997; Chavé et al., 2005; Návar, 2009a). The theoretical WBE model calls for the specific gravity value for the entire tree (West et al., 1999). The Návar (2010b) reduced semi-empirical model requires the wood specific gravity value to escalate the \(a\)-scalar intercept value. The \(\rho_w\) values are conventionally measured and reported figures can be found in recently-reported compilations. There remains the question to be solved whether \(\rho_w = \rho\). Therefore, further information is required on easy ways to estimate \(\rho\). The physical assessment of a weighted \(\rho\) value for the entire tree is derived with model [23]:

\[
\rho = \frac{M_b \rho_{bw} + M_{br} \rho_{br} + M_l \rho_l}{M}
\]

(23)

Where: \(M_b\rho_{bw}\) = bole mass x bole specific gravity; \(M_{br}\rho_{br}\) = branch mass x branch specific gravity; \(M_l\rho_l\) = leaf mass x leaf specific gravity; \(M\) = total aboveground biomass.

The mean \(\rho_w\) value is difficult to assess, since it changes from the bark to the pith and from the bole base to the tip (Parolin, 2002). Miles and Smith (2009) compiled a comprehensive dataset containing standard specific gravity values for bole wood and bark for 156 North American tree species that can be preliminarily explored. Standard \(\rho_w\) value is measured at DBH. Therefore, this as well as any other data source requires a correction factor since \(\rho_w = f(H)\) (Silva-Arredondo and Návar-Cháidez, 2009). One approximation to solve for the change of \(\rho_w\) with \(H\) is mathematically described in model [24]:

\[
\rho_b = \frac{M}{V} = \frac{Mc + Mh + Ms}{Vc + Vh + Vs} \cdot \cdot.
\]

(24)

\[
M_{c,h,s} = 0.7854 \int_{h=0}^{h=H} \left[ d = f(h) \cdot \rho_w = f(h,D) \right] \bar{c}h
\]

\[
V_{c,h,s} = 0.7854 \int_{h=0}^{h=H} \left[ d = f(h) \right] \bar{c}h
\]

Where: \(M\) = mass, \(V\) = volume, \(c\) = bark, \(h\) = hardwood, \(s\) = softwood, \(h\) = relative tree height, \(H\) = total tree height, \(d\) = relative diameter, \(D\) = diameter.

Taper functions that relate bole diameter to relative bole tree height \((d = f(h))\) are available for most worldwide timber commercial tree species. Preliminary exploration of the bole wood and bark specific gravity data values reported by Miles and Smith (2009) show that they have similar specific gravity mean values (0.483 and 0.487) for all 156 North American tree species, although there are significant differences between these biomass components within each reported tree species. The scientific literature hardly reports leaf specific gravity values. However, leaf biomass accounts for by approximately 20% of the total aboveground biomass for 110 young trees of five species of northwestern USA (Delaney, 2007) and for 55 young trees of five pine species of Durango, Mexico (Návar et al., 2004). This ratio would probably steadily diminish with tree age and this relationship is also needed for most tree species.

Preliminary empirical evaluations of the entire tree basic specific gravity can be derived from the relationship between re-calculated \(a\) vs. \((a_v \times a_h)\) or \(a\) vs. \(C\) with a slope describing \(\rho\). Mathematically, \(\rho = M/(C \times D^{2.38})\). These relationships are species specific and require
sufficient allometric data to obtain coefficients with minimum variance. Site features may also influence this statistical function. Therefore, additional studies are needed to further advancing the knowledge on simple ways to estimate $\rho$ and $C$ with the goal of developing improved non-destructive methods of M assessment. In the meantime bole wood specific gravity is an estimator of the entire tree specific gravity.

3. Plot aboveground biomass assessments

3.1 Introduction
The application of allometric equations to forest inventory tree data is the standard methodology for the plot, stand M assessment since allometric equations that straightforward calculate plot aboveground biomass are hard to find. One equation for tropical dry forests of Jalisco, Mexico reported by Martínez-Yrízar et al. (1992) and a second one for semi-arid sub-tropical Tamaulipan thornscrub matorral of northeastern Mexico published by Návar et al. (2002b) were found in a brief scientific literature review. One major drawback of these equations is that they harvested all standing trees, including un-inventoried trees ($d < 7.5$ cm) in small plots (5 m x 5 m). Both equations use basal area as independent variable and the second one draw on also mean stand H and tree diversity, S, as an index of the stand wood specific gravity variation.

The conventional physics equation (model [19]) is an independent method for calculating tree and plot biomass. Commercial and research forest inventories conventionally report timber volume, $V$, at the plot scale. The entire tree specific gravity is at the present difficult to calculate with reported information, but an estimator can be used instead, the bole wood specific gravity, $\rho_w$, that is conventionally measured and reported in most wood technology studies. One shortcoming of this procedure is that branch volume is rarely integrated into bole volume estimates. On the other hand, Silva-Arredondo and Návar-Cháidez (2009) reported that the mean $\rho_w$ value for trees should be taken at 0.22H for temperate trees of northern México when guidelines for $\rho_w$ measurements recommend taking core samples at diameter at breast height, and therefore M estimates must use a correction factor to assess less biased M values.

A second independent approach to evaluate plot M multiplies plot volume times a biomass expansion factor, BEF. The BEF values previously calculated by the ratio of $M/V$ are available for several tree species and for several forests (Gracia et al., 2004; Lehtonen et al., 2004; FAO, 2007; Návar-Cháidez, 2009; Silva-Arredondo and Návar, 2009). Whenever FEB, $V$ and $\rho_w$ data are available, M estimation procedures described above can be used as contrasting methods since they are partially independent methods of plot M assessment. Brown (1997) coupled these variables to come up with plot M density with the following equation:

$$M = ER \cdot \rho_w \cdot BEF$$

Where $ER = \text{stand timber volume (m}^3 \text{ha}^{-1})$; $\rho_w = \text{mean standard bole wood specific gravity}$ for the tree species dominating the stand (Mg m$^{-3}$); and $BEF = \text{biomass expansion factor}$ (dimensionless).

The BEF value of equation [25] is dimensionless and it only expands bole plot M to the entire aboveground tree biomass (boles, branches and leaves). Brown (1997) interpolated this equation for complex forests by weighting it for tree species or genera that constitute the forest.
The conventional BEF dimensional values reported in the recent scientific literature take the following form:

$$BEF = \frac{M(\text{Mg}) or (\text{Mg ha}^{-1})}{V(m^3) or (m^3 \text{ ha}^{-1})} = BEF \left[ \frac{\text{Mg}}{m^3} \right]$$

(26)

Preliminary analysis of BEF values point to the local calibration by mean stand H, D, age, density, etc. to be recommended as methods of plot M assessment (Brown, 2002; Gracia, 2004; Lehtonen et al., 2005; Návar-Cháidez, 2009; Silva-Arredondo & Návar-Cháidez, 2009; Návar, 2010d). Therefore they are currently empirical factors of local use. The FAO (2007) compiled dimensional BEF figures for worldwide forests and they are shown in Table 2.

![Table 2](https://www.intechopen.com)

<table>
<thead>
<tr>
<th>Region/Sub-region</th>
<th>Aboveground Biomass (Mg)</th>
<th>Total tree biomass (Mg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eastern Africa</td>
<td>2.3</td>
<td>2.9</td>
</tr>
<tr>
<td>North Africa</td>
<td>2.1</td>
<td>2.7</td>
</tr>
<tr>
<td>Western and Central Africa</td>
<td>1.3</td>
<td>1.7</td>
</tr>
<tr>
<td>Africa</td>
<td>1.5</td>
<td>1.9</td>
</tr>
<tr>
<td>Eastern Asia</td>
<td>0.7</td>
<td>0.9</td>
</tr>
<tr>
<td>Southern Asia</td>
<td>1.4</td>
<td>1.8</td>
</tr>
<tr>
<td>Western and Central Asia</td>
<td>0.9</td>
<td>1.1</td>
</tr>
<tr>
<td>Asia</td>
<td>1.1</td>
<td>1.4</td>
</tr>
<tr>
<td>Europe</td>
<td>0.7</td>
<td>0.8</td>
</tr>
<tr>
<td>Caribbean Countries</td>
<td>2.0</td>
<td>2.6</td>
</tr>
<tr>
<td>Central-America</td>
<td>1.4</td>
<td>1.8</td>
</tr>
<tr>
<td>North America</td>
<td>1.0</td>
<td>1.1</td>
</tr>
<tr>
<td>North and Central America</td>
<td>1.0</td>
<td>1.2</td>
</tr>
<tr>
<td>Australia New Zealand</td>
<td>1.4</td>
<td>2.0</td>
</tr>
<tr>
<td>South America</td>
<td>1.1</td>
<td>1.5</td>
</tr>
<tr>
<td>World</td>
<td>1.0</td>
<td>1.3</td>
</tr>
</tbody>
</table>

Total tree biomass = boles, branches, foliage and roots.

Table 2. Biomass expansion factors to assess below and total standing stand aboveground biomass as a function of bole volume (Source: FAO, 2007)

Reported BEF values are practical for regional aboveground and total tree biomass calculations. For specific, local biomass projects, regional BEF factors must be applied whenever they are available since they can vary notoriously from place to place by changes in the forest structure (Brown, 2002).

Most studies that evaluate standard plot aboveground biomass use a single mathematical function, which is frequently the most popular, the one developed for worldwide studies, or the one locally developed, although there is a wide range of allometric equations available for regional and world-distributed forest communities (Brown et al., 1989; Brown, 1997; Chavé et al., 2001; 2003; 2005; Ketterings et al., 2001; Návar et al., 2004; Návar, 2009a,b). Therefore, when contrasting plot M estimates with different allometric equations or methods, they will always show variations.
3.2 Contrasting plot M assessments

Research studies that contrast plot M estimates by different allometric models or different methods of estimation are scarce in the scientific literature. Figure 2 exemplifies the large variation expected when evaluating tree M for tropical rainforest, tropical dry forests, as well as for the hybrid IAN 710 *Hevea brasiliensis* trees. Figure 4 also shows large differences in M evaluations for similar tree species. Therefore, the selection of an allometric model is the most important source of uncertainty when assessing plot M (Chavé et al., 2004; Návar et al., 2010).

Several other uncertainty sources can be expected when computing tree and consequently plot M. The allometric equation has an intrinsic error given by the standard deviation of the tree M estimate that is related to the number, diversity and diameter structure of sampled trees for biomass allometry. The second, and probably the most important, arises when an off-site equation is applied to forests with different tree diversity and diameter structure. This error source has not been evaluated since allometric equations are hardly validated and therefore it is preliminary assessed when several allometric equations estimate plot biomass using the same forest inventory dataset (Chavé et al., 2004). Three other types of uncertainties related to the forest inventory scheme are: the size and shape of sampling plots, the spatial distribution of sampling units in the forest, and tree measurements within sampling plots.

Large variations in tree and consequently in plot M evaluations make the biomass stock of most forest stands to remain poorly understood. For example, for mid and high latitude forests, Fang et al. (2006) reviewed the literature and noted that inventory-based forest M stocks documented for major countries fall within a narrow range of 72–112 Mg ha$^{-1}$ with an overall area-weighted mean of 87.2 Mg ha$^{-1}$. These estimates are 0.40 to 0.71 times smaller than those (122–216 Mg ha$^{-1}$) computed in previous analysis.

For structurally-complex tropical forests, the evaluation of plot M appears to have larger variability (Fearnside and Laurance, 2003; Houghton, 2005; Saatchi et al., 2007). Chavé et al. (2003) tested four different allometric equations (Chavé et al., 2001; Chambers et al., 2001; Brown et al., 1989 (1); Brown et al., 1989 (2)) for Panamanian tropical forests and mentioned that all four equations yielded comparable but statistically different plot M estimates. The variation among equations was 26% of the mean estimate. Houghton et al. (2001) tested seven different plot M estimates for the Brazilian Amazon forests and calculated a mean deviation of 20% but individual observations deviated 45% from the mean estimate. Araujo et al. (1999) harvested and weighed all standing tree biomass in a 0.2 ha area plot of the Brazilian Amazon forest. Of 14 different biomass equations applied to this dataset, 12 biased notoriously and only two provided suitable plot M assessments, within ± 0.6% of the weighted field biomass.

3.3 An example of the application of tree allometry for the plot M assessment of Mexican tropical forests

Návar et al. (2010) evaluated plot M by fitting nine different allometric equations for eight different Mexican tropical forests. Fitted functions are reported in Table 3. The conventional physics equation of volume times the wood specific gravity was also fitted. Results showed that several allometric equations predicted significantly different mean stand M estimates for all eight data sources (Figure 7). The mean deviation between allometric equations was 10.7 Mg ha$^{-1}$ (62%) and uncertainties are a function of the forest aboveground biomass density ($\text{Sxe} = 3.92M^{0.71}$; $r^2= 0.92$). Therefore the allometric equations consistently yielded larger mean standard errors for moist and wet than for dry tropical forests.
Researcher | Tropical Forest | Equation
--- | --- | ---
Brown et al. (1989) | Wet | M=e^(-2.409+0.952(D^2 H))
Brown (1997) | Dry | M=34.47-8.0671(D^2 H)
Brown (1997) | Moist | M=e^(-2.134+2.53(ln(D))
Martínez-Yrizar et al. (1992) | Dry | M=10**(-0.5352+Log10(BA))
Návar et al. (2010) | Dry | M=0.08479(ρw0.552ΔD^2.243ΔH^0.4773)
Návar et al. (2010) | Dry | M=e^(-2.409+0.952Ln(D^2 H))
Návar (2010a) | All | M=(38.36*B-6.9045)(D=B+d+hΔ)
Návar (2010b) | All | M=(-0.0094+0.2687ρwΔD^2.38
Chavé et al., (2005) | Dry | M=0.112*(ρwD^2H)^0.916
Chavé et al., (2005) | Moist | M=ρw*e^(-1.49+2.148Ln(D)+0.207Ln(D)^2-0.028Ln(D)^3)
Chavé et al., (2005) | Wet | M=0.0776(ρwD^2H)^0.94
Chavé et al., (2005) | Wet | M=0.0776(ρwD^2H)^0.94

Where: The non-destructive method proposed by Návar (2010a) uses the H-D and V=H,D (i.e., H = aΔD^6 and V = aΔD^6H^6) equations and an empirical equation to estimate a = 38.36B-6.9045; D = diameter at breast height, BA = basal area; ρw = wood specific gravity.

Table 3. Allometric equations employed in aboveground biomass estimation for eight tropical forest communities of Mexico.

Total aboveground biomass deviations for all tropical forests inventoried between either a) allometric equations or b) allometric and the physics equation were smallest when using the non-destructive model proposed by Návar (2010a). The equations of Brown (1997) for tropical dry forests and the equation of Návar (2010a) yielded consistent plot M estimates across allometric equations and methods. Deviations as large as 2.4 orders of magnitude were found; i.e. see for example stand M assessments by the Brown (1997) and the physics equation for moist forests or the non-destructive method II and the local equation developed by Návar (2009) for tropical dry forests.

3.4 Future directions in stand M assessments
The large deviations due to the application of allometric models or methods developed off site reveal that probabilistic plot M estimates are highly likely to be skewed by more than one and sometimes up to two orders of magnitude. In the absence of harvested trees in sampling stands for the development of local plot M allometry, the choice of tree biomass models must focus on those that consistently result in similar plot M estimates and with the least deviation to the physics equation. Whenever it is possible, a mean estimate across allometric equations that do not notoriously deviate between them would probabilistically deliver a better plot M assessment. However further research is required on this issue for several complex forests before it is recommended to improve precision of plot M assessments.

3.5 The need for the development of plot allometry
In future plot allometry studies, research must center on harvesting all inventory trees from plots under, for example, by shifting cultivation, the opening of roads, or other forest disturbances. Weighting all biomass, collecting samples for fresh and dry weights and
Fig. 8. Mean and confidence interval aboveground biomass estimates for two tropical forests calculated by different allometric equations for eight tropical forest communities of Mexico.
calculating all stand M. The application of local or worldwide-developed tree allometry must be the first step to calculate stand M. In case these techniques do not provide a good fit, new tree and plot M equations should be developed that can be escalated up and down from tree levels to plot or stand scales. 

Other alternatives that expand local allometry to plot scales could be the combination of restrictive in the number of harvested trees (Zianis & Mencuccini, 2004) and non-destructive (West et al., 1997; Návar, 2010a; 2010b) methods with local tree or plot allometry. Whenever local allometric equations are not available, coupling available empirical equations, restrictive models and semi-empirical non-destructive methods for inventoried datasets and choosing an average of those that consistently produce similar plot M evaluations could improve stand M precision, according to the Central Limit Theorem. However there are a few of such studies reported in the scientific literature that discuss this issue.

4 Biomass assessments at regional scales

4.1 Introduction

Tree or plot M assessments are extrapolated over larger spatial scales to evaluate forest, regional or national aboveground biomass, AGB. Houghton et al. (2001) classify techniques available in the interpolation procedure as: a) field measurements, b) environmental gradients and c) remote sensing techniques. Field measurements are common approaches and they are mathematically just the multiplication of the mean plot M estimate times the area of each forest type (Schroeder et al., 1997; Houghton et al., 1992; Brown, 2002). Commercial and research forest inventories or a combination of both field data collection sources provide information for the AGB estimation for each forest class. This approach is time-consuming, labor intensive, expensive, and difficult to implement, especially in remote areas and it cannot provide the spatial distribution of AGB biomass for large areas because the error involved increases with the forest area. The major assumption is that a good sampling scheme represents the forest class under research. The error can be evaluated by multiplying the standard deviation, the standard deviation as a percentage of the mean, percent error, or the confidence interval over the forest area. The simple field method can increase precision by augmenting the spatial resolution of each forest type combined with the application of improved sampling schemes. AGB assessments augment precision when this method is applied for compact forest classes such as old-growth forests and forests plantations.

The environmental gradient method should be the recommended technique when there are systematic plot M changes over large forest areas. So far, annual rainfall has proved to be the best predictor variable for Amazonian rainforests (Saatchi et al., 2007) as well as for Mexican tropical forests (Návar et al., 2010c). In general, non-linear models account for by between 50 to 60% of the total stand M variation and they are constrained to attain a final steady constant M value indicating that highest M figures do not have further increments with additional annual rainfall. However, for seasonal or dry forests the relationship is almost linear stressing the importance of the moisture available in places with ample light and radiation (Malhi et al., 2006). In this case, the integration of the non linear equation relating stand M to annual rainfall multiplied by the area evaluates AGB. This relationship can be discretized at different spatial resolutions by calculating forest areas with similar annual rainfall that are, in general, areas with the same altitude above sea level. The error involved in this relationship is associated with the standard error of this relationship which is also multiplied by the same area.
Remote sensing techniques are currently indirect methods of AGB assessments. Remote sensing based methods for AGB assessments are classified as: a) fine spatial resolution (aerial photographs, IKONOS), b) medium spatial resolution data (Landsat, Spot, TM/ETM+), c) coarse spatial resolution data (AVHRR, IRS-IC, WiFs), d) methods based on radar data (Radar, Lidar) (Lu, 2006). Tree M can be better computed when using fine spatial resolution methods as well as radar data. Stand M is better evaluated when the image spatial resolution is from medium to coarse. Remote sensing is also the standard methodology used to classify forests according to vegetation types, and/or cover classes (Houghton et al., 2000; 2001; Saatchi et al., 2007) and takes advantage of the high correlations between spectral signatures, vegetation parameters, transformed images, and image textures to evaluate AGB using field measurements or environmental methods (Lu, 2006). Interpolation techniques are classified as: multiple regression analysis, nonparametric k-nearest neighbor technique, neural networks, or through indirect relationships between remotely sensed forest attributes and biomass (Popescu et al., 2003; Zheng et al., 2004).

Satellite data may eventually be developed to straightforward estimate AGB, but neither optical nor radar data have yielded consistent results in forests with moderate to high biomass (Nelson et al., 2000). Optical and radar data saturate with canopy closure or at relatively low levels of biomass, respectively (Rignot et al. 1997; Nelson et al. 2000). A promising new approach is the use of lidar, which yields a measure of tree height that is related to biomass (Means et al. 1999). Recent 3D models with lidar suggest that the data are highly correlated with aboveground biomass in coniferous forests of northwestern USA (Lefsky et al., 1999a; Means et al., 1999) as well as in deciduous forests of eastern USA (Lefsky et al. 1999b). A NASA satellite equipped with a lidar designed to measure tree heights, biomass, and topography, the Vegetation Canopy Lidar (VCL), is currently taking information and a new tree height worldwide map is already on the NASA homepage.

4.2 Uncertainties of AGB evaluations
The variety of remote sensed data, spatial resolutions, tree and forest attributes, and interpolation techniques make AGB assessments variable. Therefore, the largest AGB uncertainty over large regions results from the extrapolation technique used. Houghton et al. (2001) evaluated seven different M interpolation methods (three field measurements, two methods based on environmental gradients and two methods using remote sensing techniques) with different spatial resolutions for the Brazilian Amazon forests. As a result, AGB estimates deviate by more than one order of magnitude, from 80 to 190 Pg and models also differed on the spatial AGB distribution (Houghton et al., 2001). Although most research has been conducted for tropical forests, Houghton et al. (2001; 2003; 2005); Jenkins et al. (2003); and Pacala et al. (2001) pointed out that forest biomass for the mid-and high latitudes in the Northern Hemisphere is also uncertain due to error in the estimation of tree and plot biomass and its increment. For instance, biomass estimates of Russian forests varied from 112.0 Pg to 140.4 Pg by different authors (Alexeyev et al., 1995; Isaev et al., 1995) although the same forest data sources but different methods were used.

4.3 An example of AGB assessment for Mexican tropical forests and contrasting results
For Mexican tropical forests, Návar et al. (2010c) contrasted two field measurements (red and blue bars), one environmental gradient (green bar), and one Forest Inventory Analysis
(FIA) to assess AGB. In addition, AGB estimates reported by De Jong et al. (2008); Houghton et al. (1999) and De Fries et al. (2002); Brown (1997) and Achard et al. (2004); and IPCC (2006) integrated a contrast analysis. Návar (2010c) calculated that AGB assessments vary between 1.54 with the FIA to 3.00 Pg with the improved spatial resolution analysis. Other evaluations ranged from 3.84 to 4.34 Pg (Figure 9), which are larger than AGB assessments conducted by the author of this report.

Fig. 9. Mean aboveground biomass estimates for Mexican tropical forests calculated by different methods of interpolation (Red = rain, green = dry deciduous, purple = moist deciduous, and blue = moist evergreen forests). Návar-Cháidez (2009) reported biomass expansion factors, BEF, for dry tropical forests of Sinaloa, Mexico as 1.46 (0.022) that are quite consistent with the BEF values reported by FAO (2007) for Latin American tropical forests. Using the total standing volume value calculated in the Mexican Forest Inventory of 2004-2006, AGB biomass expands to 1.54 (0.023) Pg.

The FIA estimate was 2.8 orders of magnitude smaller than the AGB estimate made by IPCC (2006). Assuming all these eight AGB statistics yield a mean of 3.26 Pg, deviations as large as 1.72 Pg (52.76%) are found between methods of interpolation at the regional scale with a mean absolute deviation of 0.82 Pg (25% of the mean value).

4.4 Future directions in regional AGB evaluations
The application of a single interpolation methodology from tree or plot M to larger spatial scales is also highly likely to skew regional AGB figures. Therefore, there is an urgent need for combining available methods of AGB computation to understand the magnitude and sources of variation. Future studies must focus on using at least a second independent interpolation approach and an average estimate and its deviation should be reported. However, I am tempted to recommend the coupling of all three current methodologies of AGB calculation. These are: field techniques, collected in most forest inventory studies; develop relationships between environmental variables (annual precipitation,
evapotranspiration, water and energy balance, Bowen ratio, etc.) and plot M stock estimates and fluxes that are calculated with proved allometric studies; with forests classified by field and remote sensing techniques in several classes and gradients within classes. Although the combination of these techniques requires high expenditure of resources, a diversity of professional skills, and improved current technologies, more precise AGB approximations would be expected that may eventually reduce costs of regional biomass assessments. A single, independent approach could be to develop tree data (diameter and canopy height) with LIDAR techniques; use empirical, locally-derived volume and canopy height equations \( v = f(D, H); \ H = f(D) \) to be fitted to the semi-empirical shape-dimensional, nondestructive model to evaluate tree M at the spatial scale of interest. This is a matter of future studies.

5. Conclusions

Many tree species and worldwide forests do not have updated local allometry. They call for the development and application of local allometry. However, there are a great number of allometric equations reported for worldwide species with a major emphasis on temperate North American and European tree species. Available equations can be classified by the way equation parameters are estimated as empirical, theoretical and semi-empirical non-destructive methods of tree M assessment. Empirical models commonly calculate statistical coefficients by least square techniques in linear, log-linear, non-linear, multiple linear and non-linear, seemingly unrelated linear and non-linear and exponential mathematical formats where diameter at breast height or basal diameter, top height, canopy area and wood specific gravity are independent variables that individually or in combination explain tree M of harvested trees. Conventional mathematical allometric models have intrinsic variations larger than 20% of the tree M but new empirical equations that contain H and \( \rho_w \) as independent variables, in addition to the conventional normal diameter, are improving tree M evaluations since the intrinsic error has been reduced to 16%. Theoretical and semi-empirical non-destructive methods are in the early stages of development and require further testing and refinement although they can be preliminarily recommended as non-destructive approaches of tree and stand M computation. In spite of the wealth on biomass allometry for several trees and forests, conventional plot M evaluations exhibit variations as large as two orders of magnitude when contrasting on and off-site equations. One potential procedure to reduce uncertainty is combining the conventional empirical, semi-empirical non-destructive, and restrictive methods to improve precision when computing tree and plot M for forests that do not convey local biomass allometry. However, research on understanding variations by coupling these methods are lacking elsewhere. There is a variety of interpolation techniques of tree or plot M to forests, regional and national scales but they display variations close to three orders of magnitude when assessing AGB stocks. Coupling tree allometry with FIA evaluations, environmental gradient analysis and remote sensing techniques may reduce this variation in future AGB studies. Modern remote tree data collection techniques (LIDAR) combined with empirical, locally-derived functions to estimate timber volume as a function of diameter and canopy height and canopy height as a function of diameter plugged into the non-destructive semi-empirical, shape-dimensional analysis model could improve AGB evaluations at the spatial scale of interest.
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Due to demands placed on natural resources globally and subsequent deterioration of the environment, there is a need to source and develop appropriate technology to satisfy this requirement. For decades mankind has largely depended on natural resources such as fossil fuels to meet the ever increasing energy demands. Realizing the finite nature of these resources, emphasis is now shifting to investigating alternate energy source governed by environmentally friendly principles. The abundance of biomass and associated favorable techno-economics has recently changed global perceptions of harnessing biomass as a valuable resource rather than a waste. To this end this book aims to make a contribution to exploring further this area of biomass research and development in the form of a compilation of chapters and covering areas of ecological status of different types of biomass and the roles they play in ecosystems, current status of biomass utilization and deriving energy and other value added products from biomass. In this context biomass can be defined as large plants and trees and different groups of microorganisms. This book will serve as an invaluable resource for scientists and environmental managers in planning solutions for sustainable development.

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