1. Introduction

While the position of a robot link can be measured accurately, measurement of velocity and acceleration tends to result in noisy signals. In extreme cases, these signals could be so noisy that their use in the controller would no longer be feasible (Daly & Schwartz, 2006). In order to overcome the problem of noisy velocity measurements and to guarantee that the error between the time-varying desired position and the actual position of the robot system goes asymptotically to zero for a set of initial conditions, a controller/observer scheme, based on position measurements, can be used. In this sort of schemes, the incorporated observer is used to estimate the velocity signal and sometimes the acceleration signal.

Another approach consists in using the Lyapunov theory to design a controller/filter to guarantee the tracking of the desired trajectory, no matter if an estimate of the velocity and acceleration can be possible with the obtained design.

In the perspective of control engineering, the approach of using only joint position measurements in either a controller/observer or a controller/filter to achieve tracking of a desired joint trajectory is denominated output–feedback tracking control of robot manipulators.

Recently, attention has been paid to the practical evaluation of output–feedback tracking controllers. In the paper by Arteaga & Kelly (2004) a comparison of several output–feedback tracking controllers is made, showing that those schemes which incorporate either an observer or filter are better than those which incorporate a numerical differentiation to obtain an estimation of the joint velocity.

The work by Daly & Schwartz (2006) reported the experimental results concerning three output–feedback tracking controllers. They showed the advantages and disadvantages of each control scheme that was tested.

On the other hand, saturation functions have been used in output–feedback tracking control schemes to guarantee that the control action is within the admissible actuator capability. See the papers by Loria & Nijmeijer (1998), Dixon et al. (1998), Dixon et al. (1999) and, more recently, Santibáñez & Kelly (2001). Although much effort was done to derive those controllers and very complex stability analyses were necessary, as far as we know, no experimental evalu-
ation of output–feedback tracking controllers that contain saturation functions in its structure has been reported. Considering that output–feedback tracking controllers can be more efficient than full–state feedback tracking controllers, specially if noisy velocity measurements are present, and taking into account the philosophy of including saturation functions in an output–feedback tracking control design, the objective of this paper is to present an experimental comparison between output–feedback tracking controllers that do not have saturation functions in its structure and controllers that do have saturation functions. The experiments were carried out in a two degrees–of–freedom direct–drive robot, which is important from the control point of view, because the dynamics in this type of robots is highly nonlinear.

This chapter is organized as follows. The robot model and control problem formulation is presented in Section 2. Section 3 describes the experimental robot arm used in the experiments. Section 4 concerns to the description of the desired position trajectory and performance criterion. The controllers as well as the experiments on output–feedback tracking control are presented in Section 5, while Section 6 contains some discussions. Finally, concluding remarks are drawn in Section 7.

2. Robot dynamics and control goal

The dynamics in joint space of a serial–chain n-link robot manipulator considering the presence of friction at the robot joints can be written as (Canudas de Wit et al., 1996; Kelly et al., 2005; Ortega et al., 1998; Sciavicco & Siciliano, 2000):

\[ M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) + F_v \dot{q} = \tau \] (1)

where \( M(q) \) is the \( n \times n \) symmetric positive definite inertia matrix, \( C(q, \dot{q}) \) is the \( n \times n \) vector of centripetal and Coriolis torques, \( g(q) \) is the \( n \times 1 \) vector of gravitational torques, \( F_v = diag\{ f_{v1}, \ldots, f_{vn} \} \) is the \( n \times n \) positive definite diagonal matrix which contains the viscous friction coefficients of the robot joints, and \( \tau \) is the \( n \times 1 \) vector of applied torque inputs.

Assume that only the robot joint displacements \( q(t) \in \mathbb{R}^n \) are available for measurement. Then, the output–feedback tracking control problem is to design a control input \( \tau(t) \) so that the joint displacements \( q(t) \in \mathbb{R}^n \) converge asymptotically to the desired joint displacements \( q_d(t) \in \mathbb{R}^n \), i.e.,

\[ \lim_{t \to \infty} \tilde{q}(t) = 0, \] (2)

where

\[ \tilde{q}(t) = q_d(t) - q(t) \] (3)

denotes the tracking error.

3. Experimental robot system

A direct–drive arm with two vertical rigid links —see Fig. 1— is available at the Mechatronics and Control Laboratory of the Instituto Tecnológico de La Laguna, which was designed and built at the Robotics Laboratory of CICESE Research Center. High–torque brushless direct–drive motors operating in torque mode are used to drive the joints without gear reduction. A motion control board based on a TMS320C31 32–bit floating–point microprocessor from Texas Instruments is used to execute the control algorithm. The control program is written in C programming language and executed in the control board at \( h = 2.5 \) [ms] sampling
period. The maximum torque limits are $\tau_{1}^{\text{Max}} = 150 \text{ [Nm]}$ and $\tau_{2}^{\text{Max}} = 15 \text{ [Nm]}$ for motor 1 and 2, respectively.

The robot dynamics is described with details in (Reyes & Kelly, 1997; 2001). With reference to the symbols listed in Table 1, we present below the entries of the robot dynamics:

<table>
<thead>
<tr>
<th>notation</th>
<th>value</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Link 1 length</td>
<td>$l_1$</td>
<td>0.45 m</td>
</tr>
<tr>
<td>Link 2 length</td>
<td>$l_2$</td>
<td>0.45 m</td>
</tr>
<tr>
<td>Link 1 center of gravity</td>
<td>$l_{c1}$</td>
<td>0.091 m</td>
</tr>
<tr>
<td>Link 2 center of gravity</td>
<td>$l_{c2}$</td>
<td>0.048 m</td>
</tr>
<tr>
<td>Link 1 mass</td>
<td>$m_1$</td>
<td>23.90 kg</td>
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<tr>
<td>Link 2 mass</td>
<td>$m_2$</td>
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<td>Link 1 moment of inertia</td>
<td>$l_1$</td>
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</tr>
<tr>
<td>Link 2 moment of inertia</td>
<td>$l_2$</td>
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</tr>
<tr>
<td>Gravity acceleration</td>
<td>$g$</td>
<td>9.8 m/s$^2$</td>
</tr>
</tbody>
</table>

Table 1. Parameters of the manipulator

The elements $M_{ij}(q)$ ($i, j = 1, 2$) of the inertia matrix $M(q)$ are

\[
M_{11}(q) = m_1 l_{c1}^2 + m_2 \left( l_1^2 + l_{c2}^2 + 2l_1 l_{c2} \cos(q_2) \right) + l_1 + l_2,
\]

\[
M_{12}(q) = m_2 \left( l_{c2}^2 + l_1 l_{c2} \cos(q_2) \right) + l_2,
\]

\[
M_{21}(q) = m_2 \left( l_{c2}^2 + l_1 l_{c2} \cos(q_2) \right) + l_2,
\]

\[
M_{22}(q) = m_2 l_{c2}^2 + l_2.
\]
The elements $C_{ij}(q, \dot{q})$ ($i, j = 1, 2$) of the centripetal and Coriolis matrix $C(q, \dot{q})$ are

\[
\begin{align*}
C_{11}(q, \dot{q}) &= -m_2l_1l_2 \sin(q_2)\dot{q}_2, \\
C_{12}(q, \dot{q}) &= -m_2l_1l_2 \sin(q_2)(\dot{q}_1 + \dot{q}_2), \\
C_{21}(q, \dot{q}) &= m_2l_1l_2 \sin(q_2)\dot{q}_1, \\
C_{22}(q, \dot{q}) &= 0.
\end{align*}
\]

The entries of the gravitational torque vector $g(q)$ are given by

\[
\begin{align*}
g_1(q) &= (m_1l_1 + m_2l_1)g \sin(q_1) + m_2l_2g \sin(q_1 + q_2), \\
g_2(q) &= m_2l_2g \sin(q_1 + q_2).
\end{align*}
\]

The coefficients of the viscous friction are

\[F_v = \text{diag}\{2.288, 0.175\} \text{ [N m s]}.\]

Experiments showed that static and Coulomb friction at the motor joints are present and they depend in a complex manner on the joint position and velocity. We have decided to consider them as disturbances for the closed-loop system.

### 4. Desired position trajectory and performance criterion

The desired position trajectory $q_d(t)$ used in all experiments is given by

\[
q_d(t) = \begin{bmatrix} 45[1 - e^{-2.0t^3}] + 10[1 - e^{-2.0t^3}] \sin(7.50t) \\ 60[1 - e^{-1.8t^3}] + 125[1 - e^{-1.8t^3}] \sin(1.75t) \end{bmatrix} \text{ [degrees/s].}
\]

(4)

An important characteristic of the position trajectory $q_d(t)$ in (4) is that the desired velocity $\dot{q}_d(t)$ and acceleration $\ddot{q}_d(t)$ are null in $t = 0$, then the closed-loop system trajectories will not present rude transients if the robot starts at rest. It is noteworthy that the execution of the proposed trajectory $q_d(t)$ in (4) demanded a 75% of the torque capabilities, which was estimated through numerical simulation and verified with the experiments.

The time evolution of the position error $\tilde{q}$ reflects how well the control system performance is. The performance criterion considered in this chapter was the Root Mean Square —RMS— value of the velocity error computed on a trip of time $T$, that is,

\[
RMS[\dot{\tilde{q}}] = \sqrt{\frac{1}{T} \int_0^T ||\dot{\tilde{q}}(\sigma)||^2 d\sigma} \text{ [degrees/s].}
\]

(5)

In practice, the discrete implementation of the criterion (5) leads to

\[
RMS[\dot{\tilde{q}}] = \sqrt{\frac{1}{T} \sum_{k=0}^{i} ||\dot{\tilde{q}}(kh)||^2 h} \text{ [degrees/s]},
\]

where $h = 2.5$ [ms] is the sampling period and $T = 10$ [s].
5. Tested controllers

We have tested six controllers. Three of the tested controllers do not contain saturation functions, while the others do it. The main goal of the experimental evaluation was to assess the tracking performance of controllers that do not have saturation functions, i.e.,

- PD+ (Paden & Panja, 1988),
- Loría & Ortega (1995), and
- Lee & Khalil (1997),

with respect to those that have saturation functions,

- Loría & Nijmeijer (1998),
- New Design 1, and
- New Design 2.

The controllers denoted as New Design 1 and New Design 2, which are defined explicitly later, were proposed in (Moreno et al., 2008). Tools for analysis of singularly perturbed systems are used to show the local exponential stability of the closed–loop system given by those controllers and the robot dynamics.

Let us first describe the results concerning controllers without saturation functions. The first controller tested was the PD+ control (Paden & Panja, 1988), which is written as

$$
\tau = M(q)\ddot{q}_d + C(q, \dot{q})\dot{q}_d + g(q) + F_v\dot{q} + K_d\dot{q} + K_p\dot{q},
$$

where $K_p$ and $K_d$ are $n \times n$ symmetric positive definite matrices, $\dot{q} = q_d - q$ denotes the tracking error and the joint velocity measurements $\dot{q}$ are approximated via simple numerical differentiation, i.e.,

$$
\dot{q}_i(hk) = \frac{q_i(hk) - q_i(h[k-1])}{h}
$$

where $h$ is the sampling period and $k$ is the discrete time. It is well known that the approach (7) is very common in many robot control platforms to obtain an estimation of the velocity measurements. The controller was tested using the following proportional and derivative control gains

$$
K_p = \text{diag}\{3500, 1000\} [1/s^2],
K_d = \text{diag}\{45, 15\} [1/s],
$$

Let us notice that the gains (8) were obtained by trial and error until obtaining a reasonable performance in the tracking of the desired joint position $q_d(t)$, i.e., a relatively small bound of the maximum values of $\dot{q}_1(t)$ and $\dot{q}_2(t)$. Fig. 2 shows the time evolution of tracking errors $\dot{q}_1(t), \dot{q}_2(t)$, and applied torques $\tau_1(t), \tau_2(t)$. Further improvement could have been obtained in the tracking performance, but paying the price of a noisy control action, which would excite other dynamics such as vibrating modes of the mechanical structure.

Since all of the tested controllers have a proportional–derivative structure, we have used the same numerical value of the gains in (8) for all of them, while the remaining gains in each controller were selected so that a reasonable performance were obtained, as we will explain later.
The other two control schemes that do not contain saturation functions correspond to the output–feedback tracking controllers by Loría & Ortega (1995) and Lee & Khalil (1997). The Loría & Ortega (1995) controller is written as

\[ \tau = M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) + F_v\dot{q} + K_d\ddot{\bar{q}} + K_p\bar{q}, \]  \hfill (9)

where \( \bar{q} \in \mathbb{R}^n \) is obtained with the linear filter

\[ \begin{align*}
\dot{x} &= -b_f \bar{q}, \\
\bar{q} &= x + b_f \dot{q},
\end{align*} \]

with \( b_f > 0 \). The controller (9) was implemented in our system with the control gains \( K_p \) and \( K_d \) in (8) and

\[ b_f = 600.0 \ [1/s]. \]  \hfill (10)

The Lee & Khalil (1997) controller in its non–adaptive version can be written as

\[ \tau = M(q_d - x_1)\ddot{q}_d + C(q_d - x_1, \dot{q}_r)\dot{q}_d + g(q_d - x_1) \\
+ F_v[\dot{q}_d - x_2] + K_d x_2 + K_p x_1, \]  \hfill (11)

where \( \dot{q}_r = \dot{q}_d - x_2 + \lambda \ddot{q} \) and the second order high–gain observer is defined as

\[ \begin{align*}
\dot{x}_1 &= x_2 + \frac{L_1}{\epsilon} [\ddot{q} - x_1], \\
\dot{x}_2 &= \frac{L_2}{\epsilon^2} [\ddot{q} - x_1] + \ddot{q}_d \\
&\quad + M(q_d - x_1)^{-1} [C(q_d - x_1, \dot{q}_r)\dot{q}_d \\
&\quad + g(q - x_1) + F_v[\dot{q}_d - x_2] - \tau].
\end{align*} \]  \hfill (13)
The gains used for the implementation of this controller were \( K_p \) and \( K_v \) in (8), \( \lambda = 1 \) [1/s],

\[
L_1 = \text{diag} \{5.0, 40.0\} \ [1/s], \quad L_2 = \text{diag} \{50.0, 400.0\} \ [1/s^2],
\]
and \( \varepsilon = 0.1 \). We tried several sets of gains for the observer (12)–(13) until obtaining a good response in the tracking error. However, it was pretty difficult to find numerical values for which instability was avoided. As pointed out in (Daly & Schwartz, 2006), the real–time implementation of the controller/observer (11)–(13) may make estimations of the position and velocity errors inaccurate to the point of not being useful, which was confirmed during the experimental set up.

The obtained experimental results are given in Fig. 3, for the Loría and Ortega controller, and in Fig. 4, for the Lee and Khalil scheme.

![Fig. 3. Loría and Ortega controller: Tracking errors \( \tilde{q}_1(t), \tilde{q}_2(t) \), and applied torques \( \tau_1(t), \tau_2(t) \).](image)

On other hand, concerning output–feedback tracking controllers that contain saturation functions in their structure, the first controller tested was the Loría & Nijmeijer (1998) approach, written as

\[
\tau = M(q)\ddot{q}_d + C(q, \dot{q}_d)\dot{q}_d + g(q) + F_vq + K_d \text{col}\{\tanh(\tilde{\vartheta}_i)\} + K_p \text{col}\{\tanh(\tilde{q}_i)\},
\]

where \( \text{col}\{f(x_i)\} = [f(x_1) \cdots f(x_n)]^T \in \mathbb{R}^n \) for any scalar function \( f \), used along with the saturated filter

\[
\dot{x} = -b_f \text{col}\{\tanh(\tilde{\vartheta}_i)\},
\]

\[
\dot{\vartheta} = x + b_f \dot{q}.
\]
Once again, in order to keep a fair comparison with respect to the three previous controllers that do not use saturation functions, we used the numerical values of $K_p$ and $K_d$ in (8) and $b_f$ in (10). The result of the experiment is depicted in Fig. 5.

Besides, we implemented the controller denoted as New Design 1 (Moreno et al., 2008)

$$
\tau = M(q)\ddot{q}_d + C(q, \dot{q}_d)\dot{q}_d + g(q) + F_v\dot{q}_d
+ K_d \text{ col } \left\{ \frac{\dot{\vartheta}_i}{\sqrt{\delta_{di}^2 + \dot{\vartheta}_i^2}} \right\} + K_p \text{ col } \left\{ \frac{\dot{q}_i}{\sqrt{\delta_{pi}^2 + \dot{q}_i^2}} \right\},
$$

(15)

where $\delta_{pi}$ and $\delta_{di}$ are strictly positive constants, and the filter

$$
\dot{x} = -b_f \text{ col } \left\{ \frac{\dot{\vartheta}_i}{\sqrt{\delta_{di}^2 + \dot{\vartheta}_i^2}} \right\},
$$

$$
\dot{\vartheta} = x + b_f \dot{q},
$$

with $b_f > 0$ is used. As previously, we chose the numerical values of $K_p$ and $K_d$ as in (8), and $b_f$ as in (10). Parameters $\delta_{pi}$ and $\delta_{di}$ were

$$
\delta_{p1} = 0.3, \delta_{p2} = 0.75, \delta_{d1} = 1.0, \delta_{d2} = 1.0.
$$

Fig. 6 shows the results of the experiment.

Finally, the controller New Design 2 (Moreno et al., 2008)

$$
\tau = M(q)\ddot{q}_d + C(q, \dot{q}_d)\dot{q}_d + g(q) + F_v\dot{q}_d
$$
Fig. 5. Loría and Nijmeijer controller: Tracking errors $\tilde{q}_1(t)$, $\tilde{q}_2(t)$, and applied torques $\tau_1(t)$, $\tau_2(t)$.

$$\dot{\tilde{q}}_i = -b_f \col \left\{ \frac{\delta_i}{\delta_{di} + \ln(\cosh(\tilde{\vartheta}_i))} \right\},$$

which is used with the filter

$$\dot{x} = -b_f \col \left\{ \frac{\delta_i}{\delta_{pi} + \ln(\cosh(\tilde{\vartheta}_i))} \right\},$$

$$\dot{\tilde{\vartheta}} = x + b_f \tilde{q},$$

was tested under the same conditions that the New Design 1 in (15), while the parameters $\delta_{pi}$ and $\delta_{di}$ used in this case were

$$\delta_{p1} = 0.55, \quad \delta_{p2} = 1.0, \quad \delta_{d1} = 1.0, \quad \delta_{d2} = 1.0.$$  

The results are illustrated in Fig. 7.

6. Discussions

All the tested controllers assure theoretically that the position error $\tilde{q}(t)$ must vanish as time increases. In practice, Figures 3 to 8 reveal a steady-state oscillatory behavior. This is due to several factors such as the uncompensated Coulomb friction and the discrete implementation of the controller.
Fig. 6. New Design 1: Tracking errors $\tilde{q}_1(t)$, $\tilde{q}_2(t)$, and applied torques $\tau_1(t)$, $\tau_2(t)$.

<table>
<thead>
<tr>
<th></th>
<th>Controllers without saturation functions</th>
<th>Controllers with saturation functions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PD+</td>
<td>LO</td>
</tr>
<tr>
<td>$\max{</td>
<td>\tilde{q}_1(t)</td>
<td>}$ [deg]</td>
</tr>
<tr>
<td>$\max{</td>
<td>\tilde{q}_2(t)</td>
<td>}$ [deg]</td>
</tr>
<tr>
<td>$\text{RMS}[\tilde{q}]$ [deg]</td>
<td>0.418</td>
<td>0.417</td>
</tr>
</tbody>
</table>

Table 2. Performance of the controllers: PD+, Lória and Ortega (LO), Lee and Khalil (LK), Lória and Nijmeijer (LN), New Design 1, and New Design 2.

Table 2 summarizes the information about the tracking performance of the six schemes, remarking the difference between controllers that do not use saturation functions and controllers that do use them. In addition, Fig. 8 shows a bar chart of the $\text{RMS}[\tilde{q}]$ value computed for the six tested controllers. With respect to controllers without saturation function we can see that the performance of the PD+ controller (6) and the Lória and Ortega algorithm (9) is very similar, while the worst performance of the six controllers was obtained with the Lee and Khalil controller (11). On the other hand, concerning the experimental results using controllers with saturation function, we can see that the performance of the Lória and Nijmeijer scheme (14) is very similar to the one of the controllers PD+ (6) and Lória and Ortega (9). The reason is that in the situation of a very small tracking error $\tilde{q}$ the structure of the three controllers is very similar. In addition, the best performance of the six controllers was obtained with the New Design 1 and New Design 2, in equations (15) and (16), respectively, because they presented the lowest values of $\max\{|\tilde{q}_1(t)|\}$, $\max\{|\tilde{q}_2(t)|\}$ and $\text{RMS}[\tilde{q}]$. 

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The comparison reveals that all the controllers work efficiently since the tracking errors are relatively close to the performance of the PD+ controller (6), although the new controllers (15) and (16), which incorporate saturation functions, present a lower tracking error \( \tilde{q}(t) \) than other output–feedback tracking controllers, including the PD+ control (6). The explanation of this is that the new controllers (15) and (16) incorporate the extra parameters \( \delta_{pi}, \delta_{di} \), whose numerical value has effect in increasing the slope of the profile of the saturation function in the proximity of the origin. Different numerical values of \( \delta_{pi} \) and \( \delta_{di} \) lead to a similar behavior with respect to the PD+ (6), Loría and Ortega (9), and Loría and Nijmeijer (14) schemes.

7. Concluding remarks

In this work the output–feedback tracking control of robot manipulators was studied. An extensive experimental study in a two degrees–of–freedom direct–drive robot was presented, where it was shown that output–feedback tracking controllers having saturation functions in the corresponding proportional and derivative parts present better tracking performances than the controllers that have simple linear functions in the corresponding proportional and derivative parts. To the best of the authors’ knowledge, the output–feedback tracking controllers discussed in the experimental results have been tested in a real–time robot control system for the first time. The results obtained in practice suggest that output–feedback tracking controllers that incorporate saturation functions are reliable for application in industrial robots.
8. Acknowledgments

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9. References


Robot manipulators are developing more in the direction of industrial robots than of human workers. Recently, the applications of robot manipulators are spreading their focus, for example Da Vinci as a medical robot, ASIMO as a humanoid robot and so on. There are many research topics within the field of robot manipulators, e.g. motion planning, cooperation with a human, and fusion with external sensors like vision, haptic and force, etc. Moreover, these include both technical problems in the industry and theoretical problems in the academic fields. This book is a collection of papers presenting the latest research issues from around the world.

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