Simulated Regulator to Synthesize ZMP Manipulation and Foot Location for Autonomous Control of Biped Robots

Tomomichi Sugihara
Kyushu University
Japan

1. Introduction

Biped robots have potentially high mobilities. Since they have similar morphologies with the humans’ lower limbs, they are expected to grow up into the robots which can travel wherever humans can. To make biped robots catch up with such an expectation still needs to resolve many challenges. The two bases of pedipulation, namely, the legged motion control are the indirect reaction force manipulation (Fujimoto et al., 1998; Furusho & Masubuchi, 1986; Mitobe et al., 1995; Vukobratović et al., 1970) and the discontinuous grounding foot location (Kajita & Tani, 1995; Miura & Shimoyama, 1984; Raibert et al., 1984; Witt, 1970). The former is necessary to transport the center of mass (COM) of the floating multibody system, which is not mechanically connected to the inertial frame. The latter is required to reform the supporting region of the system, which determines the limitation of the physically available reaction forces. Each has a strongly nonlinear property, so that the synthesis of them is still an open problem.

A major solution against it is to refer the motion trajectory which is defined as a function of time. It can coordinate the fullbody motion involving foot location and COM transportation rather easily with physical feasibility represented by ZMP (Vukobratović & Stepanenko, 1972) and geometric constraints such as collision avoidance taken into account. Many successful biped walkers based on this approach have been appeared (Hirai et al., 1998; Huang et al., 2001; Kajita et al., 2003; Löffler et al., 2003; Nagasaka et al., 1999; 2004; Sugihara & Nakamura, 2005; Takanishi et al., 1988). Such time-slaved controls, however, are not robust against various extrinsic events. While they work in situations where sufficient knowledge about the environment and the task is given in advance, they are less promising in fields of action with many uncertainties. It is desired that the control system is designed as an autonomous system, namely, a system which does not explicitly depend on time.

Some COM control methods by manipulating the reaction force or ZMP in realtime were proposed (Fujimoto et al., 1998; Mitobe et al., 1995; Sugihara et al., 2002). They focused on the COM control under a given supporting condition, and the foot location strategies were out of the scope. Some stepwise-stable autonomous biped controllers in accordance with the point-foot contact were also proposed (Gubina et al., 1974; Kajita et al., 1992; Westervelt et al., 2004). The application to realistic robots which support themselves on their soles has been a future work. Passive dynamic walking (Collins et al., 2001; McGeer, 1990; Yamakita et al., 2000) is
another approach to design an autonomous biped controller by utilizing an inherent stability of discretized biped dynamics. It stands on the ideally perfect plastic collision between the robot and the ground, and thus, has a low stabilizing ability.

This paper proposes a control to synthesize the above ZMP manipulation control and the foot location in a consistent manner. We design a regulator based on the approximate dynamical model of a biped robot, focusing on a simple relationship between COM and ZMP. In this stage, the feasible area where ZMP can exist is unbounded against the physical constraint. In this sense, we call it the simulated regulator. When the desired ZMP is located out of the supporting region, it is modified to be within the actual region. The robot is controlled in such a way that the real ZMP tracks the desired ZMP. Simultaneously, the supporting region is deformed by a foot replacements to include the original desired ZMP in the future. The regulator gains are decided by the pole assignment method in order to give COM a slow mode and ZMP a fast mode explicitly, which matches the role of each foot. Since both the ZMP manipulation and the foot location originate from the identical simulated regulator, a totally consistent control system is made up. In addition, it is shown that a cyclic walk is automatically generated without giving a walking period explicitly by coupling the support-state transition and the goal-state transition. It does not assume a periodicity of the motion trajectory, and hence, seamless starting and stopping can be achieved.

2. Simulated COM–ZMP regulator

2.1 Linearized biped system and simulated regulator

The strict equation of motion of a biped robot takes a complicated form with tens of degrees-of-freedom. Here, we assume that an effect of the moment about COM is smaller enough to be neglected than that about ZMP due to the movement of COM. Then, the macroscopic behavior of the legged system is represented by the motion of COM. The equation of motion in horizontal direction of a biped model with such a mass-concentrated approximation as
Fig. 1(B) is expressed as follows:

\[
\begin{align*}
\dot{x} &= \omega^2 (x - x_Z), \\
\dot{y} &= \omega^2 (y - y_Z),
\end{align*}
\]

where \( p_G = [x\ y\ z]^T \) is the position of COM, and \( p_Z = [x_Z\ y_Z\ z_Z]^T \) is ZMP. \( \omega \) is defined as:

\[
\omega^2 \equiv \frac{\dot{z} + g}{z - z_Z} \quad (\geq 0),
\]

where \( g \) is the acceleration of gravity, and \( z_Z \) is the ground level, which is known. \( z, x \) and \( y \) axes are aligned along the gravity, the forward and the leftward directions, respectively. Eq.(1) and (2) imply that COM can be controlled via manipulation of ZMP.

The coupled movement of ZMP and COM is not simple. Let us consider a case where the robot lifts up one foot from the both-standing state, for example. Note that, in such situations, a conventional distinction between swing foot and stance foot does not make sense any longer, since neither feet are swinging. However, they are obviously different from each other in terms of function. In this paper, the foot to be the swing foot is called *kicking foot*, and that to be the stance foot is called *pivoting foot*, instead.

The sequence is illustrated by Fig. 2. ZMP is required to be within the pivoting sole at the end of the phase in order to detach the kicking foot off the ground, while it moves into the sole of kicking foot in the initial phase in order to accelerate COM towards the pivoting foot. Namely, ZMP initially moves oppositely against the direction of the desired COM movement, and overtakes COM during the motion. The fact that the biped robot is a non-minimum-phase-transition system as well as the inverted pendulum underlies the requirement of such a complex manipulation of ZMP. In addition, ZMP travels faster than COM between the feet in the double support phase, as ZMP depends on the acceleration of the robot. Both modes of COM and ZMP movement are desired to be explicitly designed in accordance with the locations of feet. Then, we include ZMP in the state variable and regard the ZMP rate as the input. The linearized state equation is represented as follows:

\[
\dot{x} = Ax + bu,
\]
where the motion along $x$-axis is only considered from the isomorphism of Eq.(1) and (2), and:

\[
x \equiv \begin{bmatrix} x \\ \dot{x} \\ x_Z \end{bmatrix}, \quad A \equiv \begin{bmatrix} 0 & 1 & 0 \\ \omega^2 & 0 & -\omega^2 \\ 0 & 0 & 0 \end{bmatrix}, \quad b \equiv \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad u \equiv \dot{x}_Z,
\]

respectively. In the above equation assumed that the vertical movement of COM is slower enough to regard as $\omega \simeq \text{const.}$ than the horizontal movement. The ZMP rate is decided based on the state feedback around the referential state $^{\text{ref}}x$.

\[
u = k^T(\text{ref} x - x).
\]

The gain $k$ is designed by the pole assignment method so as to embed a faster mode explicitly into ZMP movement than the mode of COM. The motion along $y$-axis is dealt with as well.

In this stage, we don’t constrain ZMP in the supporting region, so that the system is not necessarily physically consistent. In this sense, let us call it the simulated ZMP and represent it by $\hat{p}_Z = [\hat{x}_Z \ \hat{y}_Z \ \hat{z}_Z]^T$. As long as $\hat{p}_Z$ is within the supporting region, the actual desired ZMP $^d p_Z$ is set for the same position with $\hat{p}_Z$.

![Simulated ZMP and Actual desired ZMP](image)

**Fig. 3.** The concept of the simulated regulator. When the simulated ZMP $\hat{p}_Z$ lies out of the supporting region, the desired ZMP $^d p_Z$ is set for the proximity to the supporting region. At the same time, the swing foot is relocated to deform the supporting region so as to include $\hat{p}_Z$ in the future.

Fig. 3 illustrates the idea of the proposed control. The situation where $\hat{p}_Z$ lies out of the supporting region means that COM cannot be provided with the desired acceleration under the current supporting condition. In order to compromise this inconsistency between the desired control and the acceptable control, the following two maneuvers are required. One is to take a physically-feasible acceleration which is the nearest to the desired value by setting the desired ZMP $^d p_Z$ for the proximity of $\hat{p}_Z$ to the supporting region as Fig. 4 depicts. The motion continuity at the moment of landing is held by resetting the simulated ZMP $\hat{p}_Z$ for the originally desired ZMP $^d p_Z$. This idea has been already proposed by the authors (Sugihara et al., 2002). The other is to deform and expand the supporting region so as to include $\hat{p}_Z$ in the future, which is described in the following section.
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Fig. 4. Substitution of \( \hat{p}_Z \) for \( d_p_Z \) to match the actual supporting region.

2.2 Foot location control based on simulated ZMP

The deformation of the supporting region is achieved via the relocation of stance feet. Suppose ZMP is within the pivoting sole. Let us define that \( p_S = [x_S \ y_S \ z_S]^T \) and \( p_K = [x_K \ y_K \ z_K]^T \) are the tip positions of the pivoting foot and the kicking foot, respectively. They correspond to the positions of the stance foot and the swing foot during the single support phase, respectively. We decide the desired position of the foot \( d_p_K = [d_{x_K} \ d_{y_K} \ d_{z_K}]^T \) by the following procedure.

The COM acceleration which the simulated regulator requires (called the simulated COM acceleration, hereafter), and the desired COM acceleration which conforms to the actual supporting condition (called the desired COM acceleration in short, hereafter) are defined by the relative COM locations with respect to the simulated ZMP \( \hat{p}_Z \) and the originally desired ZMP \( d_p_Z \), respectively. The necessity of a relocation of grounding feet arises in case where the desired COM acceleration is inconsistent with the simulated COM acceleration. It is judged with respect to \( x \)- and \( y \)-axes independently. \( d_{x_K} \) is defined as follows:

\[
d_{x_K} = \begin{cases} 
\lambda_x \hat{x}_Z + (1 - \lambda_x)x_S & \text{(for } \iota_x < 0) \\
x_K & \text{(for } \iota_x \geq 0) 
\end{cases} 
\]

(6)

where \( \lambda_x \) is a constant to define the step magnitude (\( \lambda_x > 1 \)). The above rule means that the robot puts its swing foot on the place where the desired COM acceleration orients to the same direction with the simulated COM acceleration, if they direct counterwards to each other.

For the motion in \( y \)-axis, \( d_{y_K}' \) is firstly computed from the designed \( \lambda_y (> 1) \) as well. Then, it is converted to \( d_{y_K} \) by the following rule in order to avoid the self-collision between both feet:

\[
d_{y_K} = \bar{y} + \frac{1}{2} \left\{ d_{y_K}' - \bar{y} \pm \sqrt{(d_{y_K}' - \bar{y})^2 + a} \right\},
\]

(8)

where + is chosen for the left leg for the double sign, while – for the right leg, and \( \bar{y} \) is the inner boundary of the swing foot. The above function has a profile as shown in Fig. 6.
Fig. 5. Step ratio $\lambda_x$ to cover simulated ZMP in the future.

Fig. 6. Foot location transformation in $y$-axis for self-collision avoidance.

Fig. 7. Spatial foot trajectory (left) in $xz$-plane (right) in $xy$-plane.

A smaller constant $a$ makes the curve approach to the asymptotic lines with the break point $(d'y_K, d'y_K) = (\bar{y}, \bar{y})$.

Suppose the initial position of the swing foot is $p_{K0} = [x_{K0} \ y_{K0} \ z_{K0}]^T$, and the lift height of the swing foot $dZ_K$ is defined as:

$$dZ_K = 2h\sqrt{(1 - \theta)}$$

$$\theta \equiv \min \left\{ \frac{(dx_K - x_{K0})^2 + (dy_K - y_{K0})^2}{|x_S - x_{K0} + s|}, 1 \right\}.$$  

It generates a spatial trajectory which carries the swing foot along a half ellipsoid with a height $h$ as the leftside of Fig. 7, and makes it land on a circle with the center $(x_{K0}, y_{K0})$ and the radius $x_S - x_{K0} + s$, the bird’s-eye view of which is depicted in the right side of Fig. 7; it lands to the point with a stride $x_S - x_{K0} + s$ from the initial position as long as $d'y_K = y_{K0}$ is ensured.

The above procedure does not guarantee the time continuity of $dp_K$, so that it might jump largely at the moment when ZMP travels to the pivoting sole, or when the relative COM location with respect to the simulated ZMP comes in the opposite side of that with respect to
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Fig. 8. Block diagram of the proposed biped control system with the simulated regulator.

the desired ZMP, for instance. Then, the time sequence of $d\hat{p}_K$ is smoothened by second-order low-pass filters, for example.

Fig. 8 is a block diagram of the proposed control system described above. 'IP Observer' in the figure shows a subsystem which outputs the desired COM position $dp_G$ equivalent to the desired ZMP $dp_Z$ (Sugihara et al., 2002). One can note that both the COM controller with ZMP manipulation and the foot relocation controller branch from the identical simulated regulator and join in the inverse kinematics solver (the motion rate resolver).

3. Autonomous walk by coupled goal-state/support-state transition

Suppose the referential COM position is $refp_G = \begin{bmatrix} refx \\ refy \\ refz \end{bmatrix}^T$, the referential state of the simulated regulator in x-axis is $refx = \begin{bmatrix} refx \\ 0 \\ refz \end{bmatrix}^T$. The control in the previous section yields a step motion automatically by locating $refp_G$ out of the supporting region on purpose. This property is utilized to achieve an autonomous continual walk by coupling the referential goal state transition and the supporting state transition, namely, by repeating to set $refp$ out of the supporting region after the supporting region is deformed so as to include $refp_G$ by the stepping. More concretely, $refx$ is defined by the following equation for a given $s$ and the position of pivoting foot $x_S$ in x-axis:

$$refx = x_S + rs,$$

where $r$ is a positive coefficient ($0 < r < 1$). In cases where the robot changes the orientation, x- and y-axes are again realigned with respect to the moving direction, and the desired COM position is computed with the above Eq.(11).

4. Simulation

We verified the proposed control via a simulation with an inverted pendulum model whose mass was concentrated at the tip. The length of the pendulum was 0.27[m], which fits to the robot "mighty" (Sugihara et al., 2007) shown in Fig. 9. Note that the robot mass does not affect the behavior of the inverted pendulum. The both sole were modelled as rectangles with the length 0.055[m] to the toe edge, 0.04[m] to the heel edge, and 0.035[m] to each side. The state feedback gains were designed by the pole assignment method. The poles were -3, -6 and -10 with respect to x-axis, and -2.5, -25 and -30 with respect to y-axis. The other control
parameters were set for $\lambda_x = 2$, $\lambda_y = 3$, $a = 0.001$, $r = 0.9$ and $h = 0.01$[m], respectively. The desired swing foot position was smoothened by a second-order low-pass filter $\frac{1}{(0.02s+1)^2}$. The initial state was set for $(x, y) = (0, 0)$ and $(\dot{x}, \dot{y}) = (0, 0)$. The initial stance position of the left and the right feet were $(0, 0.045)$ and $(0, -0.045)$, respectively. From the first to the sixth step, the stride $s$ was set for 0.3[m], and the referential COM position was automatically updated by the method described in section 3. Immediately after landing the sixth step, the referential COM position was settled at the midpoint of both feet.

The loci of the referential COM position $(\text{ref} x, \text{ref} y)$, the actual COM (the tip point of the inverted pendulum)$(x, y)$, the simulated ZMP position $(\ddot{x}_Z, \ddot{y}_Z)$, the actually desired ZMP position $(d\dot{x}_Z, d\dot{y}_Z)$, the referential feet positions $(d x_L, d y_L)$, $(d x_R, d y_R)$ and the filtered positions of them $(x_L, y_L)$, $(x_R, y_R)$ are plotted in Fig. 10. It is seen that an almost cyclic continual walk was achieved without giving a walk period explicitly by an alternation of the supporting-region deformation via the pedipulation and the goal-state transition. In this example motion, the simulated ZMP and the actually desired ZMP in $y$-axis always coincided with each other, so that a sideward stepping was not resulted. The difference of COM and ZMP modes particularly appear in the movement along $y$-axis. The given pole to design feedback gains set the time-constant of the sideward kicking for about 0.1[s], which contributed to ensure about 60% of duty ratio of the swinging phase. Fig. 11 zooms a part of Fig. 10 from $t = 0 \sim 1.5$. $d\dot{x}_Z$ differs from $\ddot{x}_Z$ in $t \simeq 0.4 \sim 0.5$, $t \simeq 0.9 \sim 1.0$ and $t \simeq 1.4 \sim 1.5$. $d\dot{x}_Z$ in those terms are thought to be saturated at the toe edge of the supporting sole. $\ddot{x}_Z$ is synchronized at $t \simeq 0.5, 1.0$ when the swing foot lands on the ground, and the continuity of ZMP is held. $d\dot{x}_L$ and $d\dot{x}_R$ discontinuously jump at $t \simeq 0.15, 0.75, 1.25$ which are thought to be times when the ZMP reaches the pivoting sole. In spite of that, $x_L$ and $x_R$ keep continuous, thanks to the low-pass filters. The robot responded to the sudden stop of the reference at $t \simeq 3.0$ without bankruptcy. Fig. 12 shows some sequential snapshots of a motion of the inverted pendulum. The red ball and the green ball in the figure indicate the referential COM position and the simulated ZMP position, respectively. The magenta area is the supporting region composed from the grounding sole.
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5. Conclusion

We developed an autonomous biped controller, in which the ZMP manipulation under the current support condition and the pedipulation to deform the future support region were synthesized. Both are based on an identical simulated regulator, so that they are integrated into the total control system without any conflicts. Since the simulated regulator involves ZMP in the state variable, it is possible to give a slow mode to COM and a fast mode to ZMP, which is accommodated to the current choice of stance and kicking feet, explicitly by the pole assignment method.

The autonomous controller is promising to improve the system robustness against extrinsic events and uncertainties in the environment. The next short-term issues are to verify the absorption performance of perturbations and to examine the adaptability against rough terrains.
Fig. 12. Snapshots of an inverted pendulum motion controlled by the proposed method.

Fig. 13. Snapshots of a walking motion replayed by mighty.

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6. References


Nowadays robotics is one of the most dynamic fields of scientific researches. The shift of robotics researches from manufacturing to services applications is clear. During the last decades interest in studying climbing and walking robots has been increased. This increasing interest has been in many areas that most important ones of them are: mechanics, electronics, medical engineering, cybernetics, controls, and computers. Today's climbing and walking robots are a combination of manipulative, perceptive, communicative, and cognitive abilities and they are capable of performing many tasks in industrial and non-industrial environments. Surveillance, planetary exploration, emergence rescue operations, reconnaissance, petrochemical applications, construction, entertainment, personal services, intervention in severe environments, transportation, medical and etc are some applications from a very diverse application fields of climbing and walking robots. By great progress in this area of robotics it is anticipated that next generation climbing and walking robots will enhance lives and will change the way the human works, thinks and makes decisions. This book presents the state of the art achievements, recent developments, applications and future challenges of climbing and walking robots. These are presented in 24 chapters by authors throughout the world. The book serves as a reference especially for the researchers who are interested in mobile robots. It also is useful for industrial engineers and graduate students in advanced study.

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