Locomotion analysis of hexapod robot

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1. Introduction

Multi-legged robots display significant advantages with respect to wheeled ones for walking over rough terrain because they do not need continuous contact with the ground. In Multi-legged robots, hexapod robots, mechanical vehicles that walk on six legs, have attracted considerable attention in recent decades. There are several benefits for hexapods rover.

(a) Hexapod robot is easy to maintain static stability on three or more legs,
(b) It has a great deal of flexibility in how it can move.
(c) Hexapod robot is the most efficient one for statically stable walking. Preumount et al. 1991, observed that a larger number of legs more than six do not increase walking speed.
(d) Hexapod robots show robustness in case of leg faults
(e) Hexapods makes it possible for the robot to use one, two or three legs to work as hand and perform complex operations.

The most studied problem for multi-legged robots concerns how to determine the best sequence for lifting off and placing the feet (gait/locomotion planning). From the stability point of view, robot locomotion can be classified into dynamic locomotion, such as running and hopping, and statically stable locomotion as walking. Statically stable locomotion has the constraint that the moving body is stable at all times. The vertical projection of the centre of gravity of the robot must be within the convex of the supporting polygon linked positions of all supporting feet.

Statically stable gait is solely dependent on the design of bodies and legs. Hexapod gaits have been widely investigated as a function of shape and characteristics of the robot structure. In 1985, Kaneko et al. addressed the gait of a rectangular hexapod with decoupled freedoms where the propelling motion was generated by one degree of freedom (DOF). In 1988, Lee et al. realized an omnidirectional walking control system for a rectangular hexapod robot with adaptive suspension. A circular gait was studied for a layered hexapod robot (called Ambler) at the Carnegie Mellon University [Bares et al., 1989; Krotkov & Bares, 1991; Wettergreen, 1990] with rotating legs connected to the same vertical axis at six different heights. Hirose et al. in 1992 and 1998 and Gurocak in 1998 developed other two hexapods whose bodies were consisting of two different layers, each connected to three legs. The relative motion of the layers realized the omnidirectional robot gait in a simple way, but limiting the walking capability under leg faults. Two Lees in 2001 studied the gait of a special robot whose body was composed of three parts connected by revolute joints. Its
flexible gait allowed it to overcome complex terrains, but its configuration was quite complicate for control system design. R Hex, introduced by Uluc et al. in 2001, is another hexapod robot with half-circle legs with a simple alternate tripod gait. Most popular hexapods can be grouped into two categories, rectangular and hexagonal ones. Rectangular hexapods have a rectangular body with two groups of three legs distributed symmetrically on the two sides. Hexagonal hexapods have a round or hexagonal body with evenly distributed legs.

The gait of rectangular six-legged robots has motivated a number of theoretical researches and experiments which nowadays reached to some extent a state of maturity. In 1998 Lee et al. showed for rectangular hexapods the longitudinal stability margin, which is defined as the shortest distance from the vertical projection of center of gravity to the boundaries of the support pattern in the horizontal plane, of straight-line motion and crab walking. Song & Choi in 1990 defined the duty factor as the fraction of cycle time in which a leg is in the supporting phase and they proved that the wave gait is optimally stable among all periodic and regular gaits for rectangular hexapods when \( \frac{3}{4} \leq \beta \leq 1 \). Both the tripod gait and the problem of turning around a fixed point on an even terrain have been widely investigated and tested for a general rectangular hexapod with three DOF legs [Wang, 2005 and Su, 2004]. The so called 4+2 quadruped gaits [Huang and Nonami, 2003] have been demonstrated being able to tolerate faults [Yang & Kim, 1999]. A series of fault-tolerant gaits for hexapods were analyzed by Yang et al. [Yang & Kim, 1998a, 1998b, 2000 and 2003]. Their aim was to maintain the stability in case a fault event prevented a leg from supporting the robot. In 1975, Kugushev and Jaroshevskij proposed a terrain adaptive free gait that was non-periodic. McGhee et al. and other researchers [Porta & Celaya, 2004; Erden & Leblebicioglu] went on studying free gaits of rectangular hexapod robots.

At the same time, the hexagonal hexapod robots were studied with inspiration from the insect family, demonstrate better performances for some aspects than rectangular robots. Kamikawa et al. in 2004 confirmed the ability to walk up and down a slope with the tripod gait by building a virtual smooth surface that approximates the exact ground. Yoneda et al. in 1997 enhanced the results of Song & Choi in 1990, developing a time-varying wave gait for hexagonal robots, in which velocity, duty factor and crab angle are changed according to terrain conditions. A. Preumon et al. in 1991 proved that hexagonal hexapods can easily steer in all directions and that they have longer stability margin, but he did not give a detailed theoretical analysis. Takahashi et al. in 2000 found that hexagonal robots rotate and move in all directions at the same time better than rectangular ones by comparing stability margin and stroke in wave gait, but no experimental results were presented. Chu and Pang in 2002 compared the fault tolerant gait and the 4+2 gait for two types of hexapods of the same size. They proved theoretically that hexagonal hexapod robots have superior stability margin, stride and turning ability compared to rectangular robots.

It is also worth to mention here a work carried out by Gonzale de Santos et al. [Gonzale de Santos et al., 2007a and Gonzale de Santos et al., 2007b]. They optimized the structure of rectangular hexapods and found that extending the length of middle legs of rectangular robots helps in saving energy. This outcome can be seen as a transition from rectangular six-legged robots to hexagonal ones.
2. Definitions

Several definitions are necessary to be introduced before locomotion planning.

1) Support/stance phase: a leg is said in its supporting/stance phase when it stands on the ground and its foot does not leave the ground.
2) Transfer/swing phase: a leg is said in its transferring/swing phase when it does not stand on the ground but move in the air.
3) Gait period/cycle time, $T$: a gait period/cycle time is a complete cycle of a leg including supporting phase and transferring phase.
4) Duty factor $\beta$: the duty factor $\beta$ is the time fraction of stance phase of a leg to the cycle time $T$. $\beta = T_s/T$, where $T_s$ denotes time of supporting phase of leg $i$; $T$ denotes circle time of leg $i$.
5) Stroke length: the distance that the body moves thought the support phase of a leg.
6) Stride length: stride length is the distance the centre of gravity (COG) translates during one complete locomotion cycle.
7) Pitch length: the distance between the centers of the strokes of the isosceles legs.
8) Supporting polygon/pattern: the polygon the vertices of which are constructed on the horizontal plane by vertical projections of the foot-ground interaction points.
9) Statically stability margin (SSM): stability margin was defined for a given support polygon as the smallest of the distances from the COG. projection to the edges of the support polygon.
10) Longitudinal stability margin (LSM): the smallest of the distances from the COG. projection to the front and rear edges of the support polygon along the machine's longitudinal axis.
11) Crab Longitudinal Stability Margin (CLSM): The smallest of the distances from the COG. projection to the front and rear edges of the support polygon along the machine's motion axis.
12) Main walking direction stability margin (MDSM): the smallest of distance from projection of the C.G. to the front and rear edges of the support polygon along the main-walking direction.
13) Kinematics margin: kinematics margin is defined as the distance from the current foothold of leg $i$ to the boundary of the reachable area of leg $i$, measured in the opposite direction of body motion.
14) Periodic gait: a gait is periodic if similar states of the same leg during successive strokes occur at the same interval for all legs, that interval being the cycle time.
15) Symmetric gait: a gait is symmetric if the motion of legs of any right-left pair is exactly half a cycle out of phase.
16) Regular gait: A gait is said to be regular if all the legs have the same duty factor.
17) Body height: body height is the distance of the body center of mass from the support surface along the body vertical axis.
18) Protraction of leg: protraction is the forward movement of a leg relative to the body and ground.
19) Retraction of leg: retraction is the backward movement of a leg relative to the body with no movement of the leg relative to the ground.
20) Lateral offset: Lateral offset is the shortest distance between vertical projection of hip on the ground and the corresponding track.
21) Crab angle: it is defined as the angle from the longitudinal axis to the direction of motion, which has the positive measure in the anti-clockwise direction.

3. Mechanism of Hexapods

Typical hexapod robots can be classified into rectangular and hexagonal ones (Fig.1). Rectangular hexapods inspired from insects have six legs distributed symmetrically along two sides, each side having three legs. Hexagonal hexapods have six legs distributed axisymmetrically around the body (that can be hexagonal or circular).

![Two types of hexapod robots](image1)

![Beetle's Structure](image2)

Typically, individual legs range from two to six degrees of freedom. Fichters [Fichter, E.F. & Fichter, B.L., 1988] have made a survey on insects’ legs (Fig.2 as an example). They found that a general insect leg has four main segments: coxa, femur, tibia and tarsus. Most of the length of an insect leg is contributed by 2 long and nearly equally segments. The hinge joint...
between these segments allows one to fold back almost exactly along the other. Length of the tibia is highly correlated with that of the femur, the correlation coefficient ranges from 0.97 to 0.78. The coxa has no obvious correlation with femur or tibia. Similarly, the thigh and calf of mammals (dog for example in Fig.3) and human are almost equal. We will mainly talk about the locomotion of hexapod robots with leg structure in Fig.4. Three parts are connected together by two parallel revolute joints with rotating axes parallel to the ground, cox and knee. When all joints are at zero position, the link calf is perpendicular to the ground and the link thigh and calf are parallel to the ground. The hip is connected to the body by the waist joint that rotates around a vertical axis.

![Skeleton of a Dog](image1)

**Fig. 3. Dog's Structure**

![General structure of a 3DOF (degrees of freedom) leg](image2)

**Fig. 4. General structure of a 3DOF (degrees of freedom) leg**
4. Normal statically stable gaits

A hexapod has many types of statically stable gaits, such as regular gait, irregular gaits, periodic gaits and et al.

As for the regular periodic gaits, its gaits can be classified, according to the number of supporting legs during support phase, as 3 + 3 tripod gait with 3 supporting legs, 4 + 2 quadruped gait with four supporting legs and 5 +1 one by one gait with five supporting legs; according to the movement of legs, insect-wave gait which is the typical gait of rectangular six-legged robots, mammal-kick gait which is typical gait of rectangular quadruped robots and mixed gait which is typical multi-directional gait for hexagonal hexapod robots; the combination can be tripod insect-wave gait and so on. The typical irregular gait is so called free gait.

4.1 3+3 tripod gait

The tripod continuous gaits are characterized by having three legs standing on the ground for supporting and pushing the body forward, and the other three legs lifting off and swinging forward. In each gait period, the body moves two steps. The quickest tripod gait is when the duty factor β equals 1/2.

![Fig. 5. Insect-wave tripod gait](image)

![Fig. 6. Mammal-kick tripod gait](image)
In the initial configurations of insect-wave gait (see Fig.5) and mammal gait (see Fig.6), six legs of the robot are grouped into two and distributed along two sides as that of rectangular hexapod robots. Each group has three legs parallel. In Fig.5 (a) and Fig.6 (a). The positions of all waist joints are 0, -30, 30, 0, -30 and 30 degrees from leg 1 to leg 6, other joints angles are zeros.

The insect wave gait is characterized by a forward wave of stepping actions on each side of the body with a half-cycle phase shift between the two members of any right or left pair [63]. A scheme of the robot is sketched in Fig.5 (a), where the main direction of the movement, defined as main walking direction, is downwards, with legs swinging forward. Fig.4 (b) shows an example of legs sequence. In Fig.5 (b), the thick dashed or solid lines denote supporting legs.

In the first step, leg 1, leg 3 and leg 5 are in stance phase and push the body forward; while leg 2, leg 4 and leg 6 swing ahead. In the second step, leg 2, leg 4 and leg 6 are in support phase and are responsible for pushing the body forward; leg 1, leg 3 and leg 5 then change to swing phase. After this, the procedure repeats again from the first step to the second step. The whole cycle includes two steps and the body is moved twice. In every step, the support polygon is an isosceles triangle ΔABC. The stroke length of supporting legs must make sure the gravity center of robot stays in side the support polygon, that means not surpass the stability margin.

In the mammal-kick gait legs generally move in a vertical plane like human's kicking out and trajectories of feet are along legs (Fig.6 (b)). The scheme of mammal-kick gait is depicted in Fig.6, and it walks mainly from left to right. The waist-joints do not work during mammal straight forward walking, but for turning. The support polygon is similar as with insect-wave gait and is an isosceles triangle ΔABC. During walking the front supporting legs retract and the rare supporting legs protrude so that the body is moving forward; on the contrary, the front swing legs are protrude and the rare swing legs retract. The legs' moving sequence is the same at that in insect-wave gait. The difference is just the configurations.

In addition to the periodic tripod gaits mentioned above, we introduce here new type of mixed gait. In the initial configuration (see Fig.7) of insect-mammal mixed gait, all joint-angles are zeros. During walking, the mixed gait has a supporting area defined as a convex polygon connected all supporting legs, in the form of an equilateral triangle ΔABC or ΔDEF.
in Fig.7 (a). The dark point in Fig.7 (b) is the gravity centre of the body. In every half period, one leg kicks off and two legs wave as insect-wave gait. Fig.7 (b) describes the walking sequence and 2D configuration of legs of the mixed gait. The legs’ movement sequences are same as in other two gaits. The main walking direction is along the longitudinal axis of hip of leading leg, as shown in Fig.7.

From Fig.5 to Fig.7, it is can be seen that, for a given robot, the insect wave gait has the same size of supporting area \( \Delta ABC \) as the mammal gait; on the other hand, the mixed gait has the largest supporting area. In order to make a detail analysis, Song, Waldron and Choi in [Song and Choi, 1990] and [Song and Waldron, 1989] proved that wave gait has the optimum stability among all hexapod periodic and regular gaits in the range of \( 1/2 \leq \beta < 1 \). While this is true for rectangular hexapod robots, it does not hold for hexagonal ones. The statically stability margin (SSM) and main-direction stability margin (MDSM) of three statically stable and continuous tripod gaits are compared based on one hexagonal hexapod robot whose parameters are listed in table 1. The stability results are reported in table 2 and table 3 respectively. In table 2 and table 3, the body heights, the distance from the bottom of the bodies to the ground, keep constant as length of calf \( l_3 \); each link is assumed as a line and each joint is assumed as a point.

<table>
<thead>
<tr>
<th>Each leg</th>
<th>Body</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hip</td>
<td>Thigh</td>
</tr>
<tr>
<td>mass, kg</td>
<td>( m_1 = 0.80 )</td>
</tr>
<tr>
<td>length, m</td>
<td>( l_1 = 0.09 )</td>
</tr>
</tbody>
</table>

Table 1. Main physical parameters of hexapod robot example

<table>
<thead>
<tr>
<th>definition</th>
<th>formula</th>
<th>Hexapod robot example</th>
<th>MDSM, m</th>
<th>Kinematic Limitation, m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Insect-wave gait</td>
<td>( \min (OE, OF) ) in Fig.5 (a)</td>
<td>( \frac{\sqrt{3}r_b(r_b + l_{12})}{4l_{12} + 3r_b} )</td>
<td>0.1471</td>
<td>&gt; 0.20</td>
</tr>
<tr>
<td>Mammal-kick gait</td>
<td>( \min (OG, OA) ) in Fig.6 (a)</td>
<td>( l_{12} + \frac{r_b}{2} )</td>
<td>0.5700</td>
<td>0.2196</td>
</tr>
<tr>
<td>Insect-mammal Mixed gait</td>
<td>( \min (OE, OC) ) in Fig.7 (a)</td>
<td>( \frac{1}{2}(r_b + l_{12}) )</td>
<td>0.3750</td>
<td>0.2196</td>
</tr>
</tbody>
</table>

Table 2. MDSM of different tripod gaits (\( \beta = 1/2 \))

<table>
<thead>
<tr>
<th>definition</th>
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<th>hexapod robot example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Insect-wave gait</td>
<td>( \min (OD, OG, OF) ) in Fig.5 (a)</td>
<td>( \frac{\sqrt{3}r_b(r_b + l_{12})}{2\sqrt{3}r_b^2 + 4l_{12}^2 + 6l_{12}r_b} )</td>
</tr>
<tr>
<td>Mammal-kick gait</td>
<td>( \min (OD, OE, OG) ) in Fig.6 (a)</td>
<td>( \frac{\sqrt{3}r_b(r_b + l_{12})}{2\sqrt{3}r_b^2 + 4l_{12}^2 + 6l_{12}r_b} )</td>
</tr>
<tr>
<td>Insect-mammal Mixed gait</td>
<td>( \min (OD, OE, OF) ) in Fig.7 (a)</td>
<td>( \frac{1}{2}(r_b + l_{12}) )</td>
</tr>
</tbody>
</table>

Table 3. SSM of different tripod gaits (\( \beta = 1/2 \))
As shown in table 2, the mammal-kick gait has the biggest MDSM but it loses this advantage because of kinematics limitation; the insect-wave gait has the smallest possible stride (14.71cm for the example) along main walking direction whereas the other two gaits have the same and much bigger possible stride. Synthetically, the insect-mammal mixed gait is optimally stable for hexagonal hexapod robots when β=1/2 and has stability advantage over the other two gaits while turning because of the biggest SSM.

Small angle turnings are easy for all three gaits. However, insect-wave gait needs special gaits to realize big-angle turning as stated in [Chu & Pang, 2002], [Wang, 2005] and [Zhang & Song, 1991], the same for mammal gait [Wang et al., 2007]. They have to stop and adjust legs at first for some big-angle turnings. Fig.8 shows examples of turning 60 degrees with insect or mammal gait. From the initial configuration in Fig.8 (a), the robot spends three steps to realize 60° turning. Quadrangles are supporting polygons. On the other hand, insect-mammal mixed gait can have big advantage on big-angle turning, especially at ±60°, ±120° and 180°. With insect-mammal mixed gait, the robot just needs to reselect the leading leg for turning at ±60°, ±120° and 180°, plus adjustment of crab angle it can realize any angle turning without stopping.

In the following Fig.9, Fig.10 and Fig.11, R and S denote revolute and spherical joint respectively; f, k, c and w denote foot, knee, coxa and waist, respectively. For instance, Rc specifies that the coxa is a revolute joint; Sf tells that between foot and ground, a virtual spherical joint is assumed.
In the insect wave gait, the waist joints are the most active joints during walking, and each foot needs three DOFs. The connection between each foot and the ground can be considered as a spherical joint ($S_f$ in Fig.9). The similar as insect wave gait, in mixed gait. The connection between each foot and the ground can be considered as a spherical joint ($S_f$ in Fig.11). From the simplified structure, the mammal gait is easy to control. However, all legs with insect-wave gait have the same trajectories. It is therefore easiest to control. Just in insect-mammal gait, legs have different trajectory, but symmetric legs still have same trajectories.
3.2 4+2 quadruped gait
The rectangular hexapod robot has another type of gait, the "4+2" gait [Chu & Pang, 2002]. For this gait the legs are grouped into three groups. Every time there are four legs (two groups) standing on the ground to support the body, two other legs rise and walk ahead. In one gait period, there are three steps and the body moves only one step. The duty factor is 2/3. The hexagonal six-legged robot also has this gait with same leg sequences as that of a rectangular hexapod. One example can be:

1) Lifts leg 1 and leg 4, other legs support and push the body;
2) Leg 2 and leg 5 swing forward, all others support and push the body;
3) Leg 3 and leg 6 swing forward, the body is moved by others another step.
4) repeat procedure from 1) to 3).

This gait shows fault tolerant ability under certain conditions [Yang & Kim, 1998; Yang & Kim, 1999; Huang & Nonami, 2003; Chu & Pang, 2002], because three legs can support the body even if one supporting leg broken during walking. Chu and Pang had proved that the hexagonal robot by this gait has advantages compared with rectangular ones in stability, stride and turning ability, if the turning angle is within [-30 30] degrees.

3.3 5+1 one by one gait
The rectangular hexapod robot has another type of gait, the "4+2" gait [Chu & Pang, 2002]. For this gait the legs are grouped into three groups. Every time there are four legs (two groups) standing on the ground to support the body, two other legs rise and walk ahead. In one gait period, there are three steps and the body moves only one step. The duty factor is 2/3. The hexagonal six-legged robot also has three types’ gaits with same leg sequences as that of a rectangular hexapod.

The leg-sequence of one by one gait can be any order, but generally legs move one after another following a clockwise or anti-clockwise order.

3.4 Free gait
Free gait proposed by Kugushev and Jaroshevskij in 1975 is characterized as non-periodic, non-regular, non-symmetric and terrain adaptive. In a free gait, the leg sequence (i.e., the order in which leg transferences are executed), footholds, and body motions are planned in a nonfixed, but flexible way as a function of the trajectory, the ground features, and the machine's state. It is more flexible and adaptive than periodic and regular gaits on complicated terrain. A large number of free gaits for quadruped and hexapod robots have been developed to date. For more information, we can refer to [Pal & Jayaraian, 1990; Porta & Celaya, 2004; Estremera & Gonzalez de Santos, 2003 and 2005].

4. Fault tolerant gait
In arduous operating environments, robots may confront accidents and damage their legs; their legs may be dual-used as arms for some tasks, or some joints may suffer loss of control etc. In such cases, biped or quadruped robots would become statically unstable. However hexapods may still walk with static stable because their six legs provide redundancy. In this subsection we discuss these fault tolerant gaits.
4.1 Joint-lock
In this case, Yang [Yang, 2003] has already proposed a discontinuous tripod gait for rectangular hexapod robots.
However, with joint-lock a hexagonal hexapod may still maintain a continuous gait. The three possibilities for a single locked joint on one leg are discussed in the following.

1) Waist-joint-lock. In this case, the faulty leg cannot move in a horizontal plane, but it can swing in a vertical plane. The insect wave gait is difficult for this situation; whereas the mammal gait is still available by adjusting the other legs in parallel with the faulty leg. Also the mixed gait is possible if we chose the broken leg as the leading leg or the leg opposite as healing leg.

2) Knee or coax-joint-lock. For these two cases, the mammal gait and mixed gait are impossible to realize, but the insect gait is feasible, although not as efficient as before injury. If one whole leg is locked, the discontinuous tripod gait can be employed.

4.2 Loss of one leg
In the case of loss of one leg is due to fault or use for other tasks; two possibilities were considered in [Yang & Kim, 1998]. However, for symmetric hexagonal robot, there is only one case because the structure of every leg is the same and distributed evenly around the body. The 2+1+2 gait has same sequence as [Yang & Kim, 1998]. The difference is in the positions the leg. The legs of the gaits in [Yang & Kim, 1998] are overlapping. The symmetrical hexapod robot needs three steps to achieve this walk. During this procedure, the robot’s body moves two steps.

4.3 Loss of two legs
There are three cases where two legs are either faulty or being used for other tasks. The positions of these two unavailable legs may be opposite, adjacent or separated-by-one (two damaged separated by one normal leg). Some studies [Takahashi et al., 2000] have been done in the first case, but there is a lack of study on the other two cases.

1) The opposite-legs case. Losing two opposite legs, for example, leg $i$ and leg $j$ the hexapod robot becomes a quadruped robot. It can walk with one of quadruped gaits, which have been widely studied. For example, the craw gait (Chen et al. 2006), the diagonal gait (Hirose & Matins, 1989), mammal-type “3+1”gait (Tsujita et al. 2001), “3+1” craw gait (Chen et al., 2006) which maintains static stability at each step, and the omni-directional updated quadruped free gait in [Estremara & Gonzalez de Santos, 2002; Estremara & Gonzalez de Santos, 2005].

2) The two-separated-by-one case and adjacent case. For these two cases the two unavailable legs are on the same side therefore it is almost impossible for a general rectangular hexapod robot to have statically stable locomotion. For a hexagonal robot the insect wave periodic gait is still available. The other four legs can be adjusted to suitable initial positions, as shown in Fig.12 for example. Fig.12 (a) is the case of losing leg 1 and leg 3. Fig.12 (b) shows the case where leg 1 and leg 2 are unavailable. Following the four-leg periodic gait sequence, robots can realize statically stable walking. The crab angle will be different. For example, if leg 1 and leg 2 or leg 1 and leg 3 are unusable, the crab angle will be -π/6. Fig. 13 lists the leg sequences for a separated-by-one fault tolerant gait. At each instant, there are three or four legs supporting the body. The mass centre is inside the supporting area.
For the adjacent case, the leg sequence is similar to the separated-by-one case after adjusting to suitable initial positions.

![Diagram](image)

(a) Separated-by-one case (leg 1 and leg 3 are lost)  (b) Adjacent case (leg 1 and leg 2 are lost)

Fig. 12 Initial state of four legs

To realize statically stable walking, there are several requirements in Fig.13:

1) \( \overrightarrow{AB} = \overrightarrow{BF} = \overrightarrow{CG} = \overrightarrow{DH} = \overrightarrow{L} \);
2) \( L \geq R \cos(\pi/3) \);
3) \( \overrightarrow{GH} = \overrightarrow{HH} = \overrightarrow{b} \), the body stride;
4) \( \theta_4 \); 
5) \( R = \tan\left(\frac{\pi}{3}\right) \)

The rules for the quadruped insect wave gait are:

1) Rear legs (leg 4 and leg 5 in Fig.13) must not cross the central line (the point-dashed line in Fig.6-16) while moving ahead, so that the mass centre will also be in the subsequent supporting area.
2) Front legs (leg 1 and leg 2 in Fig.13) should not go back to the central line while the body (centre of mass) is moving ahead.
3) The stride of the swing legs is twice that of the body.
(a) Swing rear right leg, leg 5
Supporting area is $\triangle EFH$

(b) Swing front right leg, leg 6
Supporting area is $\triangle EFG''$

c) Move body
Supporting area is $\odot EFG''H''$

d) Swing rear left leg, leg 4
Supporting area is $\triangle FG'H''$

e) Swing front left leg, leg 2
Supporting area is $\triangle E'' G''H''$

(f) Move body again
Supporting area is $\odot E'' F''G''H''$

Fig. 13. Leg sequences of separated-by-one case fault tolerant gait while two legs are broken
From the initial configuration of mixed gait to the fault tolerant initial state (Fig.12), we can adjust the legs according to following procedures (Equation 1 to Equation 8). If two faults occur on two legs that separated by one, for example leg 1 and leg 3, the following procedure can be used to move the other legs from the original initial-positions to the fault tolerant initial-positions:

Leg 2 moves from \( P_2 \) to \( F \) with stride \( P_2F \) (Equation (1)) and rotates by angle \( \theta_2 \) (Equation (2)); Leg 4 moves from \( P_4 \) to \( E \) with stride \( P_4E \) (Equation (3)) and rotates by angle \( \theta_4 \) (Equation (4)); Leg 5 moves from \( P_5 \) to \( G \) with stride \( P_5G \) (Equation (5)) and rotates by angle \( \theta_5 \) (Equation (6)); Leg 6 moves from \( P_6 \) to \( F \) with stride \( P_6H \) (Equation (7)) and rotates by angle \( \theta_6 \) (Equation (8)).

\[
P_2F = \sqrt{(L \sin(\frac{\pi}{3}))^2 + (L - (l_1 + l_2) \cos(\frac{\pi}{3}))^2} \tag{1}
\]

\[
\theta_2 = \tan^{-1}\left(\frac{L - (l_1 + l_2) \cos(\frac{\pi}{3})}{L \sin(\frac{\pi}{3})}\right) - \frac{\pi}{6} \tag{2}
\]

\[
P_4E = \sqrt{((l_1 + l_2) - L \cos(\frac{\pi}{3}))^2 + (L \sin(\frac{\pi}{3}))^2} \tag{3}
\]

\[
\theta_4 = \tan^{-1}\left(\frac{L \sin(\frac{\pi}{3})}{(l_1 + l_2) - L \cos(\frac{\pi}{3})}\right) \tag{4}
\]

\[
P_5G = \sqrt{(L - (l_1 + l_2) \cos(\frac{\pi}{3}))^2 + ((l_1 + l_2) \sin(\frac{\pi}{3}) - s)^2} \tag{5}
\]

\[
\theta_5 = \frac{\pi}{6} - \tan^{-1}\left(\frac{L - (l_1 + l_2) \cos(\frac{\pi}{3})}{(l_1 + l_2) \sin(\frac{\pi}{3}) - s}\right) \tag{6}
\]

\[
\theta_6 = \frac{2\pi}{3} - \tan^{-1}\left(\frac{R \sin(\frac{\pi}{3}) - s}{l_1 + l_2 - L + R \cos(\frac{\pi}{3})}\right) \tag{8}
\]

\[
P_6H = \sqrt{(l_1 + l_2 - L + R \cos(\frac{\pi}{3}))^2 + (R \sin(\frac{\pi}{3}) - s)^2} \tag{7}
\]
For the adjacent-legs case, the only difference is for the leg between the two faulty legs, leg 3 for example. The foot tip of leg 3 will move from $P_3$ to $F$ with the following stride and rotation angle,

$$
P_3F = \sqrt{(l_1 + l_2 - L + R\cos(\frac{\pi}{3}))^2 + (R\sin(\frac{\pi}{3}))^2}
$$

$$
\theta_3 = -\frac{\pi}{3} + \tan\left(\frac{R\sin(\frac{\pi}{3})}{l_1 + l_2 - L + R\cos(\frac{\pi}{3})}\right)
$$

In the above equations, $R\sin(\frac{\pi}{3}) \leq L < l_1 + l_2$.

D. Loss of more than two legs

If more than two legs are lost, the robot is unable to maintain static stability while walking. Dynamic gaits may still be possible, such as the three-leg dynamics gait of Lee and Hirose, 2000. These will not be discussed further here.

5. Conclusion

In this chapter, the locomotion of symmetric hexapods has been studied in detail. We have presented a comprehensive study of hexagonal hexapod gaits including normal and fault tolerant ones. Gaits of rectangular and hexagonal six-legged robots have been compared from several aspects: stability, fault tolerance, terrain adaptability and walking ability. To facilitate simulations and experiments we have provided integrated kinematics of swinging and supporting legs for continuous gaits.

Hexagonal hexapod robots have been shown to be more flexible than rectangular ones. Moreover, hexagonal hexapods have many feasible gaits. In addition to the well-know insect gait and mammal gait, a new mixed gait for hexagonal six-legged robots has been proposed in this chapter which entails some features of both insect and mammal gaits. Except classified by legs movement as mentioned above, hexapod robots gaits are categorized according to the number of supporting legs during walking, as 3+3 tripod, 4+2 fault tolerant quadruped, and 5+1 one by one gaits. On account to the introduction of mixed gait, each numbered gait has one more form. Among three tripod-gait forms, the most stable is the mixed one. The mammal gait can reach the longest stride; whereas the continuous insect gait has the shortest maximum stride and poorest stability. Thanks to their six legs, hexapod robots have redundancy and fault tolerance. Gaits where one leg is lost or two opposite legs are lost have been discussed in recent times. In this chapter we have tackled also the cases in which two adjacent legs or two separated by a normal leg are damaged. Algorithms for realizing these two fault-tolerant gaits have been detailed and validated with simulations.
6. References


Climbing and Walking Robots


Nowadays robotics is one of the most dynamic fields of scientific researches. The shift of robotics researches from manufacturing to services applications is clear. During the last decades interest in studying climbing and walking robots has been increased. This increasing interest has been in many areas that most important ones of them are: mechanics, electronics, medical engineering, cybernetics, controls, and computers. Today’s climbing and walking robots are a combination of manipulative, perceptive, communicative, and cognitive abilities and they are capable of performing many tasks in industrial and non-industrial environments. Surveillance, planetary exploration, emergence rescue operations, reconnaissance, petrochemical applications, construction, entertainment, personal services, intervention in severe environments, transportation, medical and etc are some applications from a very diverse application fields of climbing and walking robots. By great progress in this area of robotics it is anticipated that next generation climbing and walking robots will enhance lives and will change the way the human works, thinks and makes decisions. This book presents the state of the art achievements, recent developments, applications and future challenges of climbing and walking robots. These are presented in 24 chapters by authors throughout the world. The book serves as a reference especially for the researchers who are interested in mobile robots. It also is useful for industrial engineers and graduate students in advanced study.

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